A

COMPENDIOUS SYSTEM

OF

Natural Philoſophy:

With NOTES,

CONTAINING THE

MATHEMATICAL DEMONSTRATIONS,

AND

Some Occaſional REMARKS.

In FOUR PARTS.

VOL. I.

By J. ROWNING, M.A.
Late Rector of ANDERBY, in LINCOLNSHIRE, and
Fellow of MAGDALEN COLLEGE, in CAMBRIDGE.

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BY THE

EDITOR OF THIS EDITION.

IN this Edition the Appendix to the firſt Part is incorporated with it, in its proper Place; many Miſtakes, partly original ones, committed by the Author, and partly ſuch as have crept into the Work, in the different Editions which it has undergone, ſince it paſſed through the Hands of the Author, have been corrected; and the Pages, which before were broken off, and began again with each Part, are continued without Interruption to the End of the Volume. The Editor has moreover, in a few Inſtances, taken the Liberty of ſupplying Geometrical Demonſtrations where the Author had given Algebraic ones; thinking them more agreeable to the general Tenor of the Work,

and that they carry with them a more direct and explicit Demonſtration, at leaſt to ſuch Perſons as this Work is chiefly intended for.

The Editor has not preſumed to correct his Author in any Inſtance which is not ſtrictly demonſtrative, and therefore although ſome Points in the Philoſophical Part are now accounted for in a different Manner from what he has done, the Editor judged it better to let them remain as they were, than to adapt them to the Faſhion of the Day; which, as it already hath, ſo it undoubtedly will hereafter change, with Reſpect to many of thoſe Points which admit not of Mathematical Demonſtration: ſuch are, the Compreſſibility or Incompreſſibility of Water, the Cauſe of the *Aurora Borealis*, of the Opacity and Tranſparency of Bodies, the Appearance and Curvature of the Tails of Comets, and ſome others, which are all merely hypothetical, or depend

on

on Experiments which are of too precarious a Nature to be confided in. The modern Way of accounting for the Phænomena of Thunder and Lightning ſeems worthy of Exception, as being founded on more deciſive Experiments; the Editor, nevertheleſs, did not think himſelf at Liberty to cancel the Chapter on this Subject; but rather to refer the Curious on this Head to Dr. *Franklin*'s *Eſſays*, or to Dr. *Prieſtley*'s *Hiſtory of Electricity*.

Two or three Miſtakes, which eſcaped the Editor's Notice until after the Work was printed off, may be mentioned here. Page 62, of Vol. I. the Author ſpeaks of a Body being projected with ſuch a Velocity, as would make it continually recede from the Earth in a *Curve*, until it gets out of the Sphere of the Earth's Attraction; after which, ſays he, it will go on in a *ſtraight Line*, without ever returning. If a Projectile does not aſcend in a ſtrait Line, or revolve in a Circle, or

an Ellipſis, it will deſcribe a Parabola, or an Hyperbola, and having once deſcribed a Part of a Curve, it will continue to deſcribe that Curve, and can never afterwards proceed in a Right Line.

The third Law of Nature, or Motion, has no Concern with what the Author is ſpeaking of, p. 66; neither are the centripetal and centrifugal Forces always equal, as he aſſerts, unleſs the Projectile deſcribes a Circle, or approaches to, or recedes from the Center with an uniform Velocity. Laſtly, his Reaſoning, reſpecting the Wedge, in p. 92, *&c.* is not true; for when the Wedge is driven forwards, not only the upper Ends of the Wood, but all the Parts thereof from C to G, and from C to H, will move with equal Velocities, in Directions at Right Angles to the Sides of the Wedge.

THE

THE PREFACE.

IT being an uſual Complaint with thoſe who are unacquainted with Geometry, that they are diſcouraged by the Mathematical Demonſtrations, from peruſing Books of Natural Philoſophy ; *I apprehended that ſome Papers I had drawn up for the Uſe of my Pupils in the Univerſity, would not be altogether unacceptable, if publiſhed in ſuch Form, that the Propoſitions, or Subſtance of the Book, might be read without Interruption from the Demonſtrations. I have therefore, in the following Treatiſe, laid down the ſaid Propoſitions, and endeavoured to prove them in a familiar and eaſy Manner, without Geometry, by Way of* Text : *And for the Sake of thoſe*

who are ſkilled in that Science, have added the Demonſtrations, with ſome occaſional Remarks, by Way of Notes. *And whereas the Writers on this Subject have appropriated to themſelves a Stile too* technical *for Beginners, I have, in* Hopes *of being more eaſily underſtood, ſometimes choſe a plainer and more popular Way of Expreſſion, though perhaps not always ſo accurate.*

In the Introduction to the firſt Part, Notice is taken of the Method of Philoſophiſing *made Uſe of by* Des Cartes, *and others before him, ſo far as the Deſign of this* Compendium *required. I ſhall add here a few Conſiderations relating to the Method which prevails at this Time.*

In the preſent Method of Philoſophiſing, all Matter is conſidered (with reſpect to its Subſtance) as homogeneous, *or of the ſame Kind; and no other Cauſe or Principle of Action in Matter*

is

is allowed of, but what is well eſtabliſhed by Facts.

Some Philoſophers admit Elementary Fire, *as they call it, among their Principles; or, which comes to the ſame Thing, they conſider Fire as endowed with active Powers diſtinct from thoſe of other Matter.* Keill, *in his Letter to Dr.* Cockburn, De Legibus Attractionis, aliiſque Phyſices Principiis, *makes Uſe of three Principles,* viz. 1. Empty Space. 2. The infinite Diviſibility of Quantity. 3. The Attraction of Matter. *And affirms, that all* Phyſics *depend thereon.*

The firſt of his Principles the Reader may perhaps think ridiculous; but he may conſider, that at that Time the Notion of a Plenum *was not wholly exploded: The laying down* Empty Space *as a firſt Principle was only calling out for* Elbow-Room *and a* clear Stage. ——*But not to trouble the Reader with what others have done, I have choſen*

choſen, and every where ſtuck to three; *and as oft as a* Phænomenon *occurred, which I could not account for by them, I have given it up as a Difficulty; not deſpairing, but that when all the Circumſtances of the Phænomenon ſhall be thoroughly known, thoſe Principles alone may be found ſufficient. It ſeems not conſiſtent with the Regard a Philoſopher ſhould have to the Uniformity of Nature, every where obſervable, to call in a new Principle at every knotty Point. Thoſe which I make uſe of are,*

Firſt, Attraction of Gravitation. *That is, a Diſpoſition in Bodies to move towards each other, even when at great Diſtances aſunder.*

Secondly, Attraction of Coheſion. *That is, a like Diſpoſition in Bodies to move towards each other, but diſtinct from the former, in as much as it is obſerved to take Place only when the Bodies are very* near *together.*

Thirdly, Repulſion, *or a Diſpoſition in Bodies, whereby in ſome Caſes they endeavour*

endeavour to avoid, or fly from each other.

The firſt of theſe is Matter of daily Obſervation. Thus a Ball let go from the Hand falls to the Ground.

The ſecond may be ſeen in the following Inſtance. A ſmall Portion of a Fluid forms itſelf into a Sphere or Drop: Which can only happen from a Diſpoſition in the Particles of which it conſiſts, to come as near as poſſible to each other.

An Inſtance of the third is this. If Air incloſed in a Bladder, be ſqueezed into a leſs Compaſs, the Air within, when the Preſſure is taken off, reſtores the Bladder to its former Size: A plain Indication that the Particles of which the Air conſiſts, endeavour to avoid or fly from each other (a).

Theſe Diſpoſitions in Bodies are not the Reſult of any Mechanical Cauſe

(a) *See another Inſtance of this Diſpoſition, in Vol.* II. *Page* 67, *Note* (m) *of this* Compendium.

whatever;

whatever; that is, ſuch as may ariſe from the Effluvia *of Bodies, or the Action of any other* material Subſtance (b): *They are therefore the Act of an* immaterial Cauſe, *in Virtue of which* inactive Matter *performs the Offices for which it was deſigned.*

From

(b) Demonſtration. *In the firſt Place it is well known, that if* Gravity *acts upon Bodies with the ſame Degree of Intenſeneſs, whether they be in Motion or at Reſt, it may be demonſtrated, that Bodies, when projected, will deſcribe* Parabolas; *and that when vibrating in* Cycloids, *their Vibrations will be* iſocronous, *&c. In the next Place it is as well known, that Bodies, when projected, do deſcribe* Parabolas, *and that when vibrating in* Cycloids, *their Vibrations are* iſocronous, *&c. From which two Propoſitions it demonſtratively follows, that if Gravity be the Cauſe of the above-mentioned Effects, it muſt act upon Bodies with the ſame Force, whether they be in Motion or at Reſt.*

Again, it is well known, that, if Attraction of Coheſion *acts upon Rays of Light with the ſame Degree of Intenſeneſs, whatever be the Velocity they move with, it may be demonſtrated, that the Ratio of the Sine of the Angle of Incidence to the Sine of the Angle of Refraction, will be given. But in Refraction of Light, the Ratio of thoſe Sines is given in Fact; if therefore* Attraction of Coheſion *be the Cauſe of the Refraction of Light, it muſt act upon Rays of Light with the ſame Intenſeneſs, whatever Velocity they move with.*

But no Effluvia *of Bodies, no* material Subſtance, *and in ſhort no* material Cauſe *whatever, can act with the ſame Intenſeneſs, or have the ſame Effect upon a Body in Motion, as upon the ſame Body at Reſt; becauſe as it is very*

From the firſt and third Principle (c), *that is, from the Attraction of Gravitation and Coheſion, together with the Properties of Matter enumerated in the firſt Chapter of this* Compendium, *which Properties muſt be always ſuppoſed, the Elaſticity or Spring of the Air, and from thence the Nature and Propagation of Sound, are accounted for. And from the Spring of the Air conſidered as being augmented by Heat, and diminiſhed by Cold, as it is obſerved to be, and the*

very well known to Mathematicians, to whom I addreſs this Note, Body *can only act upon* Body, *according to the Sum or Difference of their Motions. It remains therefore, that the two Diſpoſitions herein mentioned, are not the Reſult of any* material Cauſe *whatever: Which is one Part of the Propoſition to be demonſtrated.*

As to the other Diſpoſition in Bodies, their Repulſion, *ſince Rays of Light are alſo affected by it, as it appears they are by an Experiment of Sir* Iſaac Newton's *referred to in the foregoing Note, it may very reaſonably be ſuppoſed, though we don't at preſent know the exact Law of its Action, to affect Bodies in Motion after the ſame Manner that it would do the ſame at Reſt, and that it therefore is alſo the Reſult of no* material Cauſe *whatever.*

(c) *The Law or Manner wherein theſe Principles are obſerved to act in different Circumſtances, are determined from Facts, in Part* I. *Chap.* 3. *The Law of the third, ſo far as it relates to the Air, will be found in Part* II. *Chap.* 3. *of this* Compendium.

Air's

Air's being at the ſame Time affected by the firſt Principle, viz. *that of Gravitation, the* Phænomena *of the Winds are explained. By the ſecond Principle, the Coheſion of Matter, the various Degrees of Hardneſs obſervable in it, the Diſſolution of Bodies by Fluids, with other* chemical *Operations; and in particular the* Phænomena *of Fermentation, and conſequently the Cauſes of Thunder and Lightning*, &c. *By this Principle alſo the riſing of Fluids in ſmall Tubes, and from thence the Aſcent of Sap in Vegetables are accounted for; all which Particulars, except the two firſt, are treated of in the ſecond Part of this* Compendium: *As alſo the Refraction of Light, and conſequently all that Train of* Phænomena *depending thereon, which is the Subject of the third Part. By the firſt Principle, the ſeveral Circumſtances relating to falling Bodies, and to the Motion of Projectiles, together with the Doctrine of* Pendulums, *(treated of in the firſt) and likewiſe all thoſe which relate to*

the

the Preſſure of Fluids (treated of in the ſecond Part) are determined. And above all is deduced that moſt curious Doctrine of Central Forces, *juſt touched upon in the firſt, but largely and fully explained in the fourth Part, by which (aſſuming that the heavenly Bodies were at firſt put into Motion by their Creator) we are enabled to aſſign the Cauſe of the Continuation thereof, with all its Modifications and Irregularities; to determine the neceſſary Shape of thoſe Bodies; and to account for the ebbing and flowing of the Sea,* &c. *Difficulties, too great ſeemingly for human Reaſon to ſurmount!*

Some of the Phænomena *which I have not been able to give a ſatisfactory Account of, from the above-mentioned Principles, are the Reflection of Light, its Emiſſion from luminous Bodies, and the Formation and Aſcent of Vapour. This may be only owing to the Want of better Acquaintance with the Circumſtances*

ſtances of thoſe Phænomena; *that is, more ſufficient* Data, *or Facts to found their Solution upon: So that we are not to conclude immediately, that the Principles are inſufficient; but rather to wait with Patience: The Diligence of others may render that eaſy, which our utmoſt Efforts at preſent are not able to ſurmount.*

However, as a Reader unacquainted with Studies of this Kind, may wonder that ſo many of the Phænomena *or Appearances of Nature, ſhould be accounted for by ſo few Principles; and becauſe it may be a Means of giving him ſome Inſight into the Subject of theſe* Sheets, *I will here lay down the following Propoſitions, which are immediately deducible from the Principles, and alſo nearly connected with the* Phænomena *to be accounted for by them; by Means of which, he will more readily perceive the Connection or Relation between the one and the other.*

PRO-

PROPOSITION I.

Matter is an *unactive* Subſtance; that is, it is quite indifferent either to Motion or Reſt; inſomuch that the leaſt Force imaginable acting upon it will move it, if at Reſt; and alter its Courſe when moving: And further no Body has a Power either of putting itſelf into Motion, or of ſtopping or altering its Courſe when it does move. Notwithſtanding which, every Body acts upon all others (provided their Diſtances be not too great) perpetually endeavouring to put them into Motion.

Hence we have the true Idea of the Gravity or Weight of Bodies belonging to the Earth, and their Diſpoſition to deſcend towards it on all Occaſions. Thoſe Bodies are equally indifferent to Motion or Reſt; but by Virtue of the firſt Principle, that is, by their Attraction of Gravitation, (as above defined)

they are drawn towards the Sun, the Moon, the reſt of the Planets, and the Earth; but towards this laſt more ſtrongly than towards any of the reſt; and ſo they tend, gravitate, *or are* heavy *towards that. The Reaſon that they are attracted more forcibly towards the Earth than towards thoſe other Bodies, is, that although it be one of the Laws of this Kind of Attraction, that it operates according to the Quantity of Matter in Bodies, and therefore the Attraction of the Sun ſhould be the moſt prevalent, in as much as that Body contains the moſt Matter; yet it is another Law of that Principle, that it acts more ſtrongly according to the Nearneſs of Bodies to each other: This latter Conſideration in the preſent Caſe, overbalances the former; and ſo the Bodies about us tend towards the Earth.*

PROPOSITION II.

If a large round Body be covered every where with ſmaller ones to an equal

equal Height or Diſtance from its Surface; and if thoſe ſmaller ones tend towards the large Body, by Virtue of the firſt Principle, and are, at the ſame Time, diſpoſed to fly from each other by Virtue of the third, that is, by a repelling Power inherent in them; and ſuppoſing farther, that when they touch or are very near each other, their Diſpoſition to avoid each other exceeds their Tendency to the large Body, and when they are at a certain greater Diſtance from each other, that Diſpoſition is leſs than their Tendency to the large Body: Then will thoſe ſmaller Bodies keep at certain Diſtances from each other, and conſtitute an *elaſtic*, *compreſſible* Subſtance ſurrounding that large Body, and *gravitating* towards it on all Sides.

Hence an Idea of the Nature and Condition of the Atmoſphere *ſurrounding the Earth, with all its Properties.*

N. B. *When I ſay a Body* tends *to another, I do not mean that it* moves *towards it, but only that it would move*

towards it, if nothing prevented. Thus, a Bird while mounting aloft into the Air, tends *towards the Earth, as much as one that is falling down; for the one would fall as well as the other, if nothing prevented.*

Proposition III.

If, while the above-mentioned ſmaller Bodies are in the State ſuppoſed in the foregoing Propoſition, any one, or more of them be made to move (ſuppoſe for Inſtance half Way) towards the next, it will by Virtue of the third Diſpoſition, that is, its *repulſive* Force, drive or impel thoſe it comes nearer to, cloſer together; which Bodies, when that other moves back again (as it will immediately do, being repelled by them) will recede from each other again: That is, a Kind of tremulous Motion will be communicated to them by that other, and for the like Reaſon, by them to the next; and ſo on through the whole, or at leaſt to a great Diſtance from where it began.

Hence

Hence we may form an Idea how Sound *is excited by the Tremors of a Body during its Vibration, and propagated through the Air.*

PROPOSITION IV.

One of the Laws by which the ſecond Principle, or the Attraction of Coheſion, is obſerved to act, is, that Bodies act upon one another, not in Proportion to the Quantity of Matter they contain, as by the firſt Principle; but only according to the Largeneſs of their Surfaces, and the Nearneſs of the Surface of one Body to that of another.

From hence we may underſtand, that ſuch Particles of Bodies as are flat or ſquare, and ſo ſituated among each other as to touch or be very near one another in many Points, will conſtitute what we call an hard Body, *and thoſe Particles*

which are more round, or ſo ſituated that leſs Portions of their Surfaces are near together, will attract one another with a leſs Force, and ſo form a ſofter Body; *thoſe which are round, or nearly ſo, will attract one another ſtill leſs, and alſo ſlide more eaſily over one another, and ſo form what we call a* fluid Body.

PROPOSITION V.

When two Bodies meet together, if the Particles which conſtitute the one, be diſpoſed, by their Attraction of Coheſion, to move towards thoſe of the other with a greater Degree of Force, than the Particles of either Body are diſpoſed to move towards themſelves; thoſe of the firſt will leave it, and run in among thoſe of the ſecond: And for the ſame Reaſon, thoſe of the ſecond will ſever from that, leave it, and enter in between thoſe of the firſt. And if the

the Motion with which this is done, be very violent, and the Bodies be of the inflammable Kind, their Particles by thus rubbing and clashing one against another will be sufficiently heated to take Fire, and will burst out into Flame.

Hence Dissolutions, Fermentations, Explosions, Eruptions of Volcano's, Thunder, Lightning, Aurora Borealis.

With all other Phænomena *of that Tribe.*

Proposition VI.

If a Pipe, open at both Ends, and of a very small Bore, have one End dipped into Water, the Water will run up into the Pipe above the Surface of the Water on the Outside (being drawn up by the Tendency it has by the Principle of Attraction of Cohesion, towards that Part of the inner Surface of the Pipe, which is just above it, as it rises)

till that inner Surface which is ſtill juſt above it, be loaded with as great a Weight of Water as that Tendency can ſupport.

Hence a right Notion of the Aſcent of Sap *in Vegetables; the* Suction of Fluids by Spunges: *With all other* Phœnomena *reducible to that Head.*

Proposition VII.

If a Body moving right forwards, but obliquely with reſpect to the Surface of another Body, at length comes ſo near that Body as to be diſpoſed by the Principle of Attraction of Coheſion to tend towards it; inſtead of continuing to go right on, it will turn out of its Way towards that Body before it comes at it; and conſequently will ſtrike or enter it in a nearer Place, and in a leſs oblique Direction, than it would have done, in caſe it had gone right on. If it enters the Body, it ſtill keeps turning

ing out of its Courſe the ſame Way as before, till it has got ſo far within it, that there ſhall be as many Particles of the Body behind it to attract it backwards, as there are before it near enough to attract it forwards: After which it goes right on in its laſt acquired Direction, till it comes near the other Side; for while it is ſurrounded with as many Particles to attract it one Way as another, it is the ſame Thing as if it were not attracted at all. When it has got ſo near the other Side, that there are fewer Particles before it to attract it forwards, than there are behind it, near enough to attract it backwards, it then begins to turn out of its Courſe towards the Inſide of the Body; that is, from that Side of the Body towards which it is going: and continues to bend its Courſe the ſame Way, till it has got ſo far out of the Body, that there are no Particles of the Body behind it, near enough to it to attract it any more.

After

After which it purſues an undiſturbed Courſe in the Direction it acquired laſt of all.

Hence we have a juſt Idea of the Refraction of Light *with all the* Phænomena *ariſing therefrom; which are no other than ſo many Caſes of this Propoſition.*

Proposition VIII.

If ſeveral Bodies be moving right forwards, and at length be attracted by another Body, as ſuppoſed in the foregoing Propoſition, but ſome with greater Degrees of Force or Intenſeneſs than others; thoſe which are attracted with the greateſt Force, will turn the fartheſt out of their Way towards that Body; and conſequently if all of them, before this happened, were moving in one Direction, they will be made to part from each other, and move different Ways.

Hence

Hence an Idea of the different Refrangibility of the Rays of Light.

PROPOSITION IX.

If a Body be projected or thrown from another Body towards which it has a Tendency, in conſequence of the firſt Principle, its Motion will be retarded continually; that is, it will move ſlower and ſlower: If it be projected towards that Body, its Motion will be continually increaſed; and unleſs it be made to move directly to or from it, its Courſe will always keep bending towards it, ſo that it ſhall deſcribe a Curve, concave, or hollow, on the Side next the Body.

Hence all the Phænomena *of* falling Bodies, *and of* Projectiles.

PROPOSITION X.

If a Body, that by the firſt Principle tends towards another Body, is ſuffered to

to move towards it on the Surface of an inclined Plane, its Motion (as in the foregoing Propoſition) will be continually increaſed; and if it is made to move from it on the ſame Plane, its Motion will be retarded continually, but leſs in Proportion to the Obliquity of the Plane: (that is, leſs in Proportion as the Plane deviates from the Perpendicular) the Interpoſition of the Plane preventing in ſome Meaſure the Effect which its Tendency to the other Body would otherwiſe produce. And the Velocity it acquires by rolling down one Plane, will, by Virtue of its *Inactivity*, or that Diſpoſition Bodies have to continue their State of Motion or Reſt, enable it to roll up another fitly diſpoſed.

Hence the Solution of the Phænomena *of* Bodies deſcending on inclined Planes, *and the* Vibration of Pendulums.

PRO-

PROPOSITION XI.

If a Body, attracted towards another, be cauſed to move to or from it, in any other Direction than ſuch as paſſes through the Center of that other Body, the moving Body, when left to itſelf, will begin to bend its Courſe towards that other Body; and if the Direction it moves in, and the Velocity it moves with, be properly adjuſted to each other, it will move quite round that Body without touching it or coming to it; and if that Adjuſtment be ſuch, that the Body ſhall return to the Place it ſet out from with the ſame Velocity and Direction it had when it was there before, it will revolve round that other Body over and over again in the ſame Path.

From

From hence we have the Solution of the Motion of the primary Planets round the Sun, *and of the* ſecondary ones round the Primary.

Proposition XII.

If a Body be revolving about another, as in the laſt Propoſition, and a third Body approaches them, towards which they both ſhall alſo tend, the Motion of the revolving Body will be diſturbed: That is, its Path will be altered, and Irregularities in its Courſe will enſue; becauſe its Tendency to that third Body in ſome Parts of its Courſe will conſpire with, and in others perhaps be oppoſite to its own Motion. And not only ſo, but the Tendency it has to the Body about which it revolves, will in ſome Situations be increaſed, and in others be diminiſhed by the Action of the third; which will alſo conduce towards altering its Courſe.

Hence

Hence the Lunar Irregularities, *and all other* Diſturbances in the Motion of the Heavenly Bodies, *on their too near Approach towards each other.*

PROPOSITION XIII.

Imagine a large Body covered all over with ſmaller ones tending to its Center: Suppoſe alſo a diſtant Body, towards which they all tend, but the little ones with leſs Degrees of Force than they do towards the Body they cover. Then will ſuch of thoſe ſmaller Bodies, as are neareſt to the diſtant one, loſe Part of their Tendency to the Body they cover; and ſo will thoſe ſmaller ones which are fartheſt off, or placed on the oppoſite Side the large Body. But, as to thoſe ſmaller Bodies, which are at the ſame Diſtance from the diſtant Body with the Center of the large Body itſelf, their

Tendency

Tendency to the Body they lie upon, will be increaſed. The reſt will have their Tendency increaſed or diminiſhed more or leſs, according to their Nearneſs to thoſe whoſe Tendency is increaſed or diminiſhed the moſt (*d*).

Hence ariſes the Difference in the Weights which Bodies *have upon the Earth's Surface, at the Approach and Departure of the Heavenly Bodies, (but chiefly of the Moon,) to or from that Side of the Earth where the Bodies are; and conſequently the* ebbing and flowing of the Sea, *the Water riſing where its Weight or Tendency to the Earth is diminiſhed, and ſinking at the ſame Time in thoſe Places where its Weight is augmented. That the Approach and Departure of the Moon ſhould cauſe a greater Difference in the Weight of Bodies on the Earth, than the Approach*

(*d*) What is affirmed in this and the foregoing Propoſition, depends on a Train of Reaſoning too long to be inſerted here. To underſtand it thoroughly, read Chapters 18th and 19th of Part the Fourth.

and

and Departure of the other Heavenly Bodies, is owing to the Nearneſs of the Moon to the Earth; which Conſideration in this Caſe overbalances the Conſideration of her Smallneſs, the above-mentioned Effects depending in a great Meaſure, on the Proportion the Diameter of the Earth bears to the Diſtance of the Heavenly Bodies.

PROPOSITION XIV.

If a Body, whoſe Parts tend to the Center thereof, conſiſts wholly of a Fluid, or be partly ſolid and partly fluid, provided ſome of the Fluid be at the Surface, and very diſtant Parts thereof communicate with each other; and the Body have no Motion about its Axis, it will ſettle into a ſpherical Form, the mutual Tendency of its Parts towards each other, contracting it into the leaſt poſſible Shape. But if it revolves about its Axis, all its

Parts will endeavour to fly off from that Axis; but ſuch as are fartheſt from the Axis, more than the reſt: Conſequently thoſe Parts in its Surface, which are the fartheſt from the Extremities of that Axis, being alſo fartheſt from the Axis itſelf, will have a greater Endeavour to fly off, than ſuch as are nearer thoſe Extremities; beſides, as is evident, the former will endeavour to fly off directly from the Center, but the latter not ſo. The above-mentioned Endeavour therefore in the former will take off a much greater Degree of their Tendency to the Center than the Endeavour of the latter will; and ſince the ſame may be ſaid of thoſe which are at any other aſſignable equal Diſtances from the Center, all thoſe which lie between the Center of the Body, and ſuch, as are fartheſt from the Extremities of the Axis, will have their Tendency to the Center much more diminiſhed, than

than thoſe will, which lie between the Center and the ſaid Extremities: Theſe latter Parts therefore will preſs in towards the Center, overbalance the former, and raiſe them to a greater Diſtance from it than they were at before, reſtoring thereby an *Equilibrium* of the Parts of the Body one among another. On which Account the Body will aſſume a flattiſh or oblate Form. That is, ſuppoſing Lines drawn through the Middle of its Ais at right Angles therewith, thoſe Lines will be lengthen'd and the Axis itſelf will be ſhorten'd.

Hence the Figures of the Heavenly Bodies.

PROPOSITION XV.

The Impetus or Force wherewith a Body in Motion endeavours to proceed forwards, depends not only on the Quantity of Matter in that Body, but

likewiſe on the Swiftneſs the Body moves with: Thus, the Stroke of an Hammer is not only according to the Bulk or Weight of its Head, but is alſo according to the Swiftneſs of the Motion it ſtrikes with. If therefore two Bodies of equal Quantities of Matter be ſuſpended at the Ends of a Lever of equal Arms, each of them when the Lever turns on its Center, having equal Degrees of Swiftneſs or Velocity, will therefore have the ſame Impetus or Force whereby they endeavour to proceed (being in like Circumſtances with reſpect to both thoſe Things, which alone can give the one a Force or Tendency to move with, ſuperior to the other) and conſequently neither of them will preponderate. If one of the Bodies be larger than the other, the larger Body having the ſame Velocity with the other, but more Matter, will have the Advantage, and preponderate. If the Arms of the Lever are unequal,

and

and the Bodies equal, that Body which is at the greateſt Diſtance from the Center of Motion, moving the quickeſt, will have the Advantage over the other that Way, and overpoiſe it. So that the leaſt Body or Power, imaginable, may be made to equiponderate, over-poiſe, or keep in Motion the greateſt, by being applied to ſuch a Machine, and in ſuch Manner, that when the Machine moves, what it wants in Weight or Force, may be made out by the Velocity it has compared with the Velocity the Body has at the ſame Time, which is to be equiponderated, over-poiſed, or moved by it.

This holds equally in all Machines, *and is the* Foundation *of their* Theory.

PROPOSITION XVI.

Imagine the Surface of a large round Body to be covered every where, or in

Part, with ſmaller ones to an equal Height, and that theſe ſmaller ones tend towards the large Body by the firſt Principle; imagine alſo the whole Maſs of ſmaller ones divided into Columns reaching from Top to Bottom; thoſe Columns, if their Baſes be equal, will equiponderate, or be an equal Counterpoiſe to one another; and ſo they will, if their Baſes be unequal: For in this Caſe the Columns being of unequal Size in Diameter, if a larger Column ſubſides, the lower Parts of that Column (to find Room for themſelves) will raiſe a ſmaller Column farther than the larger one ſettled in the ſame Time, and in ſuch Proportion that, what the little Column wants in Weight, will be made out to it in Velocity; and conſequently, according to what was ſhewn in the foregoing Propoſition, the little Column will be a juſt Balance to the greater.

Farther,

Farther, if in the above-mentioned Suppoſition, there be a Body among thoſe ſmaller ones, heavier than a Bulk of them equal to its own Bulk, a Column of which that Body is a Part, will be heavier than any other Column of an equal Baſe; it will therefore ſubſide, permitting the Body to come to the Bottom: if the Body be lighter than a Bulk of the ſmaller ones equal to its own Bulk, a Column, of which it makes a Part, will be lighter than any other; the Body therefore will be buoy'd upwards, till it riſes ſo far out above the Surface, that it, together with the Column below it, may be a Counterpoiſe to another Column of equal Baſe.

Hence the Effects of the Preſſure of Fuids *upon one another, and upon* Solids *immerſed in them.*

PROPOSITION XVII.

Imagine the Surface of a large round Body covered every where, or in Part, with ſmaller ones to an equal Height; and that theſe ſmaller ones tend towards the large one by the firſt Principle, and that they are at the ſame Time diſpoſed to fly from one another by Virtue of the third, conſtituting thereby an elaſtic Subſtance ſurrounding that large Body, as in Propoſition the ſecond; and let them be divided into Columns, as in the laſt Propoſition. And let it be farther ſuppoſed, that the Diſpoſition in thoſe ſmaller Bodies, whereby they endeavour to depart from each other, is capable of being increaſed by Heat; and that at the Bottom of ſome of theſe Columns, that Diſpoſition is actually increaſed, but no where elſe, or at leaſt not in ſo great a Degree: then

then will the Bodies, where that Diſpoſition is increaſed, diffuſe themſelves into a larger Space, and ſo taking up more Room than an equal Number in the neighbouring Columns, a Column of which they are a Part, will become lighter than a neighbouring one of an equal Baſe. For, ſince the Bodies in the lower Parts of this Column are more diſtant from each other, than ſuch as are in other Columns, this Column cannot contain ſo many of them; that is, it cannot be ſo heavy as another of equal Baſe, unleſs it be longer; that is, unleſs the uppermoſt Parts thereof ſtand out above the Tops of the neighbouring Columns; but this they will not do; for by Virtue of the Tendency thoſe Parts have to the large Body, they will immediately (like Water raiſed above the Banks, which before confined it) ſpread themſelves every Way. This Column therefore, which, according

cording to the foregoing Propoſition, before this happened, was a Counterpoiſe to thoſe, which are round about it, being now become lighter, is no longer ſo. The Conſequence of which is, that the lowermoſt Parts of the neighbouring Columns, will preſs in under this from all Sides to reſtore the Equilibrium. Neither can the Equilibrium be reſtored, ſo long as the Place we have been conſidering remains hotter, than thoſe which are round about it. For, ſince the Bodies that come in, will ſpread themſelves into a larger Space by Means of the Heat they receive there, and fill up more Room, than the like Number in another Column of equal Baſe, the Column to which they belong, will, for the Reaſons above-mentioned, always be lighter than another of equal Baſe. And conſequently, according to the Tenour of the foregoing Propoſition, the neighbouring Columns will

will overpoiſe it, whatever Dimenſions, as to their Baſes, we ſuppoſe them to be of.

Imagine the like to happen to a Column or Columns of the Earth's Atmoſphere, and the lower Parts of the neighbouring Columns ruſhing in accordingly at the Bottom from all round, and you have an adequate Idea of the Cauſe and Nature of the Winds; *every Stream of the Particles of the Atmoſphere ruſhing in, as above, being a diſtinct Wind blowing from that Point of the Compaſs from which they come. And if you conceive the Center of that warmer Space to ſhift its Place variouſly upon the Surface of the Earth, you then get the Idea of the ſeveral Sorts of them, as the* Trade Winds, Moonſoons, &c. *For Inſtance, if it ſhifts regularly along the ſame Path, it cauſes Trade Winds; if now forwards, and then backwards, Moonſoons,* &c.

Theſe

Theſe are the Principal Phænomena *in Natural Philoſophy that are independent of each other; the reſt are for the moſt Part, no other than ſo many particular Caſes, Circumſtances, or Conſequences of theſe, or, in ſhort, one Way or other related to them. For the Solution of which, I refer the Reader to the Book itſelf.*

From a due Conſideration of the Propoſitions here laid down, the Reader will be able to form a true Judgment *of the Nature and Buſineſs of* Natural Philoſophy; *will ſee the* Uniformity *and* Conſiſtency *of the ſeveral Parts thereof with each other, and therein the wonderful* Wiſdom *and* Contrivance *of the* ſupreme Being, *in chooſing ſo ſhort and eaſy a Method of producing ſo great a Variety of Effects.*

There is one Thing more I think proper to be taken Notice of, before I put an End

End to this Preface; viz. *That it has been a ſtanding Objection againſt all* Natural Philoſophy *in general*; *that whereas it aſcribes Effects to natural or* mechanical Cauſes, *acting by* fixed *and* unalterable Laws, *it therefore excludes a Providence and the immediate Care and Protection of the* ſupreme Being, *making him no other than an Idle Spectator of Things here below.*

In Anſwer to this, it is to be conſidered in the firſt Place, that the Principles of the Philoſophy *which is now received, are ſo far from being* mechanical Cauſes, *at leaſt thoſe which are here made Uſe of, that, as above demonſtrated* (e), *they are the very Reverſe*; *and conſequently can be no other than the continual acting of God upon Matter, either mediately or immediately. Conſequently* Natural Philoſophy, *by endeavouring to account for the* Phænomena *of*

(e) See Note (*b*) of this Preface, p. 8.

Na-

Nature by thoſe Principles, is ſo far from excluding the Deity *from being concerned in the Affairs of this World; that it tends to ſhew that none are performed without him. Neither, ſecondly, does* Natural Philoſophy *inculcate, that the Laws by which thoſe Principles act, are* fixed *and* unalterable: *the Accuſation is therefore foreign. But to conſider this Matter a little more particularly.*

When, in Natural Philoſophy, *a Principle is ſaid to act according to a particular Law, the Meaning is not, that it acts neceſſarily and unalterably ſo; but only, that it does ſo* ordinarily, *and in common Caſes. Doubtleſs the* Author, *both of Matter and of thoſe very Principles by which it acts, can, notwithſtanding thoſe Principles, cauſe it to act differently from what it would do in conſequence of them alone, and ſo by that Means produce Effects contrary to the common Courſe of Nature, whenever*

ever he ſhall think proper. That he has done ſo, when wiſe Ends require it, appears from Hiſtory. That it may be done a Thouſand Ways, unperceived by us, is evident. For Inſtance, though Lightning may be accounted for by theſe Principles; and in all Probability is ordinarily *the Reſult thereof; yet who will affirm, that in any one* particular *Caſe, thoſe Principles formed that very Lightning, or that its Courſe was directed by them? Upon the whole therefore, to preſume, that the ordinary and common Courſe of Nature is not ſometimes altered, is* haſty *and* unwarrantable.

THE

THE CONTENTS.

PART I.

MECHANICS.

THE *Introduction*, Page 1
CHAPTER I. *Of the Properties of Body*, 7
CHAP. II. *Of Vacuum*, 10
CHAP. III. *Of Attraction and Repulsion*, 12
CHAP. IV. *Of the Laws of Motion*, 20
CHAP. V. *Of falling Bodies*, 24
CHAP. VI. *Of the Descent of Bodies on oblique Planes, and of Pendulums*, 28
CHAP. VII. *Of the Vibration of a Pendulum in a Cycloid*, 35
CHAP. VIII. *Of the Centers of Oscillation and Percussion*, 47
CHAP. IX. *Of Projectiles*, 53
CHAP. X. *Of Central Forces*, 61
CHAP. XI. *Of the Communication of Motion*, 66
CHAP. XII. *Of the Mechanical Powers*, 76
Conclusion, 96

PART II.

Hydroſtatics *and* Pneumatics.

CHAPTER I. *Of the Phænomena which ariſe from the mutual Action of the Particles of Fluids upon one another,* Page 99
CHAP. II. *Of the Effects which Fluids have on Solids immerſed therein,* 123
CHAP. III. *Of the Air,* 128
CHAP. IV. *Of the Reſiſtance of Fluids,* 136
Diſſertation I. *Of Sound,* 142
Diſſert. II. *Of Capillary Tubes,* 152
Diſſert. III. *Of the Origin of Fountains,* 168
Diſſert. IV. *Of the Barometer,* 179
Diſſert. V. *Of the Origin of the Winds,* 211
Diſſert. VI. *Of the Formation and Aſcent of Vapours, and their Reſolution into Rain, Snow, and Hail,* 226
Diſſert. VII. *Of the Cauſe of Thunder and Lightning, and of the* Aurora Borealis, 239
Diſſert. VIII. *Of Fermentation,* 266

PART III.

OPTICS.

CHAPTER I. *Of the Nature and Propagation of Light,* Page 273
CHAP. II. *Of the Cauſe of Refraction, and the Law whereby it is performed,* 279
CHAP. III. *Of the Refraction of Light in paſſing through plain and ſpherical Surfaces,* 288
CHAP. IV. *Of Lenſes, and the Manner in which Rays are affected in paſſing through them,* 314
CHAP. V. *Of the Eye,* 323
CHAP. VI. *Of the Nature of Viſion,* 326
CHAP. VII. *Of the Appearance of Objects ſeen through Media of different Forms,* 342
Diſſertation I. *Of the Horizontal Moon,* 356

VOL. II.

CHAPTER VIII. *Of the Manner wherein Light is reflected,* Page 3
CHAP. IX. *Of the Reflection of Light from plain and ſpherical Surfaces,* 6
CHAP. X. *Of the Appearance of Bodies ſeen by Light reflected from plain and ſpherical Surfaces,* 24
CHAP. XI. *Of the different Refrangibility in the Rays of Light; of the Colours which the diſtinct Species of them are diſpoſed to excite; and of the Cauſe of that Variety of Colours which is obſervable in Bodies,* 40
CHAP. XII. *Of the Qualifications in Bodies, which diſpoſe them to reflect the Rays of different Colours,* 53
CHAP. XIII. *Of the Cauſe of Opacity and Tranſparency in Bodies,* 60
Diſſert. II. *Of the Cauſe of Refraction of Light,* 64
Diſſert. III. *Of Microſcopes and Teleſcopes,* 75
Diſſert. IV. *Of the Rainbow,* 95
Of the Camera Obſcura, and the Magic Lantern, 116

PART IV.

ASTRONOMY.

The Introduction, Page 119
CHAPTER I. *Of the Bodies which compoſe the Solar Syſtem, and their real Motions,* 127
CHAP. II. *Of the fixed Stars,* 141
CHAP. III. *Of ſuch of the apparent Motions of the heavenly Bodies, as ariſe from the Motion of the Earth about its Axis,* 149
CHAP. IV. *Of the apparent Motion of the Sun, ariſing from the Earth's revolving about it,* 151
CHAP. V. *Of the Earth's annual Parallax, the Nutation of the Poles, and the Preceſſion of the Equinoctial Points,* 168

CHAP. VI. *Of the Phænomena which arise from the Motion of the Earth, and of the inferior Planets, Mercury and Venus, conjointly,* 174
CHAP. VII. *Of the Phænomena which are owing to the Motion of the Earth, and that of the superior Planets, Mars, Jupiter, and Saturn conjointly,* 182
CHAP. VIII. *Of the Phænomena of the Moon,* 187
CHAP. IX. *Of the Eclipses of the Sun and Moon,* 202
CHAP. X. *Of the Phænomena of the Satellites of Jupiter and Saturn; their Eclipses and Occultations: And also of Saturn's Ring,* 209
CHAP. XI. *Of the Comets,* 214
CHAP. XII. *Of the Parallax of the Heavenly Bodies,* 229
CHAP. XIII. *Of the Refraction of the Atmosphere, and the Crepusculum or Twilight,* 233
CHAP. XIV. *Of the Doctrine of the Sphere,* 241
CHAP. XV. *The Description of the Orrery and the Globes,* 263
CHAP. XVI. *Of the Equation of Time,* 294
CHAP. XVII. *Of the Division of Time,* 297
CHAP. XVIII. *Of the Forces necessary to retain Bodies revolving in circular and other Orbits,* 320
CHAP. XIX. *The Lunar Irregularities, the Precession of the Equinoctial Points, the Nutation of the Poles of the Earth, and the Ebbing and Flowing of the Sea, accounted for,* 366
CHAP. XX. *Of the Figures of the Heavenly Bodies,* 391

A COMPENDIOUS SYSTEM OF Natural Philoſophy.

PART I.

The INTRODUCTION.

SO wild and extravagant have been the Notions of a great Part of Philoſophers, both ancient and modern, that it is hard to determine, whether they have been more diſtant in their Sentiments from Truth, or from one another; or have not exceeded the Fancies of the moſt fabulous Writers, even Poets and Mythologiſts. This was owing to a precipitate Proceeding in their Enquiries, and a Neglect of Geometry and Experiment; without the Aſſiſtance of which, it is impoſſible the Powers of natural Agents ſhould be diſcovered.

THE Manner of Philoſophizing among the Ancients was to aſcribe to Bodies certain *arbi-*

trary Properties, ſuch as beſt ſerved their Purpoſe in accounting for the Phænomena * of Nature; from whence proceeded ſo many various Sects of Philoſophers; every one aſſigning a different Cauſe to the ſame Appearance, as his particular Genius and Imagination led him.

THE chief Agreement obſervable among moſt of them, conſiſts in this, *viz.* that they conceived all Bodies, as Compoſitions of Air, Earth, Fire, and Water, or ſome one or more of them, from whence theſe acquired the Name of Principles, or Elements, which they ſtill retain.

EPICURUS advanced a little farther, and aſſerted, that though Bodies conſiſted of ſome one or more of theſe, yet that they were not ſtrictly Elements, but that theſe themſelves conſiſted of Atoms; by an accidental Concourſe of which (as they were moving through infinite Space in Lines nearly parallel) all Things received their Form and Manner of Exiſtence †.

DES CARTES has contrived an Hypotheſis very different from the reſt: He ſets out with a

* By a Phænomenon of Nature, is meant any Motion or Situation of Bodies among one another, which offers itſelf to the Notice of our Senſes, and is not the immediate Reſult of the Action of an intelligent Being.

† For the Opinions of the ancient Philoſophers, conſult *Diogenes Laertius*, and *Stanley*'s Lives.

Suppo-

Suppoſition that the Univerſe at firſt was entirely full of Matter; that from this Matter, when firſt put in Motion, there would neceſſarily be rubbed off (by the grinding of the ſeveral Parts one againſt another) ſome Particles ſufficiently fine to paſs through the hardeſt and moſt ſolid Bodies without meeting with any Reſiſtance: Of theſe conſiſts his *Materia ſubtilis*, or *Materia primi Elementi*. He imagined, that from hence alſo would reſult other Particles, of a globular Form; to which he gave the Name of *Materia ſecundi Elementi*. Thoſe which did not ſo far loſe their firſt Figure, as to come under the Denomination of *Materia primi* or *ſecundi Elementi*, he called *Materia tertii Elementi*; and maintained, that all the Variety, which appears in natural Bodies, was owing to different Combinations of thoſe Elements.

He likewiſe ſuppoſes, that God created a certain Quantity of Motion, and aſſigned it to this Maſs of Matter; and that that Motion (being once created) could no more be annihilated, without an omnipotent Hand, than Body itſelf: In Conſequence of which, he was obliged to teach, that the Quantity of Motion is always the ſame: So that, if all the Men and Animals in the World were moving, yet ſtill there would be no more Motion than when they were at Reſt, the Motion which they had not, when at Reſt, being transferred

to the Æther. So unaccountable are the Notions of this great Philoſopher, that it is ſurprizing his Doctrine ſhould have met with ſuch univerſal Reception, and have got ſo ſtrong a Party of Philoſophers on its Side.

DES CARTES has been ſaid, by a late Writer *, to have joined to his great Genius an exquiſite Skill in Mathematics, and, by mixing Geometry and Phyſics together, to have given the World Hopes of great Improvements in the latter. But this Writer ought to have conſidered, that what he looked upon, in DES CARTES's Book of Principles, as Demonſtrations, are only Illuſtrations, there not being a Demonſtration from Geometry in all his Philoſophical Works †.

THE preſent Method of Philoſophizing, eſtabliſhed by Sir ISAAC NEWTON, is to find out the Laws of Nature by Experiments and Obſervations. To this, with a proper Application of Geometry, is owing the great Advantage the preſent Syſtem of Philoſophy has over all the preceding ones, and the vaſt Improvement it has received within the laſt

* Mr. *Wotton*, in his Reflections on ancient and modern Learning.

† See this Subject diſcuſſed more at large in *Keil*'s Introduction to his Examination of Dr. *Burnet*'s Theory.

Age.

Age. It is indeed in vain to imagine, that a Syſtem of Natural Philoſophy can be framed by any other Method; for without Obſervations it is impoſſible we ſhould diſcover the Phænomena of Nature; without Experiments we muſt be ignorant of the mutual Actions of Bodies; and without Geometry we can never be certain whether the Cauſes we aſſign be proportionate to the Operations we would account for, as the various Syſtems of Philoſophy built on other Foundations evidently ſhew.

THIS Way of ſearching into Nature was firſt propoſed by my Lord BACON *, proſecuted by the *Royal Society*, the *Royal Academy* at *Paris*, the Honourable Mr. BOYLE, Sir ISAAC NEWTON, *&c.*

WHAT wonderful Advancement in the Knowledge of Nature may be made by this Method of Enquiry, when conducted by a Genius equal to the Work, will be beſt underſtood by conſidering the Diſcoveries of that excellent Philoſopher laſt mentioned. To Him it is principally owing, that we have now a rational Syſtem of Natural Philoſophy; 'tis He, who, by purſuing the ſure and unerring Method of Reaſoning from Experiment and

* See his *Novum Organum.*

Obſervation, joined with the moſt profound Skill in Geometry, has carried his Enquiries to the moſt minute and inviſible Parts of Matter, as well as to the largeſt and moſt remote Bodies in the Univerſe, and has eſtabliſhed a Syſtem not ſubject to the Uncertainty of a mere Hypotheſis, but which ſtands upon the ſecure Baſis of Geometry itſelf.



CHAP.

CHAP. I.

The Properties of Body.

IT being the Deſign of *Phyſics*, or Natural Philoſophy, to account for the Phænomena of the Material World, it is neceſſary to begin with laying down the known Properties of Body.

THESE are, 1. Solidity. 2. Extenſion. 3. Diviſibility. 4. A Capacity of being moved from Place to Place. A Paſſiveneſs or Inactivity.

1. SOLIDITY, called alſo Impenetrability, is that Power which Body has of excluding all others out of its Place.

THAT Body, as ſuch, muſt be endued with this Property, follows from its Nature, for otherwiſe two Bodies might exiſt in the ſame Place, which is abſurd. The ſofteſt are equally ſolid with the hardeſt; for we find, by Experiment, that the Sides of a Bladder, filled with Air or Water, can by no Means be made to come cloſe together *.

2. THAT

* At *Florence* a hollow Globe of Gold was filled with Water, and then exactly cloſed; the Globe thus cloſed was put into a Preſs driven by the Force of Screws; the Water, finding no Room for a nearer Approach of its Particles towards each other, made its Way through the Pores of that cloſe

2. THAT Body is extended, is ſelf-evident, it being impoſſible to conceive any Body which has not Length, Breadth, and Thickneſs; that is, Extenſion.

3. IT is no leſs evident, that Body is diviſible; for, ſince no two Particles of Matter can exiſt in the ſame Place, it follows that they are really diſtinct from each other, which is all that is meant by being diviſible.

IN this Senſe, the leaſt conceivable Particle muſt ſtill be diviſible, ſince it will conſiſt of Parts which will be really diſtinct *. To illuſtrate this by a familiar Inſtance: Let the leaſt imaginable Piece of Matter be conceived lying on a ſmooth plain Surface; 'tis evident, the Surface will not touch it every where; thoſe Parts, therefore, which it does not touch, may be ſuppoſed ſeparable from the other, and ſo on as far as we pleaſe; and this is all that

Metal, ſtanding in Drops like Dew on the Outſide, before the Globe would yield to the violent Preſſure of the Engine. V. *Acad. del Ciment.*

* This Propoſition is demonſtrated Geometrically thus: Suppoſe the Line *AD* (*Fig.* 1.) perpendicular to *BF*, and another, as *GH*, at a ſmall Diſtance from it, alſo perpendicular to the ſame Line: with the Centers *C*, *C*, *C*, &c. deſcribe Circles cutting the Line *GH* in the Points *e*, *e*, *e*, &c. Now the greater the Radius *AC* is, the leſs will be the Part *e H*. But the Radius may be augmented *in infinitum*. So long, therefore, the Part *EH* may be divided into ſtill leſs Portions; conſequently it may be divided *in infinitum*. *Q. E. D.* V. *Keil's* Introd. on Phyſ. Præl. 3, 4, 5. *Graveſande's* Elem. Math. Phyſ. L. 1. C. 4. Schol.

is

is meant, when we ſay Matter is infinitely diviſible.

How far Matter may actually be divided, may in ſome Manner be conceiv'd from hence*, that a Piece of ſmall Wire, gilt with ſo ſmall a Quantity as eight Grains of Gold, may be drawn out to a Length of thirteen Thouſand Feet, the whole Surface of it ſtill remaining cover'd with Gold †.

A Quantity of Vitriol, being diſſolved and mix'd with nine Thouſand Times as much Water, will tinge the whole, conſequently the Vitriol will be divided into as many Parts as there are viſible Portions of Matter in that Quantity of Water §.

THERE are Perfumes, which without a ſenſible Diminution of their Quantity, ſhall fill a very large Space with their odoriferous Particles, which muſt therefore be of an inconceivable Smallneſs, ſince there will be a ſuffi-

* We have a ſurprizing Inſtance of the Minuteneſs of ſome Parts of Matter, from the Nature of Light and Viſion. Let a Candle be lighted and placed in an open Plane, it will then be viſible two Miles round, conſequently was it placed two Miles above the Surface of the Earth, it would fill with luminous Particles a Sphere, whoſe Diameter was four Miles, and that before it had loſt any ſenſible Part of its Weight. The Force of this Argument will appear better when the Reader is acquainted with the Cauſe of Viſion.

† *Keil*'s Introd. ad Phyſ. Præl. 5. Religious Philoſ. Contempl. 25.

§ Mem. de l'Acad. 1706.

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cient Number in every Part of that Space, ſenſibly to affect the Organ of Smelling.

4. THAT all Matter is moveable, follows from its being finite; and to ſuppoſe it poſitively infinite is abſurd, becauſe it conſiſts of Parts *.

5. BY the Paſſiveneſs or Inactivity of Matter (commonly call'd its *Vis Inertiæ*) is meant the Propenſity it has to continue its State of Motion or Reſt, till ſome external Force acts upon it. This will be farther explained under the firſt Law of Nature, in Chapter the fourth of this Part.

CHAP II.

Of Vacuum.

I. PLACE void of Matter is called empty Space, or *Vacuum*.

II. IT has been the Opinion of ſome Philoſophers, particularly the *Carteſians*, that Nature admits not a *Vacuum*, but that the Univerſe is entirely full of Matter: In conſequence of which Opinion they were oblig'd to aſſert, that if every Thing contain'd in a Veſſel could be taken out or annihilated, the Sides of that Veſſel, however ſtrong, would come together; but this is contrary to Expe-

* See Mr. *Law*'s Tranſlation of Abp. *King de Origine Mali*. Not. 3.

rience,

rience, for the greateſt Part ‖ of the Air may be drawn out of a Veſſel by Means of the Air-Pump, notwithſtanding which it will remain whole, if its Sides are ſtrong enough to ſupport the Weight of the incumbent Atmoſphere.

III. SHOULD it be objected here, that as it is impoſſible to extract all the Air out of a Veſſel, and that there will not be a *Vacuum* on that Account; the Anſwer is, that ſince a very great Part of the Air, that was in the Veſſel, may be drawn out, as appears by the more quick Deſcent of light Bodies in a Receiver * when exhauſted of its Air, there muſt be ſome Vacuities between the Parts of the remaining Air; which is ſufficient to conſtitute a *Vacuum*. Indeed to this it may be objected by a *Carteſian*, that thoſe Vacuities are filled with *Materia ſubtilis*, that paſſes freely through the Sides of the Veſſel, and gives no Reſiſtance to the falling Bodies: But as the Exiſtence of this *Materia ſubtilis* can never be prov'd, we are not oblig'd to allow the Objection; eſpecially ſince Sir ISAAC NEWTON has found, that all Matter affords a Reſiſtance nearly in Proportion to its Denſity †.

THERE are many other Arguments to prove this, particularly the Motions of the Comets

‖ A Veſſel cannot be entirely exhauſted of its Air, becauſe the Action of the Pump depends on the Spring of that which remains in the Veſſel.

* By this Term is meant any Veſſel, out of which we extract the Air by the Air-Pump.

† *Newt.* Principia, Lib. 2. Prop. 31. & 40. & Opt. Edit. 2. Book 3. Quer. 18, 19, 20, 21. *Deſagul.* Lect. 1. Ann. 2.

through

through the Heavenly Regions without any ſenſible Reſiſtance*; the different Weight of Bodies of the ſame Bulk, *&c.* but thoſe being not yet explained, are not ſo proper to be inſiſted on in this Place.

CHAP. III.

Of Attraction and Repulſion.

I. BESIDES the forementioned Properties of Matter, it has alſo certain Powers or active Principles, known by the Names of *Attraction* and *Repulſion*, probably not eſſential or neceſſary to its Exiſtence, but impreſſed upon it by the Author of its Being, for the better Performance of the Offices for which it was deſign'd.

II. ATTRACTION is of two Kinds. 1. Coheſion, or that by which minute Bodies, (or the ſeveral Particles of the ſame Body) when placed aſunder at very ſmall Diſtances, mutually approach each other; and then adhere or ſtick together, as if they were but one. 2. Gravitation, or that by which diſtant Bodies act upon each other.

III. THE Attraction of Coheſion is prov'd from Abundance of Experiments, of which ſome of the moſt obvious are as follows:

* *Deſagul.* Lect. 1. Annotat. 8.

1. LET

1. LET a small Glass Tube (commonly call'd a Capillary Tube) open at both Ends, be dipt into a Vessel of Water, the Water will immediately rise up in the Tube to a certain Height above the Level of the external Water. This Rise of the Water in the Glass Tube is manifestly owing to the Attraction of those Particles of the Glass, which lie in the inner Surface of the Tube immediately above the Water: Accordingly the Quantity of Water raised is always found to be proportional to the Largeness of that Surface *.

2. LET two Spheres of Quicksilver be placed near each other, and they will immediately run together, and form one Globule.

IV. THE Laws of this Attraction are, 1. That it acts only upon Contact, or at very small Distances; for the Spheres, mentioned in the last Experiment, will not approach each other,

* The Heights which the Water rises to in different Tubes, are observed to be reciprocally as the Diameters of the Tubes, from whence it follows, that the Quantities raised are as the Surfaces which raise them.

Dem. Let there be two Tubes, the Diameter of the first double to that of the second, the Water will raise half as high in the first as in the second: Now was it to rise equally high in both, the Quantity in the first would be four Times as great as in the second, (Cylinders of equal Heights being as the Squares of their Diameters; 11. *El.* 14.) therefore since it is found to rise but half as high, the Quantity is but twice as much, and therefore as the Diameter; but the Surfaces of Cylinders are as their Diameters, therefore the Quantities of Water raised are also as the Surfaces. *Q. E. D.*

See the Dissertation on this Subject, Part II.

till

till they are plac'd very near. 2. It acts according to the Breadth of the Surfaces of the attracting Bodies, and not according to their Quantities of Matter. For, let there be two poliſh'd Glaſs Plates laid one upon another, in ſuch a Manner as to touch at one End, and there make a very ſmall Angle: If two unequal Drops of Oil be put between theſe Plates, at equal Diſtances from the Line of Contact, ſo that the leaſt may touch both Glaſſes, they will then both move towards the Ends that touch, becauſe the Attraction of the Surfaces inclines that Way; but the largeſt, touching the Glaſſes in moſt Points, will move the faſteſt. 3. 'Tis obſerv'd to decreaſe much more than as the Squares of the Diſtances of the attracting Bodies from each other increaſe: That is, whatever the Force of Attraction is at a given Diſtance, at twice that Diſtance it ſhall be more than four Times leſs than before *.

V. From hence it is eaſy to account for the different Degrees of Hardneſs in Bodies; thoſe whoſe conſtituent Particles are flat or ſquare, and ſo ſituated as to touch in many Points, will be hard; thoſe Particles which are more round, and touch in fewer Points, will conſtitute a ſofter Body; thoſe which are ſpherical, or nearly of that Figure, will form a Fluid †.

* *Keilii* Opera, Ed. 4to. *p.* 626.

† See *Rohault* in the Notes, p. 105, 108. See Part II. Chap. I. §. 2. in the Notes. *Newtoni.* Optic. p. 335.

VI. At-

VI. ATTRACTION of Gravitation is that by which distant Bodies act upon each other. Of this we have daily Instances in the falling of heavy Bodies towards the Earth.

VII. THE Laws of this Attraction are, 1. That it decreases, as the Squares of the Distances between the Centers of the attracting Bodies increase. Thus, a Body, which at the Surface of the Earth (*i. e.* about the Distance of four Thousand Miles from its Center) weighs ten Pounds, if it was placed four Thousand Miles above the Surface of the Earth, (*i. e.* twice as far distant from the Center) as before, would weigh four Times less; if thrice as far, nine Times less, *&c.* The Truth of this Proposition is not to be had from Experiments, (the utmost Distance we can convey Bodies to, from the Surface of the Earth, bearing no Proportion to their Distance from its Center,) but is sufficiently clear from the Motion observed by the heavenly Bodies. 2. Bodies attract one another with Forces proportional to the Quantities of Matter they contain; for all Bodies are observed to fall equally fast in the exhausted Receiver, where they meet with no Resistance. From whence it follows, that the Action of the Earth upon Bodies is exactly in Proportion to the Quantities of Matter they contain; for was it to act as strongly upon a less Body as upon a larger, the least Body being most easily put into Motion, would

would move the fasteſt. Accordingly, it is obſervable, that the Weight of a Body is the ſame whether it be whole, or ground to Powder *.

VIII. FROM hence it follows, that, was a Body to deſcend from the Surface toward the Center of the Earth, it would continually become lighter and lighter, the Parts above attracting it, as well as thoſe below; in which Caſe it is demonſtrated by Mathematicians, that the Gravity would decreaſe with the Diſtance of the Body from the Center †.

Scholium.

* *Graveſande*, Lib. 4. Chap. 11. *Cotes*'s Preface to *Newton*'s Princip.

† *Dem.* Let there be a Body as *P* (*Fig.* 2.) placed any where within a Concave Sphere, as *AB*, and let us ſuppoſe it divided into an infinite Number of thin concentric Surfaces; I ſay, the Body *P* will be attracted equally each Way by any one of theſe, *v. g.* the interior *HIKLM*. Let there be Lines as *IL*, *HK*, &c. drawn through any Point of the Body *P*, in ſuch a Manner as to form the Surface of two ſimilar Figures; ſuppoſe Cones, the Diameters of whoſe Baſes may be *IH*, *KL*, which let us ſuppoſe infinitely ſmall. Theſe Baſes (being as the Squares of the Lines *IH*, *KL*) (2. *Elem.* 12.) will be directly, as the Squares of their Diſtances from *P* (for the Triangles *IPH*, *KPL*, being infinitely ſmall, are ſimilar.) But thoſe Baſes include all the Particles of Matter in the interior Surface, that are oppoſite to each other; the oppoſite Attractions are therefore in the ſame Ratio with thoſe Baſes, that is as the Squares of the Diſtances *PK*, *PI*. But the Attraction is inverſely, as the Squares of the Diſtances of the attracting Bodies, §. 7. *i. e.* inverſely as the Squares of the ſame Diſtances *PK*, *PI*; theſe two Ratios therefore deſtroying each other, it is evident, that if the Concavity of the Sphere was filled with Matter, that alone, which lies nearer the Center than the Body, can effect it, the reſpective Actions of all the Parts, that are more diſtant, being equal,

and

Scholium. It may be proper to obſerve here, that when Philoſophers ſpeak of Bodies gravitating to, or attracting each other, that Body is ſaid to gravitate to another, which moves towards it, while the other actually is, or appears to be, at Reſt; and this other is ſaid to attract the former; though indeed the Force being mutual and equal on both Sides (as will be explain'd under the third Law of Nature) the ſame Term might be apply'd to either the gravitating or attracting Body.

It is farther to be obſerv'd, that when we uſe the Terms, Attraction or Gravitation, we do not thereby determine the Phyſical Cauſe of it, as if it proceeded from ſome ſuppoſed *occult* Quality in Bodies; but only uſe thoſe Terms to ſignify an Effect, the Cauſe of which lies out of the Reach of our Philoſophy. Thus, we may ſay, that the Earth attracts heavy Bodies; or that ſuch Bodies tend to, or gravitate towards the Earth: Although at the ſame

and in contrary Directions, ſince the ſame is demonſtrable of any of the remaining concentric Surfaces. Let us ſee then what Effect that, which lies nearer the Center than the Body, will have upon it, which may be conſidered as a Sphere, on whoſe Surface the Body is placed. The Diſtances of each Particle of Matter from the Body, (taken collectively) will be as the Diameter of the Sphere, or as the Radius, *i. e.* as the Diſtance of the Body from the Center: Their Action therefore upon the Body will be inverſely as the Square of that Diſtance; but the Quantity of Matter will be as the Cube of that Diſtance, (18 *Elem.* 12.) the Attraction therefore will be alſo in that Proportion. Now, theſe two Ratios being compounded, the Attraction will be only as the Diſtance of the Body from the Center. *Q. E. D.*

Time we are wholly ignorant whether this is effected by ſome Power, actually exiſting in the Earth, or in the Bodies themſelves, or external to both. But it is impoſſible that any Error in our Reaſonings can follow from hence; as it is evident, that all the Conſequences of ſuch Tendency muſt be the ſame, let the Cauſe be where, or what it will.

X. REPULSION is that Property in Bodies, whereby, if they are placed juſt beyond the Sphere of each other's Attraction and Coheſion, they mutually fly from each other.

IF any oily Subſtance, lighter than Water, be placed on the Surface thereof, or if a Piece of Iron be laid on Mercury, the Surface of the Fluid will be depreſs'd about the Body laid on it: This Depreſſion has been by ſome aſcrib'd to a repelling Power in the Bodies, which hinders the Approach of the Fluid towards them.

BUT it is more generally imputed to this, that the Particles of the Fluid attract each other, in theſe and the like Inſtances, more forcibly than they are attracted by the Body laid on it; and ſo recede from the Body as far as the Gravity of the neighbouring Particles of the Fluid will permit them.

IT is poſſible, in ſome Caſes, to preſs or force the Particles of different Subſtances, that ſeem to repel each other in this Manner, ſo near together, that they ſhall mutually attract each other:

other: As when we mix Oil and Water till they incorporate.

THIS Property is however exceedingly apparent in the Particles of the Air, which are endued with ſo ſtrong a repulſive Force with regard to each other, that they never could, by any Experiment yet made upon them, be compreſſed, or driven ſo cloſe together, but they would ſtill endeavour to fly from each other *.

XI. BESIDES the general Powers beforementioned, there are ſome Bodies that are endued with another, call'd *Electricity*. Thus Amber, Jet, Sealing-Wax, Agate, Glaſs, and moſt Kinds of precious Stones, attract and repel light Bodies at conſiderable Diſtances.

THE chief Things obſervable in theſe Bodies are, 1. That they don't act, but when heated. 2. That they act more forcibly when heated by rubbing, than by Fire. 3. That, when they are well heated by rubbing, light Bodies will be alternately attracted and repell'd by them, but without any obſervable Regularity whatever. 4. If a Line of ſeveral Yards in Length has a Ball, or other Body ſuſpended at one End, and the other End be fixed to a Glaſs Tube; when the Tube is heated by rubbing, the Electrical Virtue of the Glaſs will be communicated from the Tube to the Ball, which will attract and repel light Bodies in the ſame Man-

* See a further Proof of this Repulſive Force in Sir *Iſaac Newton's* Opticks, *B.* 3. and Query 31.

ner, as the Glaſs itſelf does. 5. If the Glaſs Tube be emptied of Air, it loſes its Electricity *.

XII. LASTLY, the Loadſtone is obſerv'd to have Properties peculiar to itſelf, as that by which it attracts and repels Iron, the Power it communicates to the Needle, and ſeveral others †.

CHAP. IV.

Of the Laws of Motion, commonly called Sir ISAAC NEWTON'*s Laws of Nature.*

I. ALL Bodies continue their State of Reſt, or uniform Motion in a right Line, till they are made to change that State by ſome external Force impreſſed upon them.

THIS Law is no other, than that univerſal Property of Bodies, called Paſſiveneſs or Inactivity; whereby they endeavour to continue the State they are in, whatever it be. Thus a Top only ceaſes to run round on Account of the Reſiſtance it meets with from the Air, and the Friction of the Plane whereon it moves. And a Pendulum, when left to vibrate in *vacuo*, where there is nothing to ſtop it, but the Friction ariſing from the Motion

* See *Hauksbee*'s Experiments. *Philoſoph. Tranſact.* N° 326.

† Several Solutions of theſe Properties of *Electricity* and *Magnetiſm* have been attempted by different Philoſophers, but all of them ſo unſatisfactory as not to deſerve a particular Account in this Place. See *Chambers*'s Dictionary in *Electricity*, and *Des Cartes* Opera Philoſophica. P. IV. §. 133. with ſeveral others referr'd to in *Quæſtiones Philoſophicæ*, *Deſagul.* Lect. I. §. 33.

of

of the Pin on which it is ſuſpended, continues to move much longer, than one in the open Air.

II. THE Change of Motion produced in any Body, is always proportional to the Force, whereby it is effected; and in the ſame Direction, wherein that Force acts.

THIS is an immediate Conſequence of this Axiom; The Effect is always proportional to its Cauſe. For Inſtance, if a certain Force produces a certain Motion, a double Force will produce double the Motion; a triple Force triple the Motion, *&c.* If a Body is in Motion, and has a new Force impreſſed on it in the Direction wherein it moves, it will receive an Addition to its Motion, proportional to the Force impreſſed; but if the Force acts directly contrary to its Motion, the Body will then loſe a proportional Part of its Motion: Again, if the Force is impreſſed obliquely, it will produce a new Direction in the Motion of the Body, more or leſs different from the former, in Proportion to its Quantity and Direction *.

III. RE-

* This Caſe is expreſſed more accurately by Mathematicians thus. If the Proportion and Direction of two Forces, acting upon a Body at the ſame Time, be repreſented by the Sides of a Parallelogram, the Diagonal of that Parallelogram will repreſent the Proportion and Direction of their united Forces.

Dem. Let the Body *A* (*Fig.* 3.) be impell'd with the Force, which would carry it to *E*, in the ſame Time that another, acting upon it in the Direction *AD*, would carry it to *D*. Imagine

III. REACTION is always contrary, and equal to Action; or the Actions of two Bodies upon each other, are equal, and in contrary Directions.

THUS, ſuppoſe a Stone, or other Load to be drawn by an Horſe; the Load reacts upon the Horſe, as much as the Horſe acts upon the Load; for the Harneſs, which is ſtretched equally between them both Ways, draws the Horſe towards the Stone, as much as it does the Stone towards the Horſe; and the progreſſive Motion of the Horſe is as much retarded by the Load, as the Motion of the Load is

gine that while the Body paſſes to *E*, the Line *AD* (in which the Body moves by the other Force) moves to *EB*, in a Direction parallel to itſelf; when the Body has advanced to *G* in the Line *AE*, the Line *AD* will have got to *GF*, and the Body will have paſſed over *GH*, ſuch a Part of it, as bears the ſame Proportion to the whole Line *GF*, as *AG* does to *AE*, that is *GH* (the ſhorter Side of the Parallelogram *GM*,) is, to *GF*, or, which is the ſame Thing, to *EB*, (the ſhorter Side of the Parallelogram *ED*) as *AG* (the longer Side of the former) is to *AE* the longer Side of the latter; from whence the Parallelograms are ſimilar, *El.* 6. *Def.* 1. and conſequently, by 24 *El.* 6. the Point *H* is in the Diagonal, that is, the Body will always be found in the Line *AB*. Q. E. D.

Coroll. From hence we have an eaſy Method of reſolving a given Motion into any two, or more Directions whatever; *viz.* by deſcribing a Parallelogram about the given Direction as a Diagonal, the two Sides of which will repreſent the Directions ſought. Thus, ſuppoſe a Body was impell'd in the Line *AB*, we may conceive it is acted upon by two Forces at the ſame Time, one towards *E*, the other towards *D*, or any other two whatever, provided the Lines be drawn of ſuch Length, that when the Parallelogram is compleated, the given Line *AB* ſhall be its Diagonal.

promoted

promoted by the Endeavour of the Horſe *. This will be better explained from the following Inſtance; let a Perſon, ſitting in a Boat, draw another Boat equally heavy towards him, they will both move towards each other with equal Velocities: Let the Boat he ſits in be the lighteſt, and it will move the faſteſt; becauſe the Action being equal on both Sides, the ſame Quantity of Motion will be given to each Boat, that is, the leſſer will have the greater Velocity †.

We have a farther Confirmation of this from Attraction. Suppoſe two Bodies attracting one another, but prevented from coming cloſe together by ſome other Body placed between them: If their reſpective Actions, by which they tend towards each other, were not equal on both Sides, then would the intermediate Body be preſſed more one Way than the other, and ſo all three would begin to move of themſelves the ſame Way; but that three Bodies ſhould be put into Motion after this Manner, when no foreign Force acts upon

* It may be thought perhaps (two equal and contrary Forces deſtroying one another) the Horſe will in this Caſe not be able to move at all, becauſe the Load draws him back, as much as he draws the Load forwards. But it is to be obſerved, that the Strength of the Horſe is not properly exerted upon the Load, but upon the Ground; and conſequently the Ground reacting and continuing at Reſt, puſhes the Horſe forward with juſt ſo much Force as the Horſe exerts, above what is counteracted by the Load.

† See the Diſtinction between Motion and Velocity, Chap. 9.

them, is contrary to Experience, conſequently whatever different Degrees of Force, any two Bodies may be capable of exerting, their mutual Actions on each other are always equal. This may be tried with a Loadſtone and Iron; which, being put into proper Veſſels, contiguous to one another, and the Veſſels made to float on the Surface of the Water, will be an exact Counterbalance to each other, and remain at Reſt, whatever be the attractive Power of the Loadſtone, or the Proportion of their reſpective Magnitudes.

THESE Laws receive an abundant additional Proof from hence, *viz.* that all the Concluſions that are drawn from them, in Relation to the Phænomena of Bodies, how complicated ſoever their Motions be, are always found to agree perfectly with Obſervation. The Truth of which ſufficiently appears in all Parts of the *Newtonian* Philoſophy *.

CHAP. V.

The Phænomena of Falling Bodies.

I. THE Laws of Nature being thus explained, we proceed to account for ſome of thoſe Phænomena, which are ſolvable by them.

* See theſe Laws explain'd more at large by *Cheyne*, in his Principles of Philoſophy. *Keil*'s Introd. ad Phyſ. Præl. 11, 12.

II. To

II. To begin with thoſe of falling Bodies. Conſtant Experience ſhews, that Bodies have a Tendency towards the Earth, which is call'd Gravity, the Laws of which were enumerated in Chap. III. §. 7.

III. THE Height Bodies can be let fall from, bears ſo ſmall a Proportion to their Diſtance from the Center of the Earth, that it cannot ſenſibly alter their Gravity; which therefore may be conceived, as acting conſtantly and uniformly upon them, during the whole Time of their Fall: From whence they muſt neceſſarily acquire, at every Inſtant, an equal Degree of Velocity, which on that Account will conſtantly increaſe, in Proportion to the Time the Body takes up in falling.

IV. THE Spaces Bodies fall through in different Times, reckoning from the Beginning of their Fall, are as the Squares of thoſe Times; thus, a Body will fall four Times as far in two Minutes, as it does in one, and nine Times as far in three, ſixteen Times as far in four, *&c.**

V. FROM

* In order to demonſtrate this Propoſition, it will be neceſſary to lay down the following Theorem, *viz.*

That the Space a Body paſſes over, with an uniform Motion, is in a Ratio compounded of the Time and Velocity. For the longer a Body continues to move uniformly, the more Space it moves over; and the faſter it moves during any Interval of Time, the farther it goes; therefore the Space is in a Ratio compound of both, that is, is had by multiplying one into the other.

Coroll. Therefore the Area of a Rectangle, one of whoſe Sides repreſents the Celerity a Body moves with, and the other the Time of its Motion, will expreſs the Space it moves through.

This

V. FROM this Propoſition it follows, that a Body falls three Times as far, in the ſecond Portion of Time, as it does in the firſt; five Times as far in the third; ſeven Times in the fourth and ſo on in the Series of odd Numbers: For otherwiſe it could not fall four

This being premiſed, let the Line *AB* (*Fig.* 4.) repreſent the Time a Body takes up in falling, and let *BC* expreſs the Celerity acquir'd by its Fall; farther let the Line *AB* be divided into an indefinite Number of ſmall Portions, *ei*, *im*, *mp*, &c. and let *ef*, *ik*, *mn*, *pq*, &c. be drawn parallel to the Baſe. Now it is evident from §. 3. (*viz.* that the Velocities are as the Times in which they are acquir'd) that the Lines *ef*, *ik*, *mn*, *pq*, &c. being to each other (4 *El.* 6.) as the Lines *Ae*, *Ai*, *Am*, *Ap*, &c. will repreſent the Celerities in the Times repreſented by theſe: That is, *ef* will be as the Velocity of the Body in the ſmall Portion of Time *ei*, and *ik* will be as the Velocity in the Portion of Time *im*; in like Manner *pq* will be as the Velocity in the Portion of Time *po*, which Portions of Time being taken infinitely ſmall, the Velocity of the Body may be ſuppos'd the ſame, during any whole Portion: And conſequently, by the Corollary of the foregoing Theorem, the Space run over in the Time *ei* with the Velocity *ef*, may be repreſented by the Rectangle *if*: In like Manner the Space run over in the Time *im*, with the Celerity *ik*, may be expreſs'd by the Rectangle *mk*; and that run over with the Celerity *mn* in the Time *mp*, by the Rectangle *pn*; and ſo of the reſt. Therefore the Space run over in all thoſe Times will be repreſented by the Sum of all the Rectangle, that is, by the Triangle *ABC*, for thoſe little triangular Deficiencies, at the End of each Rectangle, would have vaniſhed, had the Lines *ei*, *im*, *mp*, &c. been infinitely ſhort, as the Times they were ſuppoſed to repreſent. Now as the Space, the Body deſcribes in the Time *AB*, is repreſented by the Triangle *ABC*, for the ſame Reaſon the Space paſs'd over in the Time *Ao* may be repreſented by the Triangle *Aor*; but theſe Triangles, being ſimilar, are to each other, as the Squares of their homologous Sides *AB* and *Ao* (20 *El.* 6.) That is, the Spaces repreſented by the Triangles are to each other, as the Squares of the Times repreſented by the Sides. *Q. E. D.*

Spaces

Spaces in two Minutes, and nine in three, as the Propoſition aſſerts *.

VI. The Spaces, deſcrib'd by falling Bodies in different Times, are as the Squares of the laſt acquir'd Velocities. For by §. 4. the Spaces are as the Squares of the Times, and by §. 3. the Velocities are as the Times; therefore the Spaces are alſo as the Squares of the Velocities.

VII. The Space a Body paſſes over, from the Beginning of its Fall in any determinate Time, is half what it would deſcribe in the ſame Time moving uniformly with its laſt acquir'd Velocity †.

VIII. In like Manner, when Bodies are thrown up perpendicularly, their Velocities decreaſe, as the Times they aſcend in increaſe; their Gravity deſtroying an equal Portion of their Velocity every Inſtant of their Aſcent.

* This may alſo be ſhewn in the following Manner. Let the Triangle *ABC*, (*Fig.* 4.) be divided into leſſer ones, as in *Fig.* 5. each equal to *Dbr*, which repreſents the Space deſcrib'd by the falling Body in *Db* the firſt Portion of Time; 'tis evident that, in *bc* the ſecond Portion of Time, there are three ſuch Triangles deſcribed, *viz.* thoſe that lie between the Lines *br* and *cs*; in *cd* the third Portion of Time, five ſuch, *viz.* all between *cs* and *dt*; in *df* the next equal Portion of Time, ſeven ſuch, &c.

† For let the Time be *AB*, (*Fig.* 4.) and the laſt Velocity *BC*, the Space the Body runs over, while it is acquiring that Velocity, is as *ABC*, but the Space it would paſs over in the Time *AB*, was it to move uniformly with the Celerity *BC*, is by the Theorem (Note pag. 25.) as the Space *ABCD*, double the former. *Q. E. D.*

IX. The

IX. THE Heights Bodies riſe to, when thrown perpendicularly upwards, are as the Squares of the Times ſpent from their firſt ſetting out, to the Moment they ceaſe to riſe. That is, if a Body is thrown with ſuch a Degree of Velocity, as to continue riſing twice as long as another, it ſhall aſcend four Times as high; if thrice, nine Times as high, *&c.*

THESE two are the Converſe of the third and fourth Sections *.

CHAP. VI.

Of the Deſcent of Bodies on oblique Planes, and of Pendulums.

WHEN a Body deſcends on an oblique Plane, its Motion is continually accelerated by the Action of Gravity, but in a leſs Degree, than when it deſcends perpendicularly; its free Deſcent in this Caſe being hinder'd by the Interpoſition of the Plane: From whence it follows, that what was ſaid in the laſt Chapter, concerning the perpendicular Deſcent of Bodies, is true of ſuch as fall on oblique Planes, Allowance being made for the Difference of Acceleration.

II. THE Effect which Gravity has upon a Body falling down an oblique Plane, is to that

* See *Keil*'s Introd. ad Phyſ. Præl. 11. *Graveſande*, L. 1. Chap. 17.

which

which it exerts upon another, falling freely, as the perpendicular Height of the Plane is to its Length *.

III. The Space, through which a Body falls down the oblique Side of a Plane, is to that through which it would fall perpendicularly in the ſame Time, as the perpendicular Height of the Plane is to its Length †.

For the Space a Body falls through in any determinate Time, whether down an inclined Plane, or not, is as the Effect of the Gravity with which it is acted upon during that Time; but the Gravity with which a Body deſcends down the oblique Side of a Plane (by the laſt Propoſition) is to that with which it falls perpendicularly, as the perpendicular Height of the Plane is to its Length: The Space therefore, which a Body falls through obliquely, is to

* *Dem.* Let *AC* (*Fig.* 6.) be the inclin'd Plane, the Body at *A*, and the Action of Gravity, whereby it endeavours to fall perpendicularly repreſented by the Line *AB*; let *AD* be perpendicular to *AC*, *AD* will then repreſent the Direction by which the Plane acts upon the Body (for all Bodies act in Lines perpendicular to their Surfaces) let then thoſe two Forces be reſolved into one in the Direction *AC*, (as ſhewn in Note to §. 4. Chap. IV.) by compleating the Parallelogram *BD*, whoſe Diagonal will be *AG*. In order to this, *BG* muſt be let fall perpendicularly upon *AC* (that it may be parallel to the oppoſite Side of the Parellelogram *AD*) conſequently (8 *Elem.* 6.) *AG* is to *AB* as *AB* to *AC*, that is, the Tendency of the Body down the Plane is to its perpendicular Tendency, as *AB* is to *AC*. Q. E. D.

† From this Propoſition it follows, that ſuppoſing *BG* (*Fig.* 6.) perpendicular to *AC*, the Body would fall from *A* to *G*, in the ſame Time another would fall from thence to *B*, for, as was obſerved (Note the laſt) *AG* is to *AB*, as *AB* to *AC*.

that

that which it would paſs through perpendicularly in the ſame Time, alſo in that Proportion.

IV. THE Velocity which a Body acquires by falling perpendicularly, is to that which it acquires by falling obliquely, in the ſame Time, as the Space of its perpendicular Deſcent is to that of its oblique one *.

V. THE Time, in which a Body deſcends through the oblique Side of a Plane, is to that in which it falls through the perpendicular Height of the ſame, as the Length of the oblique Side is to its Height †.

VI. A BODY acquires the ſame Velocity in falling down the oblique Side of a Plane, as

* Since by the Note to the laſt Section, a Body falls to *G*, (*Fig.* 6.) in the ſame Time another falls to *B*, and (by Chap. V. §. 7.) the Space a falling Body paſſes over in any Time, is half that which it would run over in the ſame Time moving uniformly with its laſt acquir'd Velocity, it follows that the Body falling down the oblique Plane would paſs over double the Space *AG*, moving uniformly with its laſt acquir'd Velocity, in a Portion of Time equal to that in which it was acquir'd; likewiſe double the Space *AB* would be paſſed over by the other Body moving uniformly with its laſt acquir'd Velocity, in a Portion of Time equal to that in which it was acquir'd; but ſince the Velocities of Bodies moving uniformly are as the Spaces they run over in equal Times, the Velocity of the Bodies in *G* and *B* are to each other as double the Lines *AG* and *AB*, that is, as the Lines themſelves, which by §. 3. are as the Spaces run through in the ſame Time, from whence the Propoſition is clear.

† *Dem.* The Square of the Time in which *AC* (*Fig* 6.) is run over, is to the Square of the Time in which *AG* is run over as *AC* to *AG*, (by Chap. V. § 4.) that is, ſince *AC*, *AB*, *AG* are continually proportional (8 *Elem.* 6.) as the Square of *AC* to the Square of *AB* (by *Def.* 10. *Elem.* 5.) therefore the Times themſelves are as the Lines *AC* and *AB*, that is, as the oblique Side of the Plane to the perpendicular Height. *Q. E. D.*

it

it would do, if it fell freely through the perpendicular Height of it *.

VII. A Body takes up the ſame Time in falling through the Chord of a Circle, whether it be long or ſhort, as it does in falling perpendicularly through the Diameter of the ſame Circle †.

VIII. UPON this is founded the Theory of Pendulums: For from hence it follows, that ſuppoſing a Pendulum could be made to vibrate in a Chord of a Circle, inſtead of an Arch, all its Vibrations would require the ſame Time, whether they were large or ſmall ‡.

IX. FROM hence we ſee the Reaſon, why the ſhorter Arches a Pendulum deſcribes, the

* *Dem.* The Square of the Velocity which a Body acquires by falling to *G*, is to the Square of the Velocity it acquires by falling to *C*, as the Space of *AG* to the Space *AC* (by Chap. V. §. 4.) that is (by 8 *Elem.* 6. and *Def.* 10. *Elem.* 5.) as *AGq* to *ABq*; conſequently the Velocity itſelf at *G* is to the Velocity itſelf at *C*, as *AG* to *AB*: But ſince *AG* is run over in the ſame Time *AB* is (ſee Note to §. 3.) the Velocity in *G* is alſo to the Velocity in *B*, as *AG* to *AB*, (by §. 4.) and conſequently ſince the Velocities both in *C* and *B* bear the ſame Proportion to that in *G*, they muſt be equal to each other. *Q. E. D.*

† *Dem.* It was demonſtrated (§. 3.) that a Body will fall from *A* to *G*, (*Fig.* 7.) on the inclin'd Plane *AC*, in the ſame Time another would fall freely to *B*, provided *AGB* is a right Angle, in which Caſe *AG* (by 31 *Elem.* 3.) is a Chord of that Circle of which *AB* is the Diameter; therefore a Body falls through the Chord, *&c.* Q. E. D.

‡ This may be illuſtrated by conceiving the laſt Figure inverted (as in *Fig.* 8.) where ſuppoſing the Ball ſuſpended in ſuch a Manner, as to ſwing in the right Line *GA* inſtead of the Arch *GA*, it would always fall through it in the ſame Time, however long or ſhort it was, for the Inclination of the Line *GA* to the horizontal Line *BC*, is not alter'd by inverting the Figure.

nearer

nearer its Vibrations come to an Equality, for ſmall Arches differ leſs from their Chords than large ones. But if the Pendulum is made to vibrate in a Curve, which Mathematicians call a *Cycloid*, each Swing will then be perform'd in the ſame Time, whether the Pendulum moves through a larger or leſſer Space. For the Nature of this Curve is ſuch, that the Tendency of a Pendulum towards the loweſt Point of it, is always in Proportion to its Diſtance from thence; and conſequently let that Diſtance be more or leſs, it will always be run over by the Pendulum in the ſame Time*.

X. The Time of the Deſcent and Aſcent of a Pendulum, ſuppoſing it to vibrate in the Chord of a Circle, is equal to the Time in

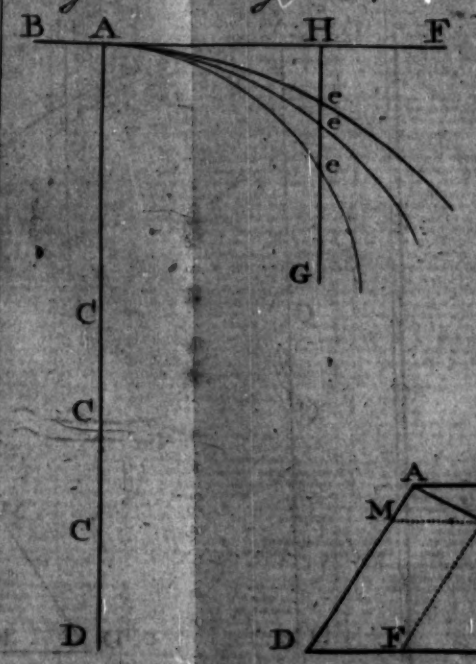
* The Deſcription of a *Cycloid*.

Upon the right Line *AB*, (*Fig.* 9.) let the Circle *CDE* be ſo placed, as to touch the Line in the Point *C*, then let this Circle roll along upon it from *C* to *H*, as a Wheel upon the Ground, then will the Point *C* in one Revolution of the Circle deſcribe the Curve *CKH*, which is called a Cycloid. Now ſuppoſe two Plates of Metal bent into the Form *HK* and *KC*, and placed in the Situation *LH* and *LC*, in ſuch Manner, that the Points *H* and *C* may be apply'd to *L*, and the Points anſwering to *K* be apply'd to *H* and *C*. This done, if a Pendulum as *LP*, in Length equal to *LH*, be made to vibrate between the Plates or Cheeks of the Cycloid *LC* and *LH*, it will ſwing in the Line *CKH*; and the Time of each Vibration, whether the Pendulum ſwings through a ſmall or a great Part of the Cycloid, will be to the Time a Body takes up in falling perpendicularly through a Space equal to *IK*, (half the Length of the Pendulum) as the Circumference of a Circle to its Diameter, and conſequently it will always be the ſame. See theſe Things demonſtrated in the next Chapter.

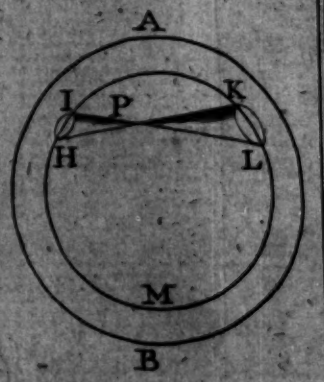
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Part I. Page 32.

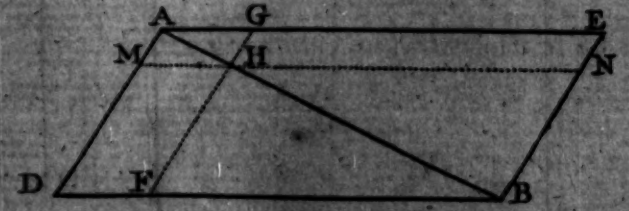
Figure 1. Page 8.



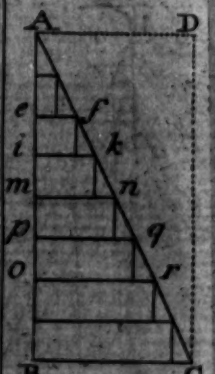
F. 2. P. 16.



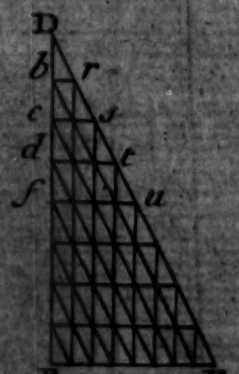
F. 3. P. 21.



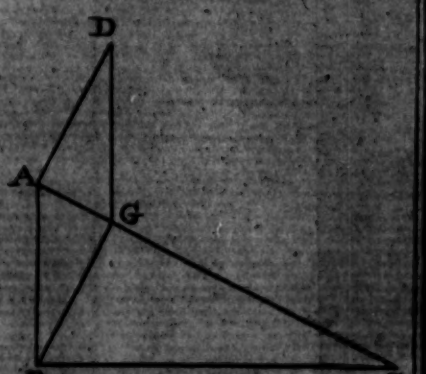
F. 4. P. 26

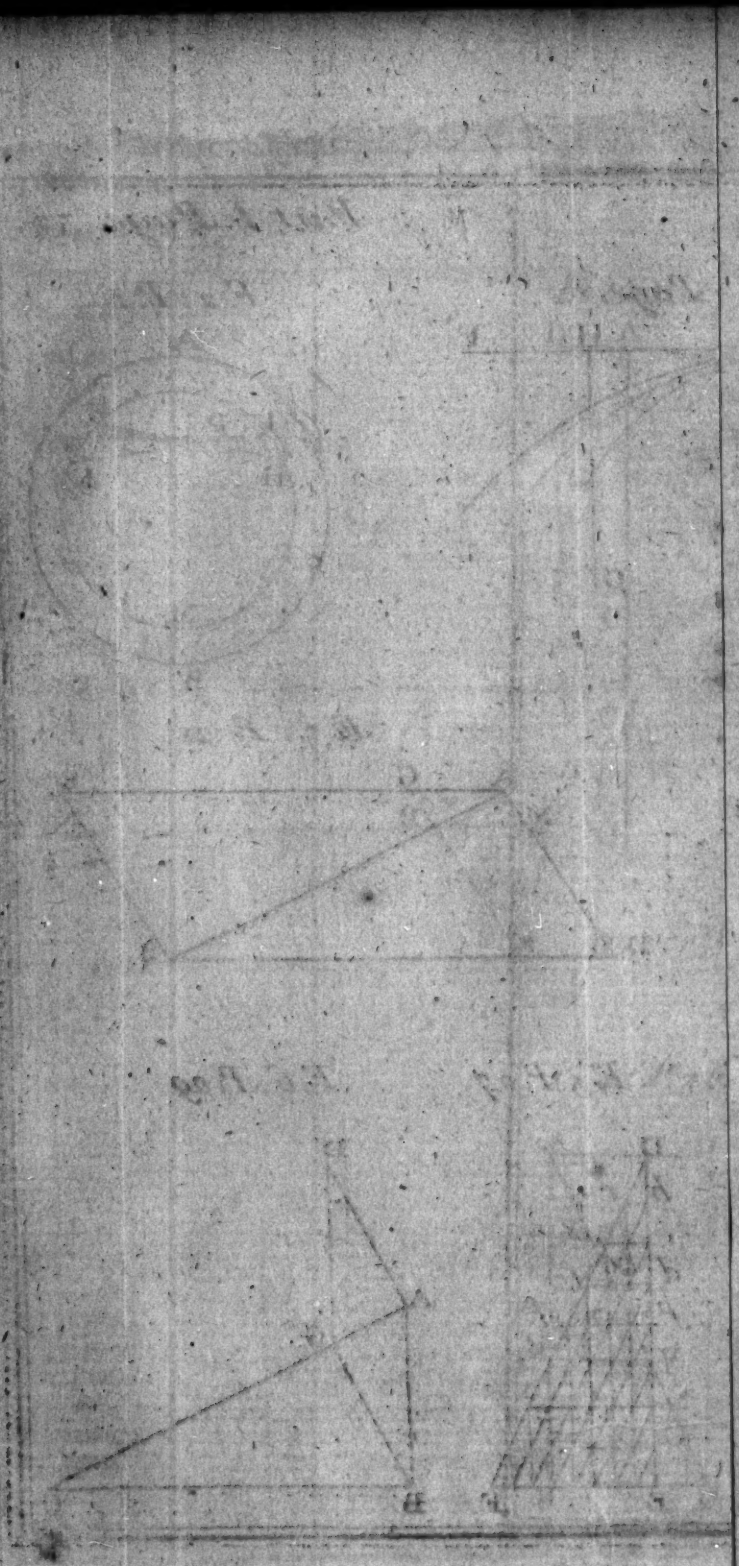


F. 5. P. 27.



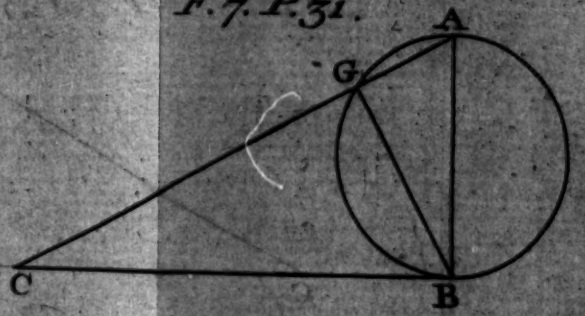
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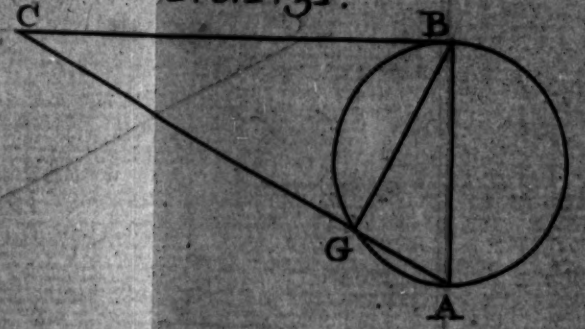


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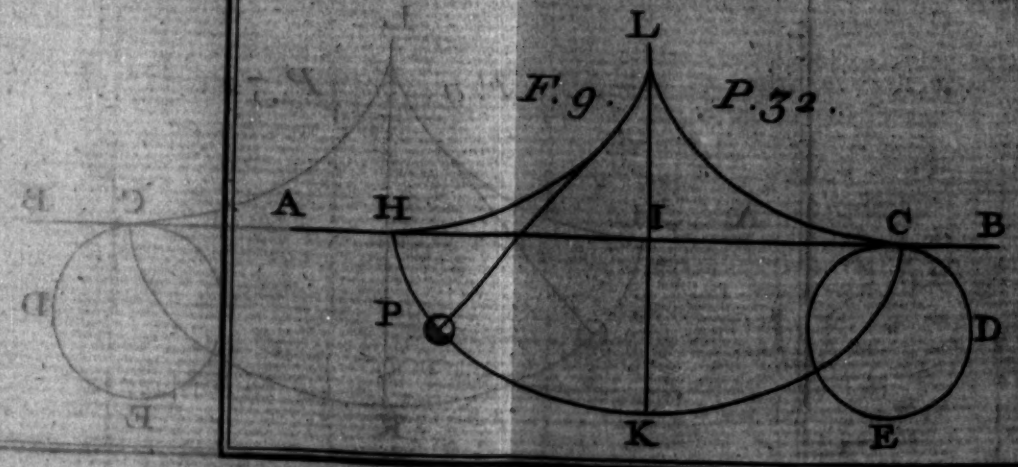
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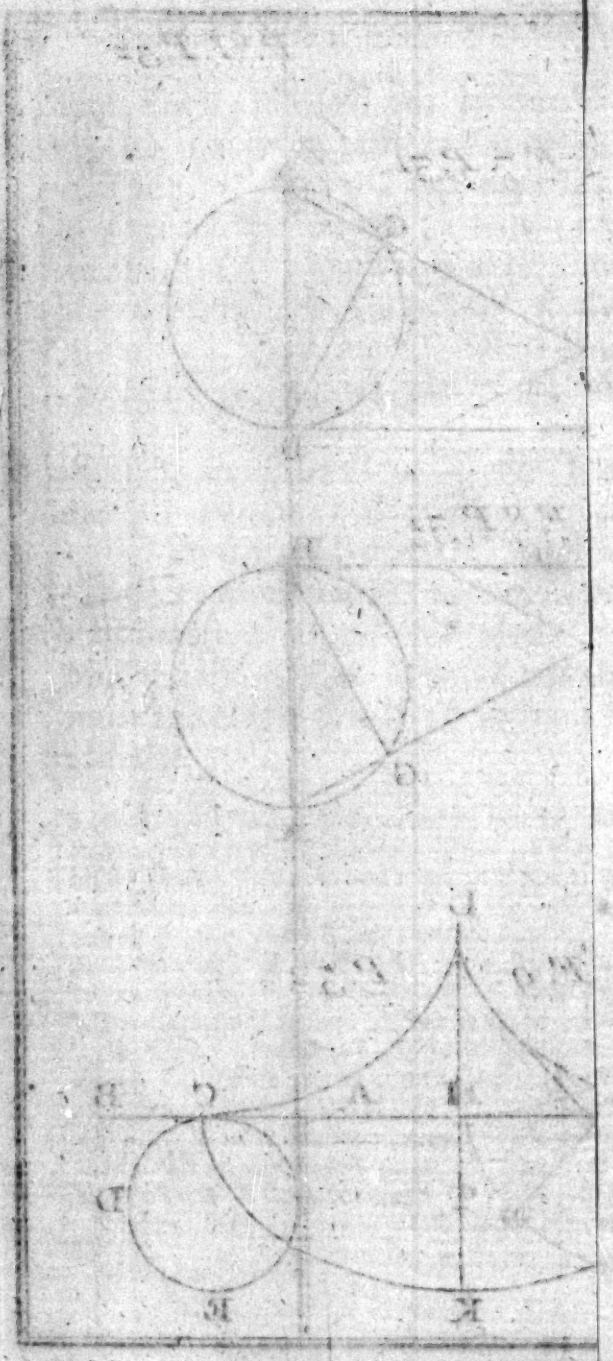


F. 8. P. 31.



F. 9. P. 32.




L
C
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which a Body falling freely would deſcend through eight Times the Length of the Pendulum.

For the Time of the Deſcent alone upon the Chord is equal to that in which a Body would fall through the Diameter of the Circle (by §. 7.); that is, twice the Length of the Pendulum: But in twice that Time (*viz.* during a whole Vibration) the body would fall four Times as far (Chap. V. §. 4.) that is, through eight Times the Length of the Pendulum.

XI. The Times, that Pendulums of different Lengths perform their Vibrations in, are as the ſquare Roots of their Lengths *.

XII. The Center of *Oſcillation*, is a Point in which if the whole Gravity of a Pendulum was collected, the Time of its Vibration would not be alter'd thereby †; this is the Point from whence

* *Dem.* Let there be two Pendulums *A* and *B* (*Fig.* 10. and 11.) of different Lengths, the Time the firſt vibrates in (ſuppoſe through a Chord) is equal to the Time in which a Body would fall freely through *DA*, the Diameter of the Circle (as demonſtrated §. 7.); in like Manner the Time *B* vibrates in, is that in which a Body would fall through *FB*. Now the Times in which Bodies fall through different Spaces are as the ſquare Roots of thoſe Spaces, that is, of *DA* and *FB*, or of their Halves *CA* and *CB*, *i. e.* of the Lengths of the Pendulums. *Q. E. D.*

† If the Ball *AB* (*Fig.* 12.) be hung by the String *CD*, whoſe Weight is inconſiderable, the Center of Oſcillation is found thus; ſuppoſe *E* the Center of the Globe, take the Line *K* of ſuch a Length, that it ſhall bear the ſame Proportion to *ED* as *ED* to *EC*, then *E H* being made equal to $\frac{2}{5}$ of *K*, the Point *H* ſhall be the Center of Oſcillation.

whence the Length of a Pendulum is meaſured, which in our Latitude, in a Pendulum that ſwings Seconds, is thirty-nine Inches and two Tenths.

XIII. THE Squares of the Times in which Pendulums, acted upon by different Degrees of Gravity, perform their Vibrations in, are to each other, inverſly as the Gravities *.

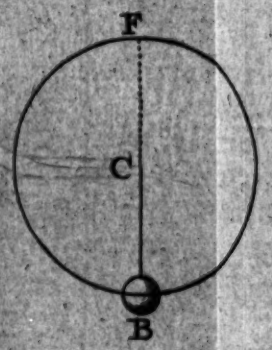
FROM hence it follows, that a Pendulum will vibrate ſlower when it is at, or near the Equator, than it will do when nearer the Poles; for the Gravity of all Bodies is leſs, the nearer

If the Weight of the Rod *CD* be too conſiderable to be neglected, divide *CD* (*Fig.* 13.) in *I*, ſo that *DI* may be equal to $\frac{1}{3}$ of *CD*, and make a Line as *G*, in the ſame Proportion to *CI*, that the Weight of the Rod bears to that of the Globe, then having found *H* the Center of Oſcillation of the Globe, as before, divide *IH* in *L*, ſo that *IL* may bear the ſame Proportion to *LH*, as the Line *CH* bears to the Line *G*; then will *L* be the Center of Oſcillation of the whole Pendulum. See the Method of finding a general Rule for determining the Center of Oſcillation in all Caſes whatever, in Chap. VIII. §. 1.

* *Dem.* The Spaces falling Bodies deſcend through, are as the Squares of the Times, when the Gravity by which they are impell'd is given (Chap. V. §. 4.); and as the Gravity, when the Time is given (for the Sum of the Velocities produced in any Time will always be as the generating Forces:) Conſequently, when neither is given, they are in a Ratio compounded of both; the Squares of the Times are therefore inverſly as the Gravities. [*For if in* 3 *Quantities* a, b, c; a *is as* bc, *then* b : $\frac{a}{c}$, i. e. *if* a *is given, as* $\frac{1}{c}$ *or as* c *inverſly.*] But if the Squares of the Times, in which Bodies fall through given Spaces be inverſly as the Gravities by which they are acted upon; then the Squares of the Times, in which Pendulums of equal Lengths perform their Vibrations, will be alſo in the ſame Ratio, on Account of the conſtant Equality between the Time of the Vibration of a Pendulum, and of the Deſcent of a Body through eight Times its Length, (§. 12.)

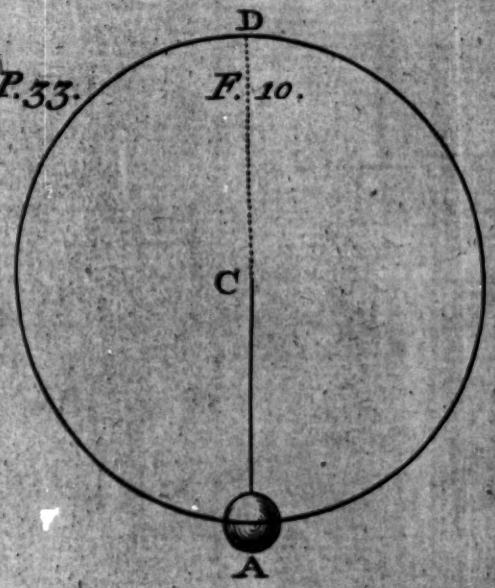
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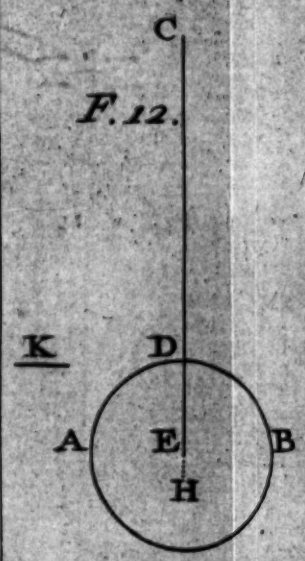
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F. 10.

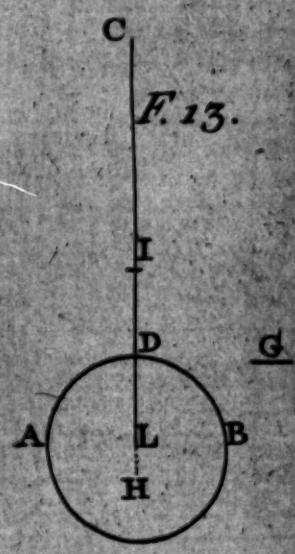


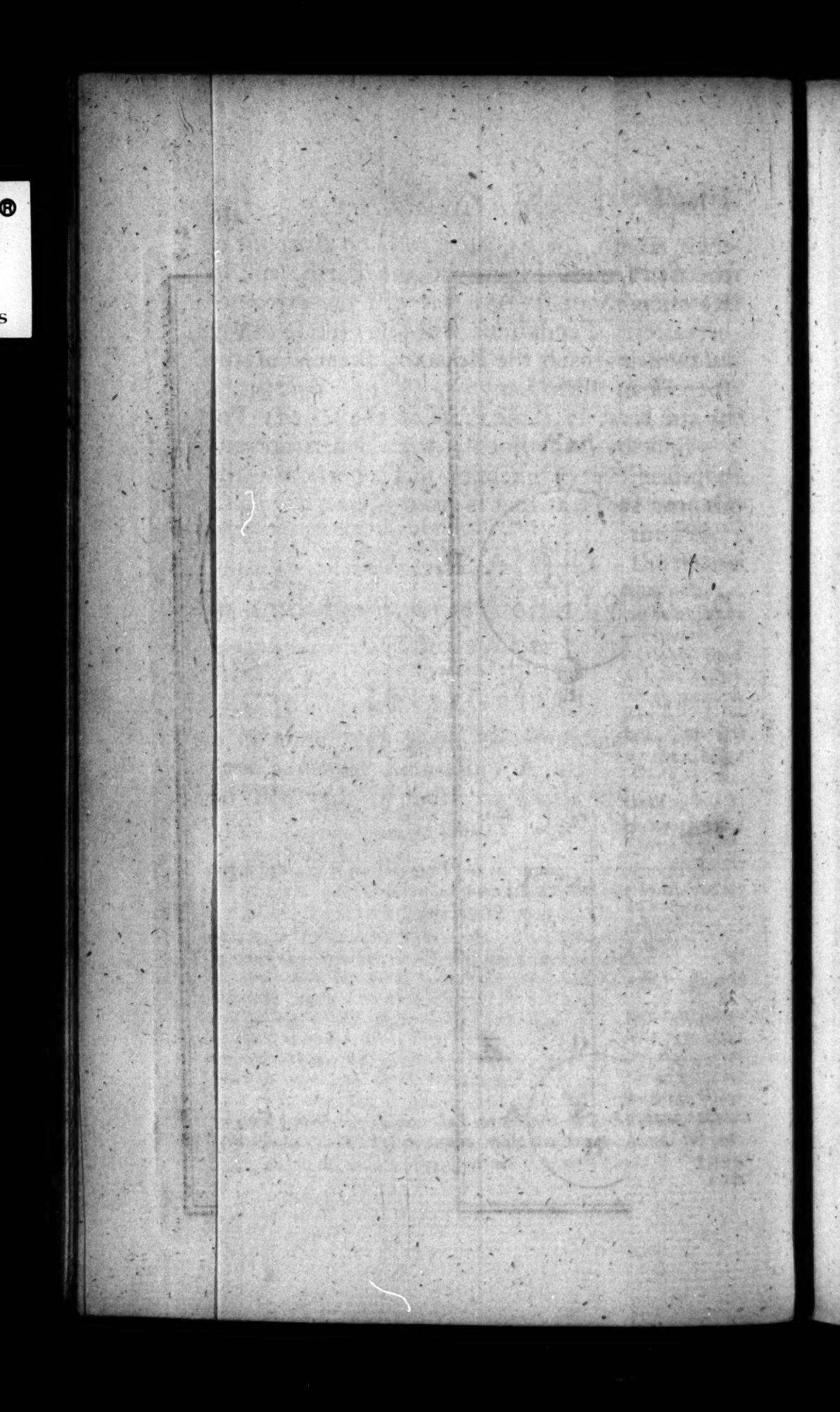


F. 12.

P. 34.

F. 13.





they are to the Equator, *viz.* on Account of the ſpheroidical Figure of the Earth, and its Rotation about its Axis, as will be explained hereafter. Pendulums will alſo vibrate ſlower as they approach the Equator, becauſe of the Increaſe of their Lengths, (§. 11.) occaſioned by the Heat in thoſe Parts of the Earth: For we find by Experiment, that Bodies are enlarged in every Dimenſion in Proportion to the Degree of Heat that is given them.

CHAP. VII.

Of the Vibration *of a* Pendulum *in a* Cycloid.

Proposition I.

IF a Pendulum be made to vibrate in a Cycloid, all its Vibrations, however unequal, will be *iſocronous*; that is, they will be performed in equal Times (*a*).

(*a*) In order to demonſtrate this Propoſition, it will be proper to lay down the following Lemma's.

LEMMA I.

If a Body deſcends from A, *along the Line* AX, (*Fig.* 14.) *by Virtue of a Force which decreaſes in Proportion as the Diſtance of the Body from* X *decreaſes; that is, if when the Body comes to* M, N, O, &c. *the Action of that Force upon the Body, be as the Diſtances* XM, XN, XO, &c. *reſpectively: And if the laſt acquired Velocity of the Body; that is, its Velocity when it comes to* X, *be expreſſed, or ſet off, by the Perpendicular* XB, *equal in Length to the Line* AX, *and its Velocities at* M, N, O, &c. *be ſet off there by the Lines* MD, NP, OQ, &c. *in Length proportional to each other, and to the Line* XB, *as the Velocities of the falling Body at* M, N, O, &c. *are to each other, and to its laſt Velocity at* X: *And*

if through the Extremities of theſe Lines, the Curve ADB *be drawn; I ſay, that Curve will be a Portion of a Circle: And the Time in which the Body will deſcend through the whole Space or Line* AX [*or any Part of it, as* MO] *will be ſuch Time as would be requiſite for it to deſcribe the whole Arch* AB [*or any Part as* DQ, *correſponding to* MO] *in, with its laſt acquired Velocity at* X.

Demonſtration of the Lemma. Parallel and contiguous to the Line MD, draw NP, in which Caſe the Line MN becomes a Point, and the Arch DP a Tangent to the Curve: Produce PD till it meets XA, produced, in T; draw the Line XD; and let fall the Perpendicular DL. Then the Lines DL and TM being parallel, the Angles PDL and DTM are equal, as being alternate (by 27 Elem. 1.); and the Angles at L and M, as being right ones; the Triangles therefore PDL and DTM are ſimilar, which for the Sake of referring to it afterwards, let us make the firſt Step of the following Proceſs

	1	The Triangles PDL and DTM are ſimilar
From the firſt Step we have this Proportion (5 Elem. 6.)	2	PL : LD : : DM : MT
By the Figure	3	LD = MN
But MD being the Velocity when the deſcending Body comes to M, the Point MN is deſcribed with that Velocity (for there is no Acceleration during the Paſſage of a Body over a Point) conſequently MN is proportional to MD; that is,	4	MN is as MD.
Comparing the ſecond, third, and fourth Steps	5	PL : MD : : DM : MT
But MD and NP being the Velocities of the deſcending Body at M and N, LP the Difference of thoſe Lines, expreſſing the Increaſe of Velocity in the Body, will be proportional to the moving Force at the Point MN; that is, by the Suppoſition, to the Diſtance XM; therefore	6	PL is as XM
Comparing the fifth and ſixth	7	XM : MD : : DM : MT
Conſequently (5 Elem. 6.)	8	The Triangles XDM and DMT are ſimilar.

And therefore, ſince their Angles at M are right ones, the Triangle TDX is (by the Converſe of Prop. 8. Elem. 6.) right angled at D. Conſequently, ſince the ſame is true of any other Point

Point of the Curve, as well as D, the Arch ADB is a Portion of a Circle (16 Elem. 3.) Which is the firſt Part.

Secondly, comparing the firſt and eighth Steps, the Triangles PDL and XMD are ſimilar; therefore	9	LD : DM : : DP : DX
ADB being a Portion of a Circle, as already proved	10	DX = XB
Comparing the 3d, 9th and 10th Steps	11	MN : DM : : DP : XB.

Since then the Point MN bears the ſame Proportion to MD, or the Velocity it is deſcribed with by the falling Body, that the Point DP does to the laſt acquired Velocity XB, it follows that the former, MN, is deſcribed in the ſame Time with the Velocity the Body has when there, that the latter, DP, might be with the laſt acquired Velocity XB. And ſince the ſame is true of every other Part of the Arch ADB, it is obvious that the Time in which the Body will deſcend through any other Part of the Space AX, [or the whole of it,] will be ſuch as would be required for it to deſcribe any correſponding Part of the Arch ADB, [or the whole of it,] with the laſt acquired Velocity XB. Which was the other Part.

Coroll. Hence it follows, that if a Body deſcends along the Line AX, by Virtue of Forces acting upon it at A, M, N, O, *&c.* proportional to the Length of the Lines XA, XM, XN, XO, *&c.* and if on X, as the Center, with the Radius XA, a Portion of a Circle, as ADB, be deſcribed; and if the Radius or whole Sine XB, be put to repreſent the Velocity of the Body when it comes to X, the other Sines MD, NP, OQ. *&c.* will repreſent the reſpective Velocities of the Body at the ſeveral Points M, N, O, *&c.* And converſely, if one of the Sines, as MD, be put to expreſs its Velocity at M, the other Sines NP, OQ. and the Radius, or whole Sine XB, will expreſs the Velocity of the Body at thoſe other Points N, O and X.

LEMMA II.

If a Body moves along the Line AX, *(Fig.* 15.) *and be urged all the Way by Forces proportional to its Diſtance from the Point* X; *whatever Point of that Line it ſets out from, it will come to the Point* X *in the ſame Time. Which Time will bear ſuch Proportion to the Time it would move over the whole Line* AX *in, with the Velocity it ſhall acquire by falling through the whole Line* AX, *as the Semi-circumference of a Circle does to its Diameter.*

Dem. Let two Bodies, A and P, ſet out from the Points, A and P, at the ſame Time; and let them be urged by Forces proportional to their Diſtances from the Point X: I ſay, thoſe Bodies will come to X at the ſame Inſtant of Time; that is, they will overtake one another at that Point. On X as a Center, and with the Radii XA and XP, deſcribe the two Quadrants AB and PQ; and draw the Line SX, and the Sines RS and MN; and let the whole Sine, or Radius XB, expreſs the Velocity the Body A will acquire by falling to X: Then, by Corollary to Lemma 1, will the Sine RS, if taken as near as poſſible to A, expreſs the firſt Velocity of the Body A. But the Force, which urges the Body A is ſuppoſed to be to that which urges the Body P, as XA to XP (or becauſe the Archs AS and PN are ſimilar) as RS to MN: As therefore RS expreſſes the firſt Velocity of A, MN will expreſs the firſt Velocity of the other Body P: And therefore, by the ſame Corollary, its Velocity, when it comes to X, will be expreſſible by XQ. Farther, the Time the Body A falls to X in, is by Lemma 1. equal to the Time the Arch AB would be deſcribed in with the Velocity XB; and the Time the other Body falls from P to X in, is equal to the Time the Arch PQ would be deſcribed in with the Velocity XQ. But a Body will be as long in moving over the Arch PQ with the Velocity XQ, as over the Arch AB with the Velocity XB, the Lines XQ and XB having the ſame Proportion to each other, that the Archs have. Therefore the Time the Body A falls to X in, is equal to the Time the other Body P would fall to that Place in. Which was the firſt Part.

Again by Lemma 1.	1	The Time a Body would fall from A to X in, is equal to the Time it would move over the Arch AB in, with its laſt acquired Velocity at X.
Axiom, or ſelf evident Propoſition	2	The Time a Body would move over the Arch AB in with the laſt acquired Velocity at X, is to the Time it would move over AX in with the ſame Velocity, as AB is to AX.
Comparing the firſt and ſecond	3	The Time a Body would fall from A to X in, is to the Time it would move over AX in with the laſt acquired Velocity, as AB is to AX.

Axiom

Axiom	4	AB is to AX as twice AB is to twice AX.
By the Figure	5	Twice AB is to twice AX as the Semicircumference of a Circle is to its Diameter.
Comparing the 3d, 4th, and 5th Steps	6	The Time a Body would fall from A to X in, is to the Time it would move over AX in with its laſt acquired Velocity, as the Semicircumference of a Circle is to its Diameter. Which was the ſecond Part.

LEMMA III.

If from the lowermoſt Point of a Circle, as X, (*Fig.* 16.) *be drawn the Chords* XQ *and* XO, *the Power of Gravity whereby it ſhall cauſe a Body to deſcend along the former, will be to the Power whereby it ſhall cauſe it to deſcend along the latter, as the Length of the former is to the Length of the latter.*

Dem. Draw the Diameter XD, the Perpendiculars QR, and OS; and join the Points QD and OD. Then (by 31 Elem. 3.) the Triangle XQD is right-angled at Q; and therefore (by 8 Elem. 6.)

	1	XR : XQ :: XQ : XD.
And for like Reaſons	2	XS : XO :: XO : XD.
But by Chap. 6. §. 2.	3	The Effect or Power of Gravity upon a Body deſcending along the Chord QX, is to that which it exerts upon another falling freely; that is, to its whole Power, as XR to XQ.
And alſo	4	The Power of Gravity upon a Body, deſcending along a Chord OX, is to its whole Power, as XS to XO.
Comparing the 1ſt and 3d	5	The Power of Gravity upon a Body deſcending along the Chord QX, is to its whole Power, as XQ to XD.
Comparing the 2d and 4th	6	The Power of Gravity upon a Body deſcending along the Chord OX, is to its whole Power, as XO to XD.

Comparing the 5th and 6th Steps	7	The Power of Gravity upon a Body descending along the Chord QX, is to the Power of Gravity upon a Body descending along the Chord OX, as XQ to XO. Q. E. D.

The Description of a Cycloid, with the Definitions relating thereto. If a Circle as FCH (Fig. 17.) be rolled along the Line AB, till it has turned once round; the Point C in its Circumference, which at first touched the Line at A, will describe the Curve Line ACXB, which Curve is called a *Cycloid.* The right Line AB is its *Base:* The middle Point X is its *Vertex:* And a Perpendicular, as XD, let fall from thence to the Base, is its *Axis:* And the Circle FCH, or any other as XGD, equal thereto, is called the *Generating Circle.*

LEMMA IV.

If on XD, (*Fig.* 17.) *the Axis of the Cycloid, as a Diameter, the generating Circle* XGD *be described; and if from a Point in the Cycloid, as* C, *the Line* CIK *be drawn Parallel to the Base, the Portion of it* CG, *will be equal to the Arch* GX.

Dem. Draw the Diameter HF, then the Circles FCH and

DGX being equal	1	KG = CI
Adding GI to each of them	2	KI = CG
By the Figure	3	KI = DF
Comparing the two last	4	CG = DF
By the Description of the Cycloid	5	The Arch CF = AF
By the Figure	6	The Arch CF = DG
Comparing the 5th and 6th	7	AF = DG
By the Description of the Cycloid	8	AFD = DGX
Comparing the 7th and 8th with the Figure	9	FD = GX
Comparing the 4th and the 9th	10	CG = GX. Q. E. D.

LEMMA V.

The same Things being supposed as in the foregoing Lemma, a Tangent to the Cycloid at the Point C, (*Fig.* 17.) *is parallel to* GX *a Chord of the Circle* DGX.

Dem. It appears from the Description of the Cycloid, that since the Angle FCH is a right one, (as it is by 31 Elem. 3.) the Chord CH is a Tangent to the Curve at the Point C, but

CH

CH is parallel to GX; a Tangent therefore at the Point C, is parallel to GX, the Chord of the Circle DGX. Q. E. D.

LEMMA VI.

Things remaining as before, if from a Point of the Cycloid, as L, (*Fig.* 17.) *the Line* LMK *be drawn parallel to the Baſe* AB, *the Arch* XL *of the Cycloid, will be double of* XM, *the Chord of the Circle correſponding thereto.*

Dem. Draw the Line SQ parallel and contiguous to LK, croſſing the Circle in R, and the Chord XM, produced, in P, then will LS, MR and MP become Points, the firſt having the Property of a Tangent to the Cycloid at LS, the ſecond that of a Tangent to the Circle at MR, and the third, the Properties of a Production of the Chord XM. Join the Points X and R, and on MP let fall the Perpendicular RO: Produce alſo the Point RM, till it meets XN, a Tangent to the Circle at X. Then will the Lines XN and QS, being each perpendicular to the Diameter DX, be parallel; and the Triangles MNX and MPR will be ſimilar; as having their Angles at M vertical, and at P and X alternate. But the Tangents NX and NM are equal (by 36 Elem. 3.) the correſponding Lines therefore, PR and RM, in the other Triangle, are ſo too: This laſt Triangle is therefore an Iſoſceles one; and therefore RO being perpendicular to its Baſe MP, MP is equal to twice MO. The Tangent LS is parallel to MP, (as being by Lemma 5. parallel to MX) and therefore equal to it, the Lines LK and SQ being parallel: It is therefore equal alſo to twice MO. But LS is the Difference between the cycloidal Archs XL and XS; and MO is the Difference between the Chords XM and XR, for ſince XO and XR are cloſe together, RQ which is perpendicular to one of them, may be conſidered as perpendicular to both: The Difference therefore between any two Arches of the Cycloid is twice that which is between two correſponding Chords of the Circle; and conſequently any Arch, as XL, is double of the correſponding Chord XM. Q. E. D.

Coroll. Since, when the Arch XL becomes XB, the correſponding Chord XM becomes XD, the Diameter of the Circle DMX; it is obvious, that the Semicycloid BX, or AX, is equal to twice DX the Diameter of the generating Circle DMX.

LEMMA VII.

If a Body deſcends in a Cycloid, the Force of Gravity (ſo far as it acts upon it in cauſing it to deſcend along the Cycloid) will be

Proposition II.

The Time in which a Pendulum, vibrating in a Cycloid, performs a Vibration, is to the

be proportional to the Distance of the Body from the lowest Point of the Cycloid.

Dem. Let the Cycloid be AXB (Fig. 18.) whose Base is AB, and its Axis DX, on which last, as a Diameter, describe the generating Circle DQX: Draw the Chords OX and QX; through the Points O and Q, and parallel to the Axis AB, draw the Lines LS and MR; draw also the Tangents LV and MY. Then because by Lemma 5. the Tangent LV is parallel to OX, and the Tangent MY parallel to QX, it's obvious that Gravity exerts the same Power or Force upon a Body descending in the Cycloid at L (because it then descends in the Tangent LV) as it would do upon the same Body descending along the Chord OX: And for the like Reason, it exerts the same Force upon it when it comes to M, that it would do if it were descending along QX: But (by Lemma 3.) the Power or Force of Gravity upon Bodies descending along the Chords OX and QX, are as the Lengths of those Chords; that is, by Lemma 6, (halves being proportional to their wholes) as the Length of the Cycloidal Archs LX and MX. The Force therefore of Gravity upon a Body descending in the Cycloid at the Point L [or any other] is to its Force upon the same, when at M [or any other Point] as the Space or Distance it has to move over in the former Case, before it gets to the lowest Point X, to that it has to run over in the latter, before it arrives at the same Point. Q. E. D.

Demonstration of the Proposition in the Text to which this Note refers.

By Lemma 7. The Force of Gravity, so far as it causes a Body to descend in a Cycloid, is proportional to the Distance of that Body from the lowest Point; imagine then that Body to be a Pendulum vibrating in the Cycloid, then whatever Point it sets out from, it will, by Lemma 2, come to the lowest Point in the same Time: And consequently, since the like is true as to its ascending from that Point, all its Vibrations, be they large or small, will be performed in the same Time. Q. E. D.

Time

Time in which a Body would fall freely thro' half the Length of the Pendulum, as the Circumference of a Circle is to its Diameter (*b*).

PROB.

(*b*) To demonſtrate this, the following Lemma's will be of Uſe.

LEMMA VIII.

If in a right-angled Triangle, as BFG, *(Fig.* 19.) *the Perpendicular* FI *be let fall from the right Angle to the Hypothenuſe* BG, *the Line* BI *multiplied by* BG *will be equal to* BFq.

Dem. By 8 Elem. 3. the Triangles BFI and BFG are ſimilar, conſequently BI is to BF, as BF is to BG; and therefore BI × BG = BF^q. Q. E. D.

LEMMA IX.

If a Body deſcends along a Curve Line, as AX (*Fig.* 20.) *it will acquire the ſame Velocity that another, or the ſame Body would do, by falling from an equal perpendicular Height in the Line* DX.

Dem. Parallel to the horizontal Line AD, draw the Lines BM and FN contiguous to each other; in conſequence of which, the Lines MN and BG may be conſidered as Points; and therefore the Velocity with which the deſcending Bodies paſs over them, as uniform; and the curve Line BG as a ſtraight Line alſo, and as a Tangent to the Curve AX at the Point BG. Things being thus, let it be ſuppoſed that the Bodies begin their Fall at B and M, or, which comes to the ſame Thing, that they have equal Velocities at thoſe Points: Then the Velocities of the Bodies being uniform and equal to each other, (for there is no Acceleration in a Point) the Lines BG and MN may repreſent the Relation which the Times they are paſſed over in bear to each other. Parallel to DX draw BF, and let the equal Lines BF and MN repreſent the Force of Gravity acting perpendicularly at thoſe Points; and let the Force BF be reſolved into two others, *viz.* BI and IF, the one parallel, the other perpendicular to the Courſe of the Body at B: It is only the former of theſe, *viz.* BI, that accelerates the Body along the Curve BG; the other, *viz.* IF, neither accelerates it nor retards it, but is wholly ſpent in preſſing the Body cloſe to the Surface BG, if it be a Surface; or in ſtretching the String which keeps the Body in the Courſe ABX, if it be a String. Now the Velocity which a Body acquires by moving over any Space, is proportional to the

Force

Force that acts upon it, multiplied by the Time that Force acts. Since then BI represents the Force in one Case, and MN the Force in the other; and BG the Time in one Case, and MN the Time in the other; it follows that the Velocity generated in one Case, is as BI × BG; and in the other, as MN × MN; or since BF and MN are equal, as the Quantities BI × BG and BF × BF, (or BF^2) which Quantities, by Lemma 8, are equal to each other. The Velocity therefore which the one Body acquires by descending along BG, is equal to that which the other acquires by falling through MN: But the Lines BM and GN being parallel, it is obvious there is the same Number of BG's in the Curve AX, as of MN's in the perpendicular DX; the Velocity therefore which a Body would acquire by falling through one, is equal to that which it would acquire in falling through the other. Q. E. D.

Demonstration of the Proposition. Let AXB (Fig. 18.) be the Cycloid which the Pendulum vibrates in. Then by Lemma 2, compared with Lemma 7, we have

	1	The Time a Body would descend from A to X in, is to the Time it would move over the same Space in with its last acquired Velocity, as the Semicircumference of a Circle is to its Diameter.
By the Corol. of Lemma 6.	2	AX is equal to twice DX.
By Lemma 9.	3	The Velocity a Body acquires by falling from A to X, is equal to the Velocity it would acquire by falling from D to X.
From the three last compared	4	The time in which a Body would descend from A to X is to the Time it would move over twice DX with the Velocity acquired by a Fall from D to X, as the Semicircumference of a Circle is to its Diameter.
By Chap. 5. §. 7.	5	The Time in which a Body would move over twice DX with the Velocity acquired by falling from D to X, is equal to the Time it would fall from D to X in.
Comparing the 4th and 5th	6	The Time in which a Body would descend from A to X, is to the Time it would fall from D to X, as the Semicircumference of a Circle is to its Diameter.

From

P.t I. P. 44.

Fig. 14 P. 35.

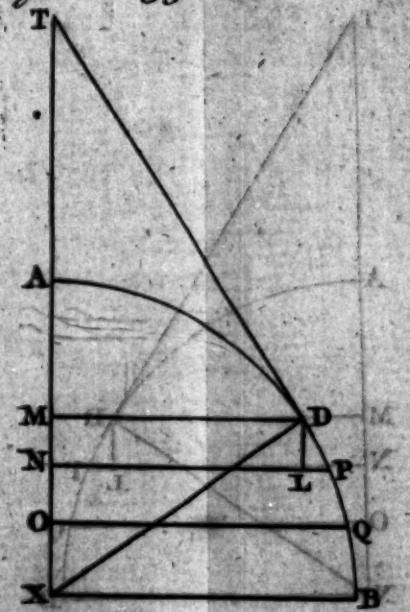


Fig. 15. P. 37.

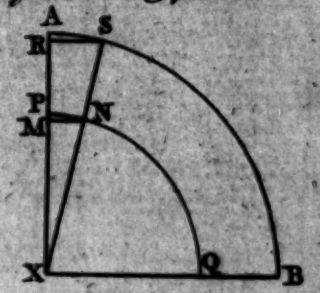
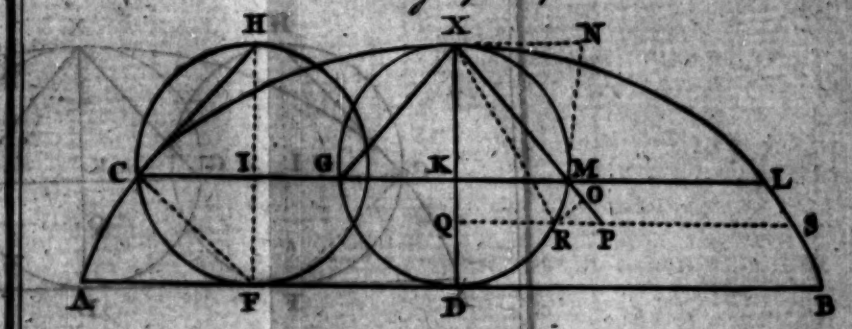
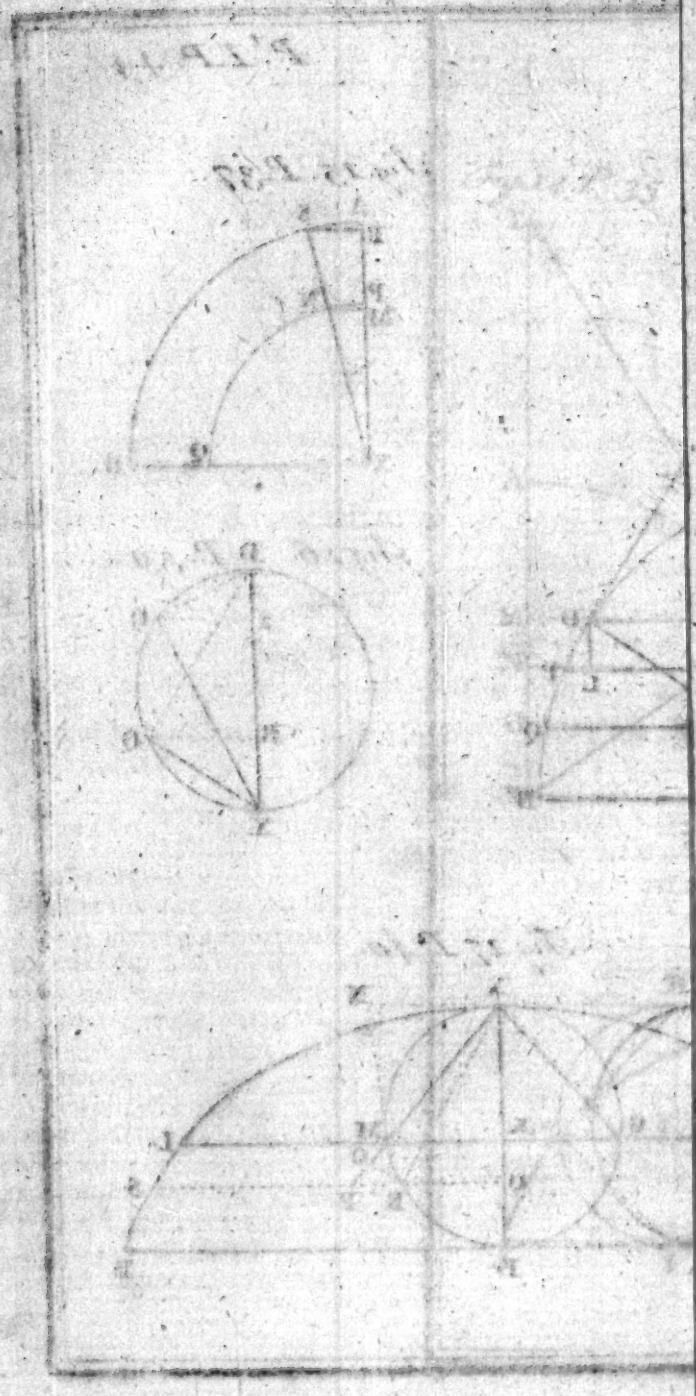


Fig 16 D P. 39.



Fig. 17. P. 40.





PROBLEM.

To make a Pendulum *vibrate in a given* Cycloid.

Solut. Let AXB (Fig. 18.) be the given Cycloid; its Baſe AB, its Axis DX, and its generating Circle DQX, as before: Produce XD to C, till DC be equal to DX: Through C draw the Line EF parallel to AB, and take CE and CF each equal to AD, or DB; and on the Line CE, as a Baſe, and with the generating Circle AGE, equal to DQX, deſcribe the Semicycloid CTA, whoſe Vertex will therefore touch the Baſe of the given Cycloid in A. And on the Line CF, alſo as a Baſe, deſcribe

From the Figure	7	The Time of Deſcent from A to X is half a Vibration.
From the Solution of the following Problem it will appear, that	8	DX is half the Length of a Pendulum, which in vibrating ſhall deſcribe the Cycloid AXB.
Comparing the three laſt Steps	9	The Time of half a Vibration is to the Time in which a Body would fall freely through half the Length of the Pendulum, as the Semicircumference of a Circle is to its Diameter.
Doubling the Antecedents of the laſt Step	10	The Time of an whole Vibration is to the Time in which a Body would fall freely through half the Length of the Pendulum, as the Circumference of a Circle is to its Diameter. Q. E. D.

an

an equal Semicycloid CB. On the Point C, hang the Pendulum CTP, equal in Length to the Line CX: And let the upper Part of the String of it, (as CT, in its preſent Situation in the Figure) as it vibrates this Way and that, apply itſelf to the cycloidal *Cheeks* CA and CB: Then will the Ball of it P oſcillate in the given Cycloid AXB. Q. E. F. (*c*).

CHAP.

(*c*) Draw TG and PH, each parallel to the Baſe AB; and join the Points AG and DH. Then by the Corollary of Lemma 6.

By the Figure (DC being equal to DX)	1	AC = 2 AE
Comparing the 1ſt and 2d Steps	2	2 AE = CX
By Conſtruction	3	AC = CX
Comparing the 3d and 4th	4	CTP = CX
From the 5th Step compared with the Figure	5	AC = CTP
(The String touching the Cycloid at T) by Lemma 5.	6	AT = TP
By Conſtruction	7	GA is parallel to TK
From the two laſt Steps compared, GATK is a Parallelogram, conſequently	8	GT is parralel to AK
By Lemma 6.	9	GA = TK and GT = AK
Comparing the two laſt Steps	10	GA = $\frac{1}{2}$ TA
Comparing the 6th and 11th	11	TK = $\frac{1}{2}$ TA
From the 12th Step compared with the Figure	12	TK = $\frac{1}{2}$ TP
Comparing the laſt Step with the Figure	13	TK = KP
From the laſt compared with the Figure	14	The parallel Lines GT and PH are equally diſtant from AD
Comparing the laſt with the Figure	15	The Arch GA = the Arch DH
	16	The Chords GA and DH are parallel, and GE = HX.

From

CHAP. VIII.

Of the Centers *of* Oſcillation *and* Percuſſion.

I. *THE Center of Oſcillation* is that Point in a Pendulum, in which, if the Weight of the ſeveral Parts thereof were collected, each Vibration would be performed in the ſame Time, as when thoſe Weights are ſeparate.

The Point or Center of Suſpenſion, is the Point on which the Pendulum hangs.

A general Rule *for finding the* Center *of* Oſcillation.

If ſeveral Bodies be fixed to an inflexible Rod, ſuſpended upon a Point, and each Body

From the 7th and 16th Steps compared with the Figure	17	KP is parallel to DH
And therefore (KD being by Conſtruction parallel alſo to PH) KDHP is a Parallelogram, conſequently	18	KD = PH
By Lemma 4.	19	GT = the Arch AG
Comparing the 9th and 19th	20	AK = the Arch AG
By the Deſcription of the Semicycloid CTA	21	AKD = AGE
From the two laſt compared with the Figure	22	KD = GE
Comparing the 18th and 22d	23	PH = GE
Comparing the 16th and 23d	24	PH = HX.

But by Lemma 4. if PH be equal to HX, P is a Point in the Cycloid AXB; the Ball of the Pendulum CTP therefore being at that Point, is in the given Cycloid. The Problem therefore was rightly ſolved. Q. E. D.

be

be multiplied by the Square of its Diſtance from the Point of Suſpenſion, and then each Body be multiplied by its Diſtance from the ſame Point; and all the former Products, when added together, be divided by the latter Products added together, the Quotient which ſhall ariſe from thence, will be the Diſtance of the Center of Oſcillation of thoſe Bodies from the ſaid Point.

Thus, if CF Fig. 21, be a Rod on which are fixed the Bodies A, B, D, *&c.* at the ſeveral Points A, B, D, *&c.* and if the Body A be multiplied by the Square of the Diſtance CA, and B be multiplied by the Square of the Diſtance CB, and ſo on to the reſt: And then if the Body A be multiplied by the Diſtance CA, and B be multiplied by the Diſtance CB, and ſo on for the reſt; and if the Sum of the Products ariſing in the former Caſe, be divided by the Sum of thoſe which ariſe in the latter, the Quotient will give CQ, the Diſtance of the Center of Oſcillation of the Bodies A, B, D, *&c.* from the Point C (*d*).

(*d*) *Dem.* That the Proceſs may be leſs complicated, let us ſuppoſe but two Bodies, as A and F, fixed to the Rod CF; and let AI and FL be the Archs which the Bodies A and F deſcribe when the Pendulum vibrates, and let the Pendulum be removed into the Situation CL. Contiguous to the Line CL draw CR; then may the Arches IP and LR be conſidered as Tangents at the Points I and L, and thoſe Tangents as inclined Planes, down which the Bodies I and L are to roll: Theſe Tangents being each perpendicular to CL, are equally inclined to the Horizon, the Bodies therefore will endeavour to roll down with equal Velocities; but this they cannot do, becauſe being fixed to the inflexible

flexible Rod, they muſt deſcribe the unequal Arches IP and LR in the ſame Time. That is, the Body L will oblige the Body I to deſcribe a leſs Arch than it otherwiſe would have done; and the Body I will occaſion the Body L to deſcribe a larger Arch than it would have done. And the Effects of the Forces by which they act thus upon each other, like thoſe of Action and Reaction, will be equal. It remains to determine theſe Effects.

In order to which, parallel to LI draw MN, and let the equal Spaces LM and IN be thoſe the Bodies would move over in the leaſt Time poſſible, had they been independent of each other. And let the Arches LR and IP be thoſe which the Bodies joined to the Rod deſcribe in the ſame Time. For the Reaſon juſt mentioned, the former of theſe, *viz.* LR, will be larger, and the latter, *viz.* IP, will be leſs than LM or IN; and the Arch which the Center of Oſcillation deſcribes will be equal to LM or IN, becauſe the Center of Oſcillation deſcribes that Arch which the Bodies would deſcribe in the ſame Time, if they were both together, and neither of them an Hindrance or Furtherance to the other. Conſequently the Center of Oſcillation is at Y, where the Lines MN and PR croſs.

Now the Motion which the Body I loſes by being retarded, is its Motion over the Arch PN; and the Motion the other Body gains by being accelerated, is its Motion over MR: The Force or Moment of the firſt of theſe Motions, is the Product of the Body I multiplied by the Space PN; and the Force or Moment of the laſt is the Product of the Body L multiplied by the Space MR. Theſe are the Forces, Moments or Actions, which retard the one Body, and promote the Motion of the other. But obſerve, that theſe Forces or Moments, in as much as they act at different Diſtances from the Center C, about which the Bodies I and L, when the Pendulum ſwings, do revolve; have each their *Mechanical* Advantage; but the one a greater than the other: For Inſtance, L has an Advantage which is as LC, its Diſtance from the *Fulchrum* C; and I only the Advantage IC. As then in determining the Effect of a Power applied to a Lever, we multiply it by its Diſtance from the *Fulchrum*; ſo the abovementioned Forces or Moments (*viz.* I multiplied by PN) and L multiplied by MR) muſt be multiplied by their reſpective Diſtances from C; and then we have I multiplied by PN multiplied by IC, and L multiplied by MR multiplied by LC for the Effects, which, as Things are circumſtanced, theſe Forces or Moments have upon the Bodies I and L. But, as obſerved above, thoſe Effects are equal, conſequently we have for the firſt Step.

II. *The Center of Percuſſion* is that Point in a Pendulum, or in an inflexible Rod moving

	1	$I \times PN \times IC = L \times MR \times LC$
But the Triangles PNY and MRY are ſimilar, conſequently	2	$PN : MR :: PY : RY$
Comparing the two laſt	3	$I \times PY \times IC = L \times RY \times LC$
Or taking the Pendulum in the Situation CPR, in which I coincides with P, and L with R, we have	4	$P \times PY \times PC = R \times RY \times RC$
Or, which is the ſame Thing	5	$A + AQ + AC = F + FQ \times FC.$

That is, in Words, if one of the Bodies were multiplied by its Diſtance from the Center of Oſcillation, and the Product ariſing from thence were multiplied by the Diſtance of the ſame Body from the Center of Suſpenſion, this laſt Product would be equal to the Product of the other Body multiplied by its Diſtance from the Center of Oſcillation, multiplied by its Diſtance from the Center of Suſpenſion. And, ſince the ſame would be true if there were more Bodies, if each Body be multiplied by its Diſtance from the Center of Oſcillation, and that Product by the Diſtance of the ſame Body from the Center of Suſpenſion, all the Products relating to the Bodies on one Side the Center of Oſcillation taken together, will be equal to all thoſe which relate to the Bodies on the other Side thereof taken together. Let then the Diſtances of any Number of Bodies, as A, B, D, F, from the Center of Suſpenſion be called a, b, d, f, reſpectively, and the Diſtance of the Center of Oſcillation Q from the Center of Suſpenſion C, be called x: And ſuppoſe the Diſtances of the Bodies A, B, D, leſs than the Diſtance CQ, or x; and that of the Body F greater, as in the Figure; then will the Diſtances of A, B and D from the Center of Oſcillation be expreſſible by $x-a$, $x-b$, and $x-d$; and the ſame Diſtance of F, by $f-x$; multiplying then each Body by its Diſtance from one Center, and the Product ariſing therefrom by the Diſtance of the ſame Body from the other Center, we ſhall have $Aax - Aaa + Bbx - Bbb + Ddx - Ddd = Fff - Ffx$, which reduced gives $x = \frac{Aaa + Bbb + Ddd + Fff}{Aa \times Bb + Dd + Ff}$. Which latter Equation is the Senſe of the Rule above laid down.

round

round a Point, with which, if the Pendulum or Rod ſtrikes againſt an Obſtacle, no Jar or Shock at the Point of Suſpenſion ſhall be occaſioned thereby.

Thus, let CF (Fig. 21.) be an inflexible Rod, having the Bodies A, B, D, *&c.* fixed in it at the Points A, B, D, *&c.* and let O be an Obſtacle againſt which, as it vibrates or ſwings round the Point of Suſpenſion C, it may ſtrike againſt: then, if there be no Jar or Shock occaſioned thereby at the Point C, the Point that ſtrikes againſt O, (as the Point Q ſuppoſe) is called the Center of Percuſſion.

Proposition.

The Center of Percuſſion is the ſame with the Center of Oſcillation; and conſequently may be determined by the ſame Rule (*e*).

Prob.

(*e*) *Dem.* From the Definition of the Center of Percuſſion above laid down, it appears, that the Forces with which the Bodies A, B and D, which would paſs above O, move; muſt be a Counterbalance to the Force of the Body F, which would paſs below it: and that the Force of F muſt be a Counterbalance to them. But the Forces wherewith thoſe Bodies move, are as their Maſſes multiplied by their Diſtances from C, their Velocities being as theſe Diſtances. Farther, when the Point Q comes to O, and is ſtopt there, the Bodies A, B and D, endeavouring to go on, ſway or bear againſt F, and F againſt them; juſt as if they were fixed to a Lever, as AF, having its Fulchrum at Q. Conſequently the Forces of the former Bodies, ſo far as they act againſt the latter, are as their Diſtances from the Point Q; and the Force of the latter, ſo far as it acts againſt

PROBLEM.

Let it be required to find the Center of Oſcillation, or Percuſſion of an inflexible Rod AB (Fig. 22.) as a Bar of Iron, or the like, every where of equal Size, and vibrating in, or revolving round the Point A, as a Center of Suſpenſion (*f*).

the former, is as its Diſtance alſo from Q: the abovementioned Forces muſt therefore be multiplied by the Diſtances of the Bodies from Q: but the former of them, as obſerved above, balances the latter; and the latter them. So many therefore of the laſt Products as relate to the Bodies above Q taken together, muſt be equal to that which relates to the Body (or Bodies) below it. But the like Products were equal to each other, when the Point Q was looked upon as the Center of Oſcillation (as in the 5th Step of the foregoing Proceſs) conſequently the Center of Percuſſion is the ſame with that of Oſcillation. *Q. E. D.*

(*f*) *Solut.* Imagine the Rod to be divided into the leaſt poſſible Parts B, C, D, *&c.* each of which call *One*. Theſe Parts we may conſider as ſo many Bodies contiguous to one another; ſo that the Center of Oſcillation or Percuſſion of theſe Bodies will be the Center of Oſcillation or Percuſſion of the whole Rod. To find this, we are by the Rule above laid down in the Text, to multiply each of theſe Bodies by the Square of its Diſtance from A. The firſt of theſe Products then will be B (or One) multiplied by AB ſquared; but one multiplied by AB ſquared, is the ſame with AB ſquared; now AB ſquared is a ſquare Area or Surface, one of whoſe Sides is AB. In like manner the Body C, when multiplied by the Square of its Diſtance from A, is a Square Area, one of whoſe Sides is AC, ſomewhat leſs than the former. Imagine this Area laid upon the former; and the next, which will be leſs ſtill, laid upon that; and ſo on till you come to the leaſt of all. Theſe will make a Pyramid, whoſe Baſe is the firſt Area, and its perpendicular Height will be equal to the Thickneſs of them all together; which Thickneſs will be as the Length of the Line BA. The Value or ſolid Content

Content of this Pyramid will be AB^q (*viz.* its Baſe) multiplied by a third Part of AB (its perpendicular Height). In the next Place we are to multiply each of thoſe Bodies by its Diſtance from A: Now the Body B (or One) multiplied by AB, gives a Line, as AB; ſo the Product of C, multiplied by its Diſtance AC, gives a Line, as AC; theſe Lines heapedone upon another (as the Areas were before) will make a Triangle, whoſe Baſe will be AB, and its perpendicular Height alſo AB; the Value, or Area of which, will be AB multiplied by $\frac{1}{2}$ AB. In the laſt Place, by the Rule, we are to divide the Sum of the Products in the firſt Caſe, by the Sum of the Products in the latter; that is, the Content of the Pyramid by the Area of the Triangle; that is, $AB^q \times \frac{1}{3}$ AB, by AB $\times \frac{1}{2}$ AB, which gives $\frac{\frac{1}{3}AB}{\frac{1}{2}}$; that is, $\frac{2}{3}$ AB, or two Thirds of AB: ſo that the Diſtance of the Center of Oſcillation or Percuſſion, (as E ſuppoſe) from A the Center of Suſpenſion, muſt be equal to two Thirds of AB, the whole Length of the Rod. *Q. E. I.*

CHAP. IX.

Of Projectiles.

IF a Body be projected in any Direction whatſoever, it would proceed on *in infinitum* in a right Line (by the firſt Law of Nature;) but being continually accelerated toward the Earth by its Gravity, it will deſerve a Curve called a *Parabola* *.

describe

* *Dem.* Let us ſuppoſe the Body thrown from *A* in the Direction *AB* horizontally (*Fig.* 23.) or obliquely (*Fig.* 24.) it would (if not attracted towards the Earth) move uniformly

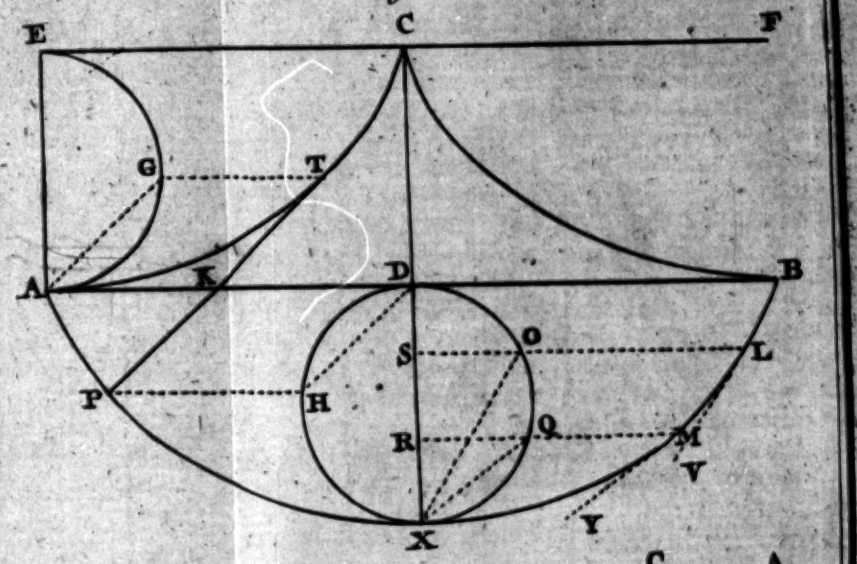
from *A* towards *B*, that is, in equal Times it would deſcribe equal Parts of the Line *AB*, as *AC*, *CD*, *DE*, &c. but, if in the firſt Portion of Time, while it would move from *A* to *C*, it would have deſcended from *A* to *G* by its Gravity, had it only been let drop from thence; it will by a Compoſition of theſe two Motions (Chap. IV. §. 2.) at the End of that Time be found in *H*, the oppoſite Angle of the Parallelogram *ACGH*. Then in twice that Time, *viz.* while it would have moved over two equal Portions, or from *A* to *D*, it would fall downwards to *M*, four Times as far as before (Chap. V. §. 4.) and will therefore be found in *I*, ſuppoſing *DI* equal and parallel to *AM*. Then again, in three Portions of Time, or while it would have moved over three Diviſions, that is, from *A* to *E*, it would have fallen downwards nine Times as far as in the firſt Portion of Time, and therefore being carried by theſe two Motions will at the End of that Time be found in *K*, ſuppoſing *EK*, or its equal *AN*, nine Times as long as *AG* or *CH*, &c. Therefore the Lines *CH*, *DI*, *EK*, &c. which are to each other as the Numbers 1, 4, 9, &c. are as the Squares of the Lines *AC*, *AD*, *AE*; (theſe being only as the Numbers 1, 2, 3.) But this is the Property of the Parabolic Curve. (See De L'*Hoſpital*, B. I. Prop. 1. Corol. 2. and Prop. 3. Corol. 1.) Conſequently the Curve *AHIK*, &c. which the Body moves in, whether thrown horizontally, or obliquely, is a Parabola. *Q. E. D.*

Lemma 1. The Quotient which ariſes from the Diviſion of the Square of the Line *AC* by the Line *AG*, *viz.* the Quantity $\frac{ACq}{AG}$ (in either of the Parabolic Curves, *Fig.* 23, or 24.) or the Square of the Line *AD* divided by *AM*, *viz.* $\frac{ADq}{AM}$, or the Square of *AE* divided by *AN*, *viz.* $\frac{AEq}{AN}$, is equal to the *Parameter* of the Point *A*, for *GHq* is equal to *AG* multiplied by the *Parameter* (De L'*Hoſpital* Con. Sect. I. Prop. 1 & 3.) therefore the Parameter is equal to *GHq* divided by *AG*, that is, $\frac{ACq}{AG}$: The ſame is demonſtrable of *AD* divided by *AM*, &c. and conſequently any of theſe Quantities may be indifferently put to expreſs the Parameter of the ſame Point.

Lemma 2. The Velocity which a Body would acquire by falling from an Height equal to the fourth Part of the Parameter of the Point *A*, is to the Velocity it would acquire by falling from *A* to *N*, as *AE* is to twice *AN*.

Dem.

Fig 18. P. 42.



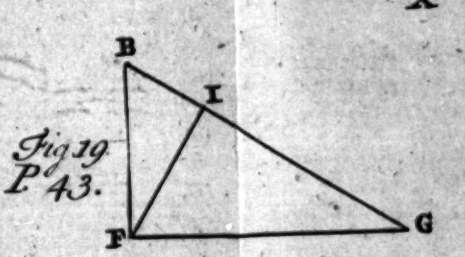


Fig 19. P. 43.

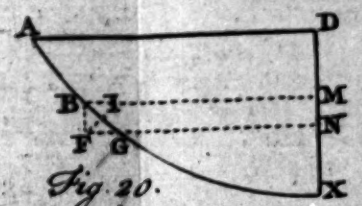


Fig 20. P. 43

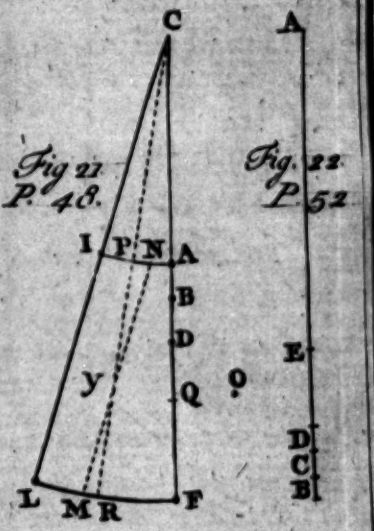


Fig 21 P. 48.

Fig. 22 P. 52

Dem. Since we are comparing the Velocity, which a Body would acquire by falling through a fourth Part of the Parameter, with that which it would acquire by falling to *N*, let $\frac{AEq}{AN}$ be made choice of to denote the Parameter. Then $\frac{\frac{1}{4}AEq}{AN}$ will expreſs a fourth Part of the Parameter. Now becauſe the Velocities, acquir'd by falling Bodies, are as the ſquare Roots of the Spaces they fall through (Chap. V. §. 6.) the Velocity, acquir'd by a Body in deſcending through $\frac{\frac{1}{4}AEq}{AN}$, is to that Velocity, which it would acquire by falling through *AN*, as the ſquare Root of $\frac{\frac{1}{4}AEq}{AN}$ to the ſquare Root of *AN*; that is, extracting the Roots of thoſe Quantities, as $\frac{\frac{1}{2}AE}{\sqrt{AN}}$ to $\sqrt{AN}$, and, multiplying each Term by $\sqrt{AN}$, as $\frac{1}{2}AE$ to *AN*, or as *AE* to twice *AN*. Q. E. D.

Prop. The Velocity a Body ought to be projected with, to make it deſcribe a given Parabola, is ſuch as it would acquire by falling through a Space equal to the fourth Part of the Parameter belonging to that Point of the Parabola, from whence it is intended to be projected.

Dem. The Velocity with which a Body muſt be projected from *A* towards *B*, to make it deſcribe the given Parabola *AHIK*, muſt be ſuch, as would carry it to *C* by an uniform Motion, in the ſame Time that it would deſcend by its Gravity from *A* to *G*; and to *E*, in the Time it would fall to *N*, &c. as was before obſerved. Now the Velocity, with which the Line *AE* is deſcribed with an uniform Motion, is to that which is acquired by the Body in falling to *N* in the ſame Time, as *AE* is to twice *AN*; becauſe (Chap. V. § 7.) its Velocity in *N* would have carried it over twice *AN* in that Time, had it alſo been uniform. But by *Lemma* 2. the Velocity a Body would acquire, by falling through a Space equal to a fourth Part of the Parameter of the Point *A*, is to that which it would acquire by falling from *A* to *N*, alſo as *AE* to twice *AN*. Since therefore the Velocity, with which the Line *AE* is deſcribed (or, which is the ſame Thing, that whereby the Body is projected) and that which a Body would acquire by falling through a fourth Part of the Parameter of the Point

A, bear one and the ſame Proportion to that Velocity which a Body would acquire by falling from *A* to *N*, they muſt be equal. *Q. E. D.*

Coroll. This affords us an eaſy Method of finding what Direction it is neceſſary to throw a Ball in with a given Velocity, in order to ſtrike an Object in a given Situation, *v. g.* Let it be required to ſtrike an Object as *K*, with a Ball thrown from *A* with a given Velocity. Here it is only neceſſary to make the Triangle *ANK* (ſuppoſe a right Line drawn from *A* to *K*) ſuch, that $\frac{NKq}{NA}$, or which is the ſame Thing, $\frac{AEq}{EK}$ in the Triangle *AEK*, may be equal to four Times the Space a Body muſt fall through, to acquire ſuch a Degree of Velocity as that with which it is intended to be thrown, and then *AE* will be the Direction ſought. In order to this we muſt lay down the following Lemma.

Lemma. Let there be a Circle as *ABC* (*Fig.* 25.) *AK* a Tangent in the Point *A*, *AB* and *KI* parallel to each other, and let the other Lines be drawn, as in the Figure, I ſay $\frac{AEq}{EK} = AB$. For the Angle *ABE* is equal to the Angle *EAK* (32 *Elm.* 3.) and the Angle *BAE* is equal to the Angle *AEK* as alternate, therefore the Triangles *ABE* and *AEK* are ſimilar; conſequently *AB* is to *AE*, as *AE* to *EK*, and multiplying the extreme Terms together, and middle Terms together, $AB \times EK = AEq$, and dividing both Sides of the Equation by *EK*, $AB = \frac{AEq}{EK}$. Q. E. D. By the ſame Method of arguing $\frac{AIq}{IK}$ may be proved equal to *AB*.

The Problem.

Let it be requir'd to ſtrike an Object in a given Situation as *K* (*Fig.* 26.) with a Ball projected from *A* with a given Velocity.

Solution. Erect *AB* perpendicular to the Horizon, and equal to four Times the Height a Body muſt fall from, to acquire the Velocity with which the Ball is to be thrown; biſect this in the Point *G*, through which draw *HC* perpendicular to *AB*, and meeting the Line *AC* (perpendicular to *AK*) in *C*. On *C* as a Center with the Radius *CA*, deſcribe the Circle *ABD*; laſtly, through *K* draw the Line *KEI* perpendicular

II. The greateſt horizontal Diſtance, to which a Body can be thrown with a given Velocity, is at the Elevation of 45 Degrees *.

III. If two Balls are thrown at different Elevations (but with equal Degrees of Velocity) the one as much above forty-five Degrees as the other below, the horizontal Diſtances (or Randoms) where they both fall, will be the ſame †.

IV. The

pendicular to the Horizon, cutting the Circle in the Points *E* and *I*; ſay *AE* or *AI* will be the Direction ſought.

For by the Lemma, $AB = \frac{AEq}{EK}$ or $\frac{AIq}{IK}$, but (*ex conſtructione*) *AB* is equal to four Times the Height a Body muſt fall from, to acquire the Velocity with which it is to be thrown, therefore its Equal, $\frac{AEq}{EK}$ or $\frac{AIq}{IK}$, is the ſame, which by the Corollary was the Thing requir'd to determine the Direction ſought; conſequently the Parabola, which the Body will deſcribe, will paſs through the Point *K*. Q. E. D.

Coroll. 1. From hence it is evident, that, if the Object to be ſtruck be placed any where in the horizontal Line *AO* (*Fig.* 27.) beyond *Q*, the Problem is impoſſible; for then *QH* will not touch the Circle, and the Ball will not reach that Point with any Direction whatever.

* And that when the Ball is directed towards *H*, it will fall on *Q* the greateſt Diſtance it can poſſibly be thrown to; but the Angle *QAH* being equal to *ABH* in the oppoſite Segment (32 *Elem.* 3.) is equal to half *AGH* at the Center (20 *Elm.* 3.) which is a right one; conſequently *QAH* is an Angle of 45 Degrees.

† *Coroll.* 2. If the Object is ſituated in the horizontal Line *AO* (*Fig.* 28.) but nearer to *A*, than the greateſt horizontal Diſtance at which it may be ſtruck, ſuppoſe in *K*; the two Directions *AE* and *AI*, with which it may be hit, are equally diſtant

IV. The Height a Body will riſe to, when thrown perpendicularly upwards, is equal to half the greateſt horizontal Diſtance it can be thrown to, with the ſame Velocity *.

From hence we may eaſily know how far a Mortar-Piece, or other ſuch Machine will carry a Ball. Let the Ball be ſhot perpendicularly upwards, note the Time of its Aſcent and Deſcent, half that is the Time of Deſcent, from whence we learn the Height from which it falls) for Bodies are obſerved to fall in the firſt Second of Time ſixteen Feet) conſequently in two Seconds they fall four Times ſixteen Feet (Chap. V. §. 4.) in three, nine Times as much, *&c.* but the perpendicular Height from whence it falls is the ſame with that to which it aſcended, conſequently (§. 4.) the double of this is equal to the greateſt horizontal Diſtance to which that Machine will carry the Ball with an equal Charge.

diſtant from the Direction *AH*; for the Angles *IAH* and *HAE* are equal, as inſiſting on equal Arches *IH* and *HE*, (28 *Elem.* 3.)

* *Coroll.* 3. The Altitude of a perpendicular Projection is equal to a fourth Part of the Height *AB*; for the Velocity, with which the Body is projected, is (*ex hypoth.*) ſuch as it would acquire by falling through a fourth Part of the Line *AB*; but a fourth Part of the Line *AB* is equal to half the Line *GH*, or *AQ* (Fig. 27.) that is, half the greateſt horizontal Diſtance to which the Body can be thrown.

See *Cotes*'s Harmonia Menſurarum, p. 87. *Keil*'s Introduct. ad Phyſ. Præl. 16.

V. The

V. The Randoms of two Projectiles, having the ſame Degrees of Elevation, but thrown with different Velocities, are as the Squares of the Velocities: For by the laſt, the Randoms are equal to double the Heights to which the Bodies thrown perpendicularly upwards will aſcend, but the Heights are (Chap. V. §. 6.) as the Squares of the Velocities, therefore the Randoms are ſo too.

VI. Supposing the Motion of the Earth, all Bodies, when thrown perpendicularly upwards, deſcribe *Parabola's*; notwithſtanding they appear both to aſcend and deſcend in the ſame right Line.

This may very eaſily be illuſtrated in the following Manner: let there be a Body carried uniformly along the Line AB (*Fig.* 29.) by the Motion of the Earth from A towards B; as it paſſes the Point C, let it be projected upwards, by ſome Force acting underneath it in the Direction CO perpendicular to the former: The Body will not thereby loſe its Motion, which it had in common with the Earth, towards B (by the firſt Law of Nature) but will be carried by two Motions, one towards B, the other towards O; let us then ſuppoſe, that in the Time it would have advanced forwards to P in the Line AB, it riſes upwards to M in the Line CO; it will then be found in D (Chap. IV. §. 2.) In like Manner, ſuppoſing it would have advanced forward to Q

while

while it riſes to N, it would then be found in E, afterwards in F, then in G, *&c.* deſcribing the Curve CGL, which (from what was demonſtrated under §. 1.) is a Parabola *.

THE Reaſon, why it appears to a Spectator to riſe and fall perpendicularly, is becauſe he is carried uniformly along with it by the Motion of the Earth in the Direction AB; *v. g.* Suppoſe the Spectator at C at the Inſtant the Body is thrown from thence, when it arrives at D, he will be moved to P, when the Body is at E he will be at Q, *&c.* as is evident from what was obſerved about the Motion of the Body in the Curve; and they will both meet in L. Therefore ſince the Spectator imagines himſelf ſtanding ſtill, and ſees the Body always perpendicularly over his Head, he muſt of Courſe think, that it riſes right up, and falls right down.

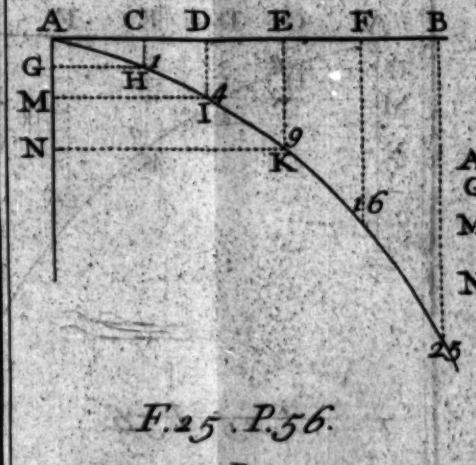
IT may be proper to obſerve here, that Experiments, relating to the Motion of projected Bodies, do not exactly anſwer the Theory, the Reſiſtance of the Air deſtroying Part of their Motion; for which a ſmall Allowance is to be made.

* *Dem.* Suppoſe the Motion the Body had in common with the Earth towards *B*, (*Fig.* 30.) and that with which it is projected towards *O*, ſuch, as being compounded (Chap. IV. §. 2.) would have produced a Motion in the Direction *CX*; it will follow from thence, that the Path deſcribed by it will be the ſame, as if it had been thrown in that Direction from a Point as *C* at Reſt; but in that Caſe it would have deſcribed a Parabola as *CGL* (§. 1.); therefore alſo in this. *Q. E. D.*

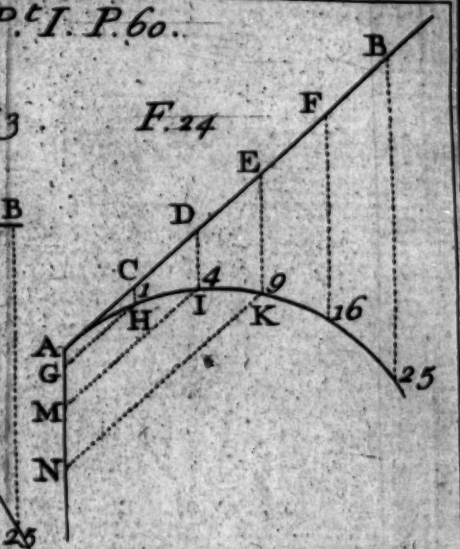
CHAP.

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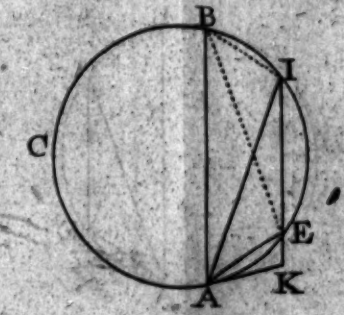
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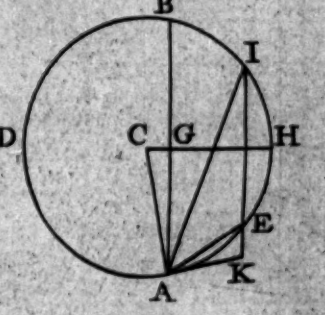
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F.25. P.56.

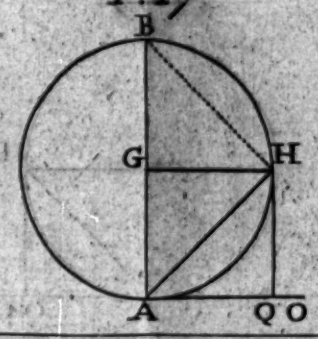


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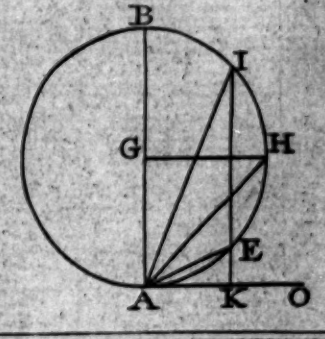


F.27

P.57.



F.28.





CHAP. X.

Of Centripetal and Centrifugal Forces.

WHEN a Body is projected in an horizontal Direction, and by its Gravity made to deſcribe a Parabola, as demonſtrated in the laſt Chapter; the Curvature of that Parabola will vary in Proportion to the Velocity with which the Body is thrown, and the Gravity which impels it towards the Earth. For the leſs its Gravity is in Proportion to the Quantity of Matter it contains, or the greater the Velocity is with which it is projected; the leſs it will deviate from a ſtraight Line, and the further it will go, before it falls to the Earth. For Inſtance, if a Bullet be ſhot out of a Cannon from the Top of a Mountain with a given Velocity in an horizontal Direction, and goes in a Curve Line, ſuppoſe to the Diſtance of two Miles from the Foot of the Mountain before it falls to the Ground; the ſame Bullet, ſhot with a much greater Velocity, would fly to a much greater Diſtance before its Fall. And by increaſing the Velocity, the Diſtance to which it is projected may be increaſed as much as you pleaſe; ſo that it will not fall to the Ground, till it is arrived at the Diſtance of ten, or thirty, or ninety Degrees; or till it has even ſurrounded the whole

whole Earth, and arrives at the very Top of the Mountain from whence it was projected: In this Caſe it will perform a ſecond Revolution, and ſo on *in infinitum*, without a new Projection, provided the Reſiſtance of the Air is taken away. And were the Sphere of the Earth's Attraction a limited one, it might be projected with ſuch Violence, that it would continually recede from the Earth in a Curve, till at length it would go out of that Sphere; after which it would go on in a ſtraight Line without ever returning. Which may thus be illuſtrated.

Let ABC (*Fig.* 31.) repreſent the Earth, M the Top of the Mountain from whence the Body is projected in the Direction MQ: It may be thrown with ſuch Force as to carry it to B before it falls, or to C, or even to go round to M, deſcribing the Circle MDM; or laſtly, it may be made to deſcribe the Curve MO, by which Means, if it gets out of the Sphere of the Earth's Attraction, ſuppoſe at O, it will go on afterwards in the infinite ſtraight Line OX, there being nothing to ſtop or alter its Courſe. Farther it may be projected with ſuch a Force from M (*Fig.* 32.) as will cauſe it continually to recede from the Earth, till it arrives at the oppoſite Point G, deſcribing the Curve MKG; and if the Point G is within the Sphere of the Earth's Attraction, the Body will return to M, deſcribing the Curve GLM exactly

actly ſimilar to MKG; and in moving nearer and nearer to the Earth, till it comes to M, will regain what Velocity it loſt in going from M to G, its Gravity conſpiring with its Motion from G to M in the ſame Degree in which it oppoſed it from M to G; conſequently the Body when at M, having recovered the Velocity with which it ſet out, will be enabled to perform a ſecond Revolution in the ſame Curve as before; and ſo on.

AGAIN, ſuppoſe it had been projected from the Point M, with a leſs Degree of Force than would have carried it round in the Circle MDM (*Fig.* 31.) but greater than would have ſuffered it to have fallen to the Earth at the oppoſite Point F, (*Fig.* 32.) it would alſo in this Caſe have arrived at the Point M, from whence it ſet out; for the Exceſs of Velocity it would have gained in F, by its Tendency towards the Earth in its Way thither, over and above that, with which it was projected from M, would be ſufficient to carry it off again from the Earth, till it arrived at M; and to make it deſcribe the Path FPM exactly ſimilar and equal to the former, loſing in its Way from F to M juſt ſo much Velocity, as it gained by paſſing from M to F; and thereby it would be enabled to perform an infinite Number of Revolutions in the ſame Curve, without requiring a ſecond Projection.

FROM

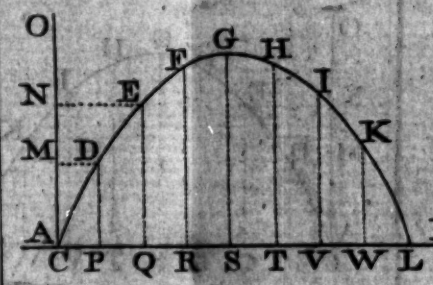
FROM whence it follows, that ſuppoſing a Body projected from a Point at any Diſtance within the Sphere of the Earth's Attraction, with a Force ſufficient to carry it half round without falling to the Surface, it is impoſſible it ſhould fall upon any Part of the other half; but will return to the Point from whence it ſet out, making continual ſucceſſive Revolutions in the ſame Curve; provided it meets with no Reſiſtance from the Medium through which it paſſes, nor any other Obſtacle to obſtruct its Motion *.

FROM hence alſo it is clear, that, the nearer the revolving Body approaches to the Earth, the faſter it moves: Its Velocity being continually increaſed during the Time of its Acceſs towards the Earth, and as much retarded during its Receſs from it. And this Acceleration and Retardation will always be ſuch, that the Body will deſcribe equal Areas in equal Times: The Meaning of which is, that if we imagine a Line conſtantly extended from the Center of the Earth to the Center of the Body, that Line will always deſcribe or paſs through equal Surfaces or Spaces in equal

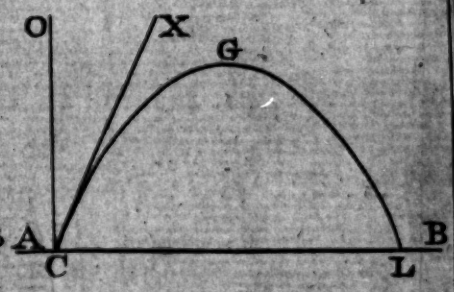
* Gravity is here ſuppoſed to be inverſly as the Squares of the Diſtance from the Earth, for 'tis poſſible that the Force, by which a Body tends towards another, may vary in ſuch a Manner at different Diſtances, that the projected Body ſhall deſcribe a ſpiral Line, continually approaching to, or receding from, that about which it revolves.

Times,

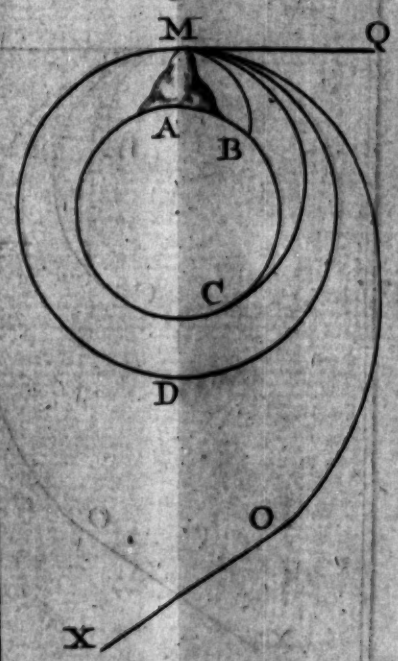
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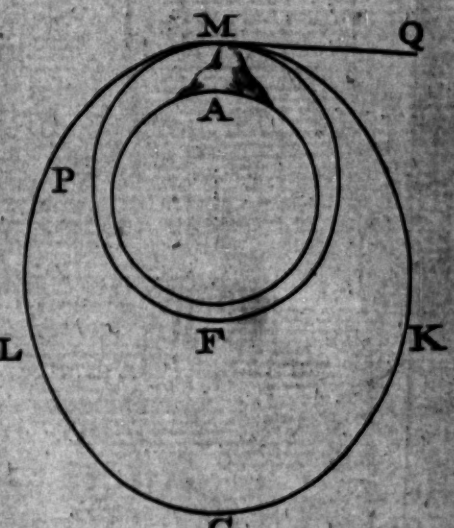
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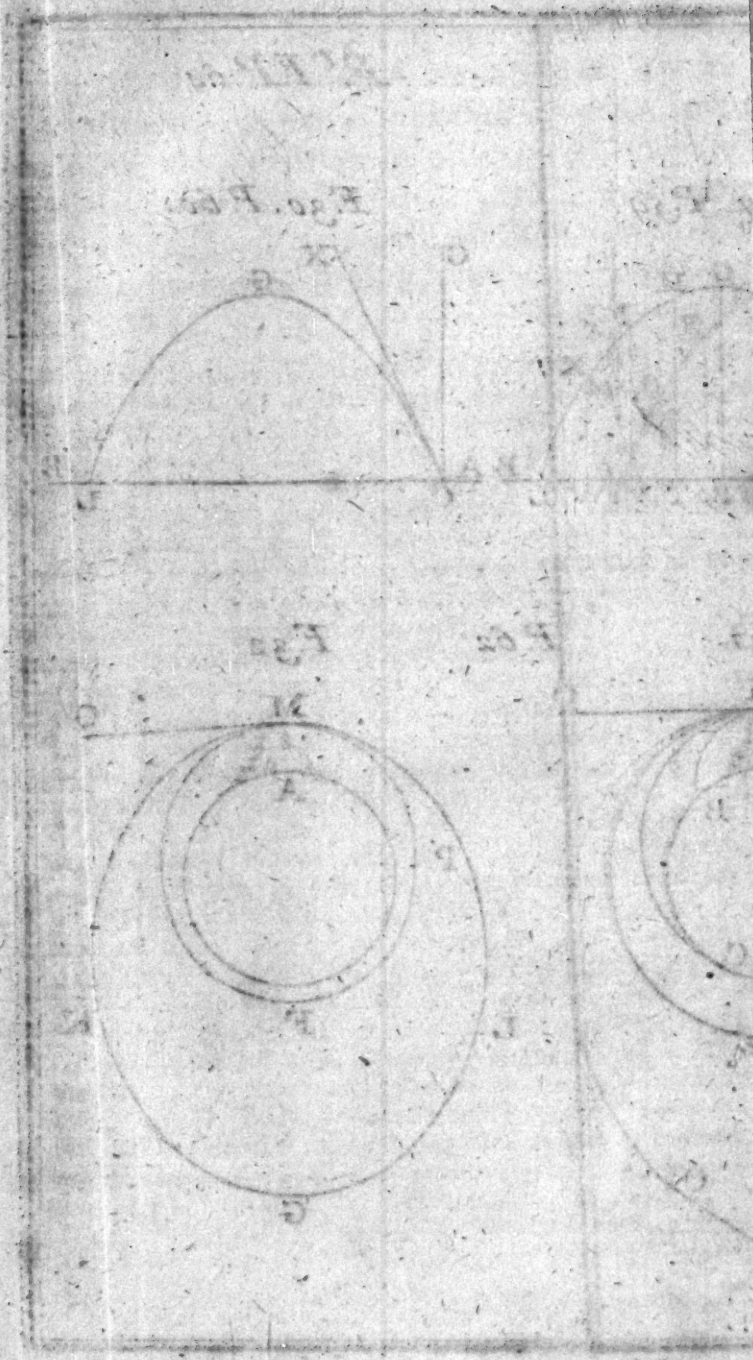


F. 31 P. 62



F. 32





Times, for it conſtantly becomes ſhorter the faſter it moves, and *vice verſâ* *.

And for the ſame Reaſon that a Body, projected with a ſufficient Velocity, may by the Force of Gravity be made to deſcribe a Curve round the Earth, and perform continual ſucceſſive Revolutions therein; it follows, that the Moon may by the ſame Force of Gravi-

* *Dem.* Let the Time in which the Body performs one Revolution be divided into equal Parts, in the firſt of which let the Body deſcribe the right Line *AB* (*Fig.* 33.) in the ſecond Part of Time, if not prevented, it would go ſtraight on to *c*, deſcribing the Line *Bc*, equal to *AB*, by the firſt Law of Nature; the Lines *SA*, *SB*, *Sc* being drawn, the Triangles *SBA*, *ScB*, will be equal to each other, their Baſes *AB* and *Bc* being equal, and their Heights *S* the ſame (38 *Elem.* 1.) When the Body arrives at *B*, let the centripetal Force by one ſingle Impulſe turn it out of the ſtraight Line *Bc* into the Line *BC*; in which let it move on uniformly without receiving a ſecond Impulſe till it comes to *C*. Let *Cc* be drawn parallel to *SB*, meeting *BC* in *C*; then at the End of the ſecond Part of Time, the Body will be found in *C*, having deſcribed the Diagonal of the Parallelogram *Nc*, (Chap. IV. §. 2.) Draw *SC*, and the Triangle *SCB* will be equal to the Triangle *ScB*, (each having the ſame Baſe *SB*, and being between the ſame Parallels *Cc* and *SB*) and therefore alſo equal to the Triangle *SBA*. For the ſame Reaſon, if the centripetal Force acts in the Points *D*, *D*, *E*, ſucceſſively, ſo as to make the Body deſcribe the ſtraight Lines *CD*, *DE*, *EF*, &c. in ſo many equal Parts of Time, the Triangles *SCD*, *SDE*, *SEF*, &c. will be all equal to one another, and to the Triangle *SAB*. Conſequently equal Areas are deſcribed in equal Times. Let us then ſuppoſe the Baſes of thoſe Triangles, *viz.* *AB*, *BC*, *CD*, *DE*, &c. diminiſhed *in infinitum*, and likewiſe the Times in which they are deſcribed; then will the Perimeter *A*, *B*, *C*, *D*, *E*, *F*, &c. become a Curve, and any Number of thoſe Triangles taken together (or their Areas) will be proportional to the Times in which they are deſcribed. *Q. E. D.*

ty be made to revolve about the Earth, or any other Planet, by the like Force, about the Sun; if the Velocities with which they move are duly adjusted to the Forces by which they are acted upon.

When a Body revolves about another in this Manner, that Force or Power by which it is prevented from flying off (as it otherwise would do in a Tangent to the Curve which it describes) is call'd the *Centripetal*; the Counter-action of this, by which it endeavours to fly off, the *Centrifugal*; these, by the third Law of Nature, being equal to each other, are called by one common Name, *Central Forces*; that with which the Body is at first projected, or continues its Motion from any Point, is the *Projectile* Force; and the Time in which it performs one Revolution, the *Periodical* Time.

These Forces, properly relating to the Motions of the Heavenly Bodies, will be more largely treated of in the fourth Part of this Treatise.

CHAP. XI.

Of the Communication of Motion.

I. BEFORE we proceed to explain the Laws, by which Bodies communicate their Motion from one to another, it is very necessary

neceſſary to make a Diſtinction between Motion and Velocity; which ought to be well obſerv'd, and is as follows:

By the Motion of a Body (ſometimes called its Quantity of Motion; ſometimes its *Momentum*) is not to be underſtood the Velocity only, with which the Body moves; but the Sum of the Motion of all its Parts taken together: Conſequently the more Matter any Body contains, the greater will be its Motion, though its Velocity remains the ſame. Thus, ſuppoſing two Bodies, one containing ten Times the Quantity of Matter the other does, moving with equal Velocity; the greater Body is ſaid to have ten Times the Motion, or Momentum, that the other has: For 'tis evident, that a tenth Part of the larger has as much as the other whole Body. In ſhort, that Quality in moving Bodies, which Philoſophers underſtand by the Term Momentum or Motion, is no other than what is vulgarly called their *Force*, which every one knows to depend on their Quantity of Matter, as well as their Velocity. This is that Power a moving Body has to affect another in all Actions that ariſe from its Motion, and is therefore a fundamental Principle in Mechanics.

II. Now, ſince this Momentum, or Force, depends equally on the Quantity of Matter a Body contains, and on the Velocity with which it moves; the Method, to determine how

great it is, is to multiply one by the other. Thus, ſuppoſe two Bodies, the firſt having twice the Quantity of Matter, and thrice the Velocity, which the other has; any two Numbers, that are to each other as two to one, will expreſs their Quantities of Matter, (it being only their relative Velocities and Quantities of Matter which we need conſider) and any two Numbers that are as three to one will expreſs their Velocities; now multiplying the Quantity of Matter in the firſt, *viz.* two by its Velocity three, the Product is ſix; and multiplying the Quantity of Matter in the ſecond by its Velocity, *viz.* one by one, the Product is one; their relative Forces therefore, or Powers, will be as ſix to one, or the Moment of one is ſix times greater than that of the other. Again, if their Quantities of Matter had been as three to eight, and their Velocities as two to three, then would their Moments have been as ſix to twenty-four, that is, as one to four.

THIS being rightly apprehended, what follows, concerning the Laws of the Communication of Motion by Impulſe, and the Mechanical Powers, will be eaſily underſtood.

The Communication of Motion.

I. In Bodies not Elaſtic.

III. THOSE Bodies are ſaid to be not *Elaſtic*, which, when they ſtrike againſt one another,

ther, do not rebound, but accompany one another after Impact, as if they were joined. This proceeds from their retaining the Impreſſion made upon their Surfaces, after the impreſſing Force ceaſes to act. For all rebounding is occaſioned by a certain Spring in the Surfaces of Bodies, whereby thoſe Parts, which receive the Impreſſion made by the Stroke, immediately ſpring back, and throw off the impinging Body; now, this being wanting in Bodies void of Elaſticity, there follows no Separation after Impact.

IV. When one Body impinges on another which is at Reſt, or moving with leſs Velocity the ſame Way, the Quantity of the Motion or Momentum in both Bodies taken together remains the ſame after Impact, as before; for by the third Law of Nature, the Reaction of one being equal to the Action of the other, what one gains, the other muſt loſe.

Thus, ſuppoſe two equal Bodies, one impinging with twelve Degrees of Velocity on the other at Reſt: The Quantities of Matter in the Bodies being equal, their Moments and Velocities are the ſame; the Sum in both twelve; this remains the ſame after Impact (§. 4.) and is equally divided between them (§. 3.) they have therefore ſix a-piece, that is, the impinging Body communicates half its Velocity, and keeps half.

V. When two Bodies impinge on each other, by moving contrary Ways, the Quantity of Motion they retain after Impact, is equal to the Difference of the Motion they had before; for by the third Law of Nature, that which had the leaſt Motion, will deſtroy an equal Quantity in the other, after which they will move together with the Remainder, that is the Difference.

Thus for Inſtance, let there be two equal Bodies moving towards each other, the one with three Degrees of Velocity, the other with five, the Difference of their Moments or Velocities will be two; this remains the ſame after Impact (§. 5.) and is equally divided between them (§. 3.) they have therefore one a-piece: That is, the Body, which had five Degrees of Velocity, loſes three, or as much as the other had, communicates half the Remainder, and keeps the other half *.

* From theſe Poſitions it is eaſy to deduce a Theorem, that ſhall ſhew the Velocity of Bodies after Impact in all Caſes whatever. Let there be two Bodies A and B, the Velocity of the firſt a, of the other b, then (§. 2.) the Moment of A will be expreſſed by Aa, and of B by Bb; therefore the Sum of both will be $Aa+Bb$; and $Aa-Bb$ will be the Difference when they meet. Now theſe Quantities (by §. 4. and 5.) remain the ſame after Impact; but knowing the Quantities of Motion and Quantities of Matter, we have the Velocity (which §. 3. is the ſame in both) by dividing the former by the latter (as follows from §. 2.) therefore $\frac{Aa+Bb}{A+B}$ or $\frac{Aa-Bb}{A+B}$ will in all Caſes expreſs the Velocity of the Bodies after Impact.

II. In

II. In Elaſtic Bodies.

VI. BODIES perfectly *Elaſtic* are ſuch as rebound after Impact with a force equal to that with which they impinge upon one another: Thoſe Parts of their Surfaces, that receive the Impreſſion, immediately ſpringing back, and throwing off the impinging Bodies with a Force equal to that of Impact.

VII. FROM hence it follows, that the Action of Elaſtic Bodies on each other (that of the Spring being equal to that of the Stroke) is twice as much as the ſame in Bodies void of Elaſticity. Therefore, when Elaſtic Bodies impinge on each other, the one loſes, and the other gains twice as much Motion as if they had not been Elaſtic: we have therefore an eaſy Way of determining the Change of Motion in Elaſtic Bodies, knowing firſt what it would have been in the ſame Circumſtances, had the Bodies been void of Elaſticity.

THUS, if there be two equal and Elaſtic Bodies, the one in Motion with twelve Degrees of Velocity impinging on the other at Reſt, the impinging Body will communicate twice as much Velocity as if it had not been Elaſtic, that is (by §. 4.) twelve Degrees, or all it had; conſequently it will be at Reſt, and the other will move on with the whole Velocity of the former.

VIII. It ſometimes happens, that in Bodies not elaſtic, the one loſes more than half its Velocity, in which Caſe, ſuppoſing them Elaſtic, it loſes more than all; that is, the Exceſs of what it loſes, above what it has, is negative, or in a contrary Direction; thus, ſuppoſe the Circumſtances of Impact ſuch, that a Body, which has but twelve Degrees of Velocity, loſes ſixteen; the overplus four is to be taken the contrary Way, that is, the Body will rebound with four Degrees of Velocity, *v. g.* Let it be required to determine the Velocity of a Body after Impact againſt an immoveable Object. Let us firſt ſuppoſe the Object and Body both void of Elaſticity: 'Tis evident the impinging Body would be ſtopp'd or loſe all its Motion, and communicate none; if they are Elaſtic, it muſt loſe twice as much (by §. 7.) and conſequently will rebound with a Force equal to that of the Stroke.

IX. It is ſufficient if only one of the Bodies is Elaſtic, provided the other be infinitely hard; for then the Impreſſion in the Elaſtic Body will be double of what it would have been, had they both been equally Elaſtic: And conſequently the Force with which they rebound, will be the ſame as if the Impreſſion had been equally divided between the two Bodies.

X. There are no Bodies, that we know of, either perfectly Elaſtic, or infinitely hard; the

the nearer therefore any Bodies approach to Perfection of Elaſticity, ſo much the nearer do the Laws, which they obſerve in the mutual Communication of their Motion, approach to thoſe we have laid down.

XI. Sir Isaac Newton made Trials with ſeveral Bodies, and found that the ſame Degree of Elaſticity always appeared in the ſame Bodies, with whatever Force they were ſtruck, ſo that the Elaſtic Power, in all the Bodies he made Trial upon, exerted itſelf in one conſtant Proportion to the compreſſing Force. He found the Celerity with which Balls of Wool, bound up very compact, receded from each other, to bear nearly the Proportion of five to nine to the Celerity wherewith they met; and in Steel he found nearly the ſame Proportion; in Cork the Elaſticity was ſomething leſs, but in Glaſs much greater; for the Celerity with which Balls of that Material ſeparated after Percuſſion, he found to bear the Proportion of fifteen to ſixteen to the Celerity wherewith they met *.

XII. We have hitherto ſuppoſed the Direction, in which Bodies impinge upon one another, to be perpendicular to the Surfaces: When it is not ſo, the Force of Impact will be leſs, by how much the more that Direction varies from the Perpendicular; for it is manifeſt that a direct Impulſe is the greateſt of all

* Newt. Princip. Phil. pag. 21.

others

others that can be given with the ſame Degree of Velocity *.

XIII. THIS is the Caſe, when Bodies impel one another by acting upon their Surfaces; but in Forces, where the Surfaces of Bodies are not concerned, as in Attraction, *&c.* we muſt not conſider the Relation which the Direction of the Force has to the Surface of the Body to be moved, but to the Direction in which it is to be moved by that Force. Here the Force of Action will be leſs, by how much the more theſe two Directions vary from each other †. My Meaning, in both Caſes,

* The Force of Oblique Percuſſion is to that of direct, as the Sine of the Angle of Incidence to the Radius.

Dem. Let there be a Plane as *AD* (*Fig.* 34.) againſt which let a Body impinge in the Point *D* in the Direction *BD*: Which Line may be ſuppoſed to expreſs the Force of direct Impulſe, and may be reſolved into two others (Chap. IV. §. 2.) *BC* and *BA*; the one parallel, the other perpendicular to the Plane; but that Force which is exerted in a Direction parallel to the Plane can no way effect it; the Stroke therefore ariſes wholly from the other Force expreſſed by the Line *BA*; but this is to the Line *BD*, as the Sine of the Angle of Incidence *ABD* to the Radius; from whence the Propoſition is clear.

If the Surface of the Body to be ſtruck is a Curve, then let *AD* be made a Tangent to *D* the Point of Incidence, and the Demonſtration will be the ſame.

† The Force of oblique Action is to that of direct, as the Co-Sine of the Angle comprehended between the Direction of the Force, and that wherein a Body is to be moved thereby, to the Radius.

Dem. Let *FD* (*Fig.* 35.) repreſent a Force acting upon a Body as *D*, and impelling it towards *E*; but let *DM* be the only Way in which it is poſſible for the Body to move; the Force

Caſes, will be underſtood from the Inſtance of a Ship under Sail. The Force by which the Wind acts upon the Sail will be leſs, by how much the more the Direction of the Wind varies from one that is perpendicular to the Surface of the Sail: But the Force of the Sail to move the Ship forward will be leſs, by how much the more the Direction of the Ship's Courſe varies from that, in which ſhe is impell'd by the Sail.

XIV. To this we may add the following Propoſition, relating to oblique Forces, *viz.* that, if a Body is drawn or impelled three different Ways at the ſame Time by as many Forces acting in different Directions; and if the Quantity of thoſe Forces is ſuch, that the Body is kept in its Place by them: Then will the Forces be to each other, as the ſeveral Sides of a Triangle, drawn reſpectively parallel to the Directions in which they act *.

CHAP.

Force *FD* may be reſolved (Chap. IV. §. 2.) into two others *FG* and *FH*, or which is equal to *GD*; but 'tis evident that only the Force *GD* impels it towards *M*. Now, *FD* being the Radius, *GD* is the Co-Sine of the Angle *FDG* comprehended between the two Directions *FE* and *GM*; from whence the Propoſition is clear.

* *Dem.* Let the Lines *AB*, *AD*, *AE*, (*Fig.* 36.) repreſent the three Forces acting upon the Body *A* in thoſe Directions, and by that Means keeping it at Reſt in the Point *A*. Then the Forces *EA* and *DA* will be equivalent to *BA*, otherwiſe the Body would be put into Motion by them (*contra Hypoth.*) But theſe Forces are alſo equivalent to *AC* (Chap. IV. §. 2.) conſequently *AC* may be made uſe of to expreſs the Force *AB*; and *EC*, which is parallel and equal to *AD*, may expreſs the Force

CHAP XII.

Of the Mechanical Powers.

I. HAVING, in the foregoing Chapter, accounted for the Communication of Motion by Impulſe; we proceed next to conſider Motion as communicated by Means of certain Inſtruments, commonly known by the Names of *Mechanical Powers*. The Uſe of theſe Powers conſiſts chiefly in managing great Weights, or performing other Works with a determinate Force.

II. THEY are uſually reckoned five, *viz.* the Lever, the Wheel and Axis, the Pulley, the Screw, and the Wedge; to which ſome add, the inclined Plane. To theſe all Machines, how complicated ſoever, are reducible.

III. THESE Inſtruments have been of very ancient Uſe; for we find that *Archimedes* was well acquainted with the Extent of their Power; as may be inferred from that celebrated Saying of his, Δὸς ποῦ στῶ, καὶ τὴν γῆν κινήσω. By which he meant, that the greateſt imaginable Weight might be moved with the ſmalleſt Power.

Force *AD*, while *AE* expreſſes its own: But *ACE* is a Triangle whoſe Sides are all parallel to the given Directions; therefore the Sides of this Triangle will expreſs the Relation of the Forces by which the Body is kept at Reſt. *Q. E. D.*

IV. THAT

IV. THAT Body, which communicates Motion to another, is called the *Power*; that which receives it, the *Weight*.

V. THAT Point in a Body, which remains at Reſt, while the Body is turning round, is called the *Center of Motion*. Beſides this, there are two other Centers in Bodies, 1. That of *Magnitude*, which is a Point, as near as poſſible, equally diſtant from all the external Parts of the Body; 2. That of *Gravity*, or that, about which all the Parts of the Body, in whatever Situation it is placed, exactly balance each other.

VI. WHEN a Body communicates Motion to another, it loſes juſt ſo much of its own, as it communicates to that other; the Action of one being equal to the Reaction of the other. See Chapter the laſt, §. 4, and 5.

VII. WHEN two Bodies have ſuch Relation to each other (ſuppoſe them fixed to different Parts of the ſame Machine) that if one be put into Motion, the other will thereby neceſſarily have ſuch a Degree of Velocity given it, that their Moments will be equal; it will then be impoſſible that one ſhould begin to move without communicating to the other a Quantity of Motion equal to its own; 'tis evident therefore from the laſt Propoſition, that if we ſuppoſe it to begin to move, in that very Inſtant it muſt loſe all its own Motion by communicating the whole to the other Body,

Body, and therefore, being left to itſelf, will remain at reſt, and communicate none at all.

Now the Moments of two Bodies are equal (Chap. XI. §. 2.) when the Velocity of the firſt is to that of the ſecond, as the Quantity of Matter of the ſecond to that of the firſt; or if we ſuppoſe their Quantities of Matter as one to three, then by the Suppoſition their Velocities are as three to one; and if we multiply the Quantity of Matter in the firſt, *viz.* one, by its Velocity three, and that of the other, *viz.* three, by its Velocity one; their Products are equal: Their Moments are therefore by the Definition (Chap. XI. §. 1. and 2.) equal. They will alſo be equal, when the Spaces the Bodies paſs over are in that Proportion; for the Times they both move in being the ſame, the Spaces will always be as the Velocities.

VIII. From hence it follows, that in any Machine, whether ſimple or compound, the Power, however ſmall, may have a Moment equal to that of the Weight; provided the Machine be ſuch, that when it is in Motion, the Velocity of the Power ſhall bear ſuch Proportion to that of the Weight, as the Weight does to the Power; for then, what the Power wants in Quantity of Matter or Weight, will be made up in Velocity; conſequently their Moments will be equal by the laſt §. and therefore, by §. 7. they will exactly balance each other; or be in *Æquilibrio*.

IX. But

IX. But if the Power bears a greater Proportion to the Weight, than the Velocity of the Weight to that of the Power, it will then have a greater Momentum than the other; ſo that though the other receives all its own Moment from it when the Machine moves, yet there will ſome remain, which, if it be ſufficient to overcome the Friction of the Machine, will keep it moving.

We proceed now to treat of each Mechanical Power in its Order, and

I. Of the Lever.

X. The Lever is a right Line (or Bar whoſe Weight in Theory is not conſidered) moveable on a Center, which is called its *Fulcrum*, or *fixed Point*.

XI. The Æquilibrium in this Machine is, when the Diſtance of the Power from the fixed Point is to that of the Weight from the ſame, as the Quantity of Matter in the Weight is to that in the Power.

For, ſuppoſing the Lever placed on its Fulcrum, with the Weight to be raiſed at one End, and the Power applied to the other; 'tis evident, the farther the Power is placed from the Fulcrum, or Center of Motion, the larger will be its Sweep when the Machine is put in Motion; that is, it will move over proportionally more Space in the ſame Time than the Weight to be raiſed: Now, if it is placed juſt

ſo

ſo many Times farther from the Fulcrum, as it is leſs than the Weight, it will move juſt ſo many Times faſter; their Moments therefore will be equal (§. 7.) and conſequently the Power and Weight will exactly balance each other, or be in Æquilibrio *. And, if the Power is ſufficiently augmented to overcome the Friction of the Machine, it will put it in Motion.

THE Lever is of three Kinds. 1. When the fixed Point is between the Weight and the Power, as in the laſt Caſe. 2. When the Weight is between the fixed Point and the Power. 3. When the Power is between the fixed Point and the Weight.

IN all which Caſes the Æquilibrium will be, when their Diſtances from the fixed Point are ſuch, that their Velocities ſhall be inverſely as their Quantities of Matter; for then (by §. 7.) being at Reſt, neither of them will communicate any Motion to the other.

THE common Scales may be conſidered as

* Geometrically thus. Let *AB* (*Fig.* 37) repreſent the Lever, *F* the Fulcrum, *W* the Weight, *P* the Power, the one ſuſpended at the Extremity of the Lever *A*, the other at *B*, and let *BF* be to *FA* as *W* to *P*; then while the Lever moves from the Situation *AB* into that of *CD*, the Point *B* which ſuſtains the Power will move as many Times farther than *A* which ſuſtains the Weight, (and conſequently as many Times faſter, ſince they perform their Motions in the ſame Time) as the Arch *BD* is longer than *AC*; that is, the Triangles *BFD* and *AFC* being ſimilar, as the Arm *BF* is longer than *AF*, which (*ex Hypoth.*) is as many Times as the Weight exceeds the Power, they will therefore (§. 7.) be in Æquilibrio. *Q. E. D.*

a Lever

a Lever of the firſt Kind, where the Weight and Power are applied at equal Diſtances from the fixed Point.

THE Steelyard is alſo a Lever of the firſt Kind, but whoſe Arms are unequal.

THE Difference between the Uſe of the Scales and the Steelyard conſiſts in this; that as in one you make uſe of a larger Power (or more Weights) to eſtimate the Weight of an heavier Body; in the other you uſe the ſame Power, but give it a greater Velocity, with reſpect to that of the Weight, by applying it farther from the fixed Point, which by §. 7. will have the ſame Effect.

II. The WHEEL and AXIS.

XII. THIS Machine is a Wheel, that turns round together with its Axis; the Power in this is applied to the Circumference of the Wheel, and the Weight drawn up by Means of a Rope wound about the Axis.

XIII. IN this there will be an Æquilibrium, when the Weight is to the Power, as the Diameter of the Wheel to the Diameter of the Axis.

'Tis evident, the Velocity of the Power will exceed the Velocity of the Weight, as many Times as the Circumference of the Wheel exceeds that of its Axis; becauſe the Spaces they paſs over in one Revolution will be as thoſe Circumferences; that is, as many Times as the

Diameter of one exceeds that of the other (the Circumferences of Circles being as their Diameters;) what therefore in this Caſe the Power wants in Weight will be made up in Velocity, from whence (§. 7.) there will be an Æquilibrium *.

The Uſe of this Machine is to raiſe Weights to greater Heights than the Lever can do, becauſe the Wheel is capable of being turned ſeveral Times round, which the Lever is not; and alſo to communicate Motion from one Part of a Machine to another; accordingly there are few compound Machines without it.

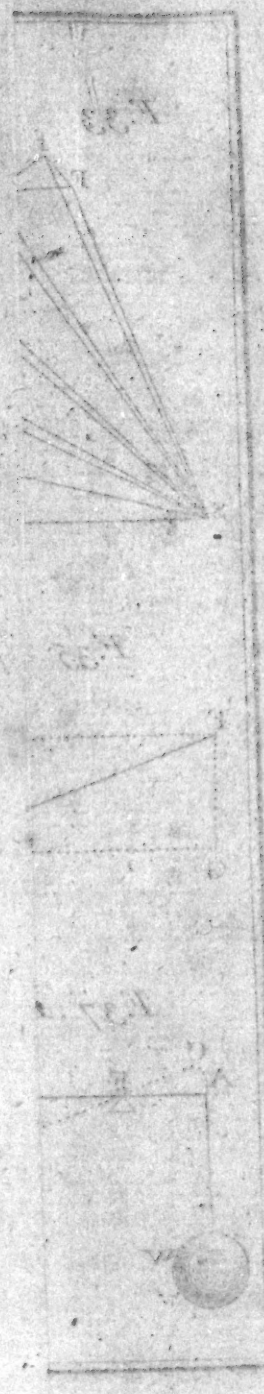
III. The Pulley.

XIV. A Pulley is an Inſtrument compoſed of one or more Wheels moveable on their Axes.

XV. A ſimple Pulley, if its Axis is fixed, is of no other Uſe, than to alter the Direction of the Power; for the Power and Weight will both move through an equal

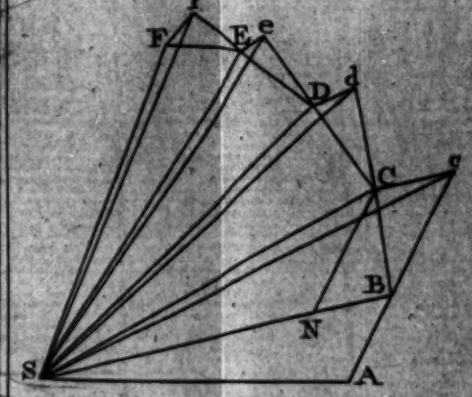
* Geometrically thus. Let *AB* (*Fig.* 38.) be the Diameter of the Wheel, *DE* that of the Axis, *W* the Weight, and *P* the Power; when the Wheel begins to move, the Point *B* and *D* will deſcribe ſimilar Arches about the Center *C*, in the ſame Manner the Point *A* and *B* in the Lever were ſhewn to do about the fixed Point *F* (*Fig.* 37.) that is, the Point *B* will move as many Times faſter than *D*, as *CB* is longer than *CD*, or *AB* than *DE*, the Motion therefore of *P* (§. 7.) will be equal to that of *W*. From whence the Propoſition is clear.

Space

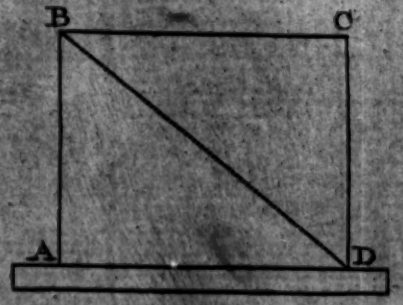


P.71. P.82

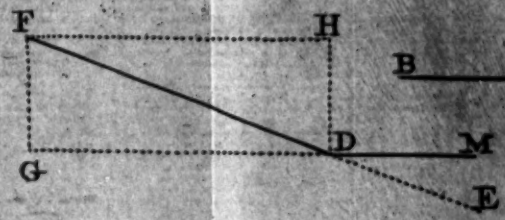
F.33 P.65.



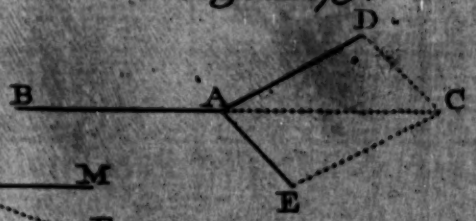
F.34 P.74



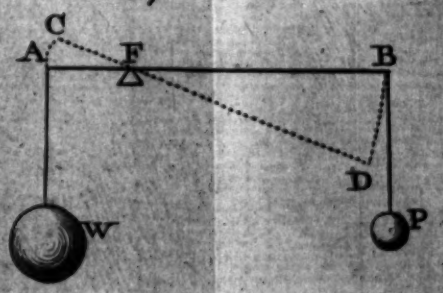
F.35. P.74.



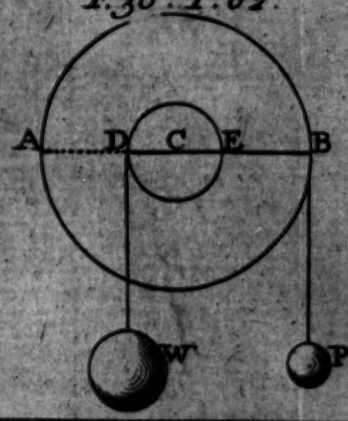
F.36. P.75.



F.37. P.80.



F.38. P.82.



Space in the ſame Time. But in a Pulley not fixed, as in *Fig.* 39. where the Rope runs under it, or in a Combination of Pullies, as in *Fig.* 40. the Æquilibrium will be, when the Power is to the Weight, as one to the Number of Ropes, that paſs between the upper and lower Pullies.

For, ſuppoſe one End of the Rope fixed in B (*Fig.* 39.) and the other ſupported by the Power P, it is evident, that in order to raiſe the Weight W one Foot, the Power muſt riſe two, for both Ropes, *viz.* BC and CP, will be ſhortened a Foot a-piece, when the Space run over by the Power will be double to that of the Weight; if therefore the Power is to the Weight as one to two, their Moments will be equal: For the ſame Reaſon, if there be four Ropes paſſing from the upper to the lower Pullies as in *Fig.* 40. the Velocity of the Power will be quadruple to that of the Weight, or as four to one, *&c.* In all Caſes therefore when the Power is to the Weight, as one to the Number of Ropes paſſing from the upper to the lower Pullies (§. 7.) there will be an Æquilibrium.

XVI. If the Pullies be diſpoſed as in Figure the 41ſt, each having its own particular Rope, the Action of the Power will be very much increaſed; for here every Pulley doubles it, wherefore the Power is four Times greater with two Pullies, eight Times with three,

ſixteen Times with four, *&c.* For, it is evident from the Conſideration of the Figure, the firſt will move half as faſt as the Power, the ſecond half as faſt as that, and ſo on; wherefore (§. 7.) the Power is doubled by each Pulley.

THE Uſe of the Pulley is nearly the ſame with that of the Wheel and Axis, but it is more portable, and eaſier to be fixed up.

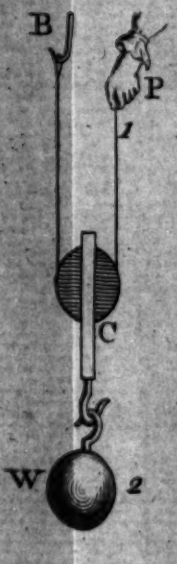
IV. The SCREW.

XVII. IN this Machine the Æquilibrium will be, when the Power is to the Weight, as the Diſtance between any two contiguous Threads or Spirals in the Screw, to the Way deſcribed by the Power in one whole Revolution. It is manifeſt, from the Form of the Machine, (*Fig.* 42.) that in one Revolution of the Screw, the Weight will be moved through a Space equal to the Diſtance of two contiguous Threads, and that the Power will run through a Space equal to the Compaſs it takes in one Revolution, therefore (§. 7.) if the Weight is to the Power in that Proportion, there will be an Æquilibrium.

THIS Machine is of great Force, and very uſeful in retaining Bodies in a compreſſed State, becauſe it will not run back, as the three foregoing will, when the Power is removed. This ariſes from the great Friction

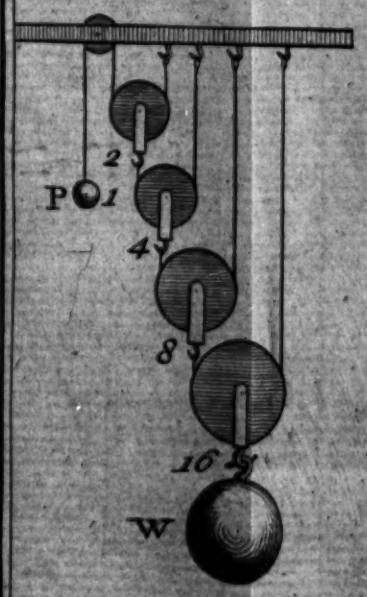
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F. 39 P. 83 F. 40

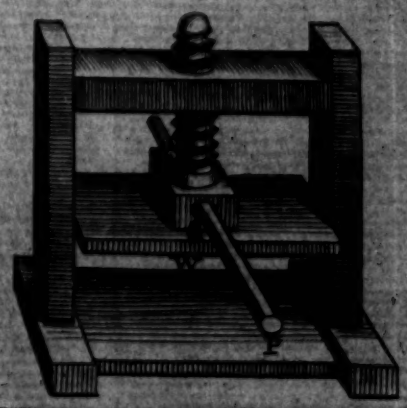




F. 41.



F. 42. P. 84.



of thoſe Parts in the Screw, which, during its Motion, ſlide upon thoſe that are at Reſt.

V. The Wedge.

XVIII. This Inſtrument is formed by two equal Rectangles, joined at their lower Baſes, and ſeparated at their upper ones, by a third; which is called the *Back* of the *Wedge*; the other two, its *Sides*.

XIX. In the foregoing Mechanical Powers we have along conſidered the Weight, as moved in the ſame Direction with that, in which it is acted upon by the Machine, as is commonly the Caſe; but in this, the Weight is generally applied in ſuch a Manner, as to be made to move in a Direction different from that in which it is protruded by the Wedge; hence it is, that Mathematicians have widely differed in their Determination of the Power of this Machine, ſome conſidering the Weight as moved by it in one Direction, and ſome in another. Nay, there are ſome, even among the late Writers, that have fallen into manifeſt Errors in their Reaſonings about it. We will therefore lay down the ſeveral Proportions they have given us, for the determining the Power of this Machine, and examine them one by one. 1. It is demonſtrated by ſome, that the Power will be equivalent to the Reſiſtance of the Weight, when it bears ſuch Proportion to it, as the Breadth of the Back

of the Wedge does to the Sum of its Sides; or, which is the ſame Thing, as half that Breadth to one of its Sides. 2. Others make it ſomewhat larger, and demonſtrate, that it ought to be, as half the Breadth of the Back to the perpendicular Height of the Wedge. 3. Some are of opinion, that there will not be an Æquilibrium in this Machine, unleſs the Power is to the Weight, as the whole Breadth of the Back to the perpendicular Height; *viz.* WALLIS, KEIL, GRAVESANDE, *&c.*

THOSE, who lay down the firſt Proportion for determining the Power of this Machine, ſuppoſe the Parts, which are ſeparated from each other thereby, to recede from the firſt Situation in Directions perpendicular to the Sides of the Wedge. Thus let BAC (*Fig.* 43.) repreſent a Wedge; P, P, two Bodies to be ſeparated by it, the one to be moved towards I, the other towards F, in the Directions CI, and CF, perpendicular to AC and CB; then 'tis evident, that when the Wedge is driven into the Situation MNO, the two Bodies will be moved to Q and Q; that is, one will have paſſed through the Space CK, the other through CL, but theſe Spaces being equal, their Velocities are the ſame as if they had both paſſed over one of them, *v. g.* CL, or, which is equal to it, DG (drawn perpendicular to CB); therefore the Power, which we ſuppoſe applied to D, moves through DC, while the

the Obſtacle moves through DG, conſequently (§. 7.) when the Power is to the Weight as DG to DC, that is, as DB to CB *, or half the Back of the Wedge to one of its Sides, they will be in Æquilibrio. This Proportion therefore, when the Parts of the Weight are ſuppoſed to be moved by the Wedge in the Directions CI and CF, is true.

2. The ſecond Proportion is alſo true, ſuppoſing the Bodies P, P, to recede from each other in the Directions CN and CM, parallel to AB the Back of the Wedge; for, when the Wedge is driven in between them, to the Situation MNO, the Bodies will have moved through a Space as CN, or, which is equal to it, DB, half the Back of the Wedge, and the Power through a Space equal to its Height as before; conſequently (§. 7.) in this Caſe, the Æquilibrium will be, when the Power is to the Weight, as half the Back of the Wedge to its Height †.

3. Thoſe,

* For (8 *Elem.* 6.) the Triangle *DCG* and *DCB* are ſimilar, and conſequently *DG* : *DC* : : *DB* : *CB*.

† The ſame may be otherwiſe demonſtrated from Section 14. Chapter XI. thus. Let there be a Body as *L* (*Fig.* 44.) drawn againſt the Wedge *ABC* by the Weight *W*, in the Direction *LF*, parallel to the Back of the Wedge *AB*; but prevented from ſliding down towards *C*, by a Plane (whoſe upper Surface we may ſuppoſe repreſented by *EF*) lying under it. I ſay, the Power will be to the Weight, when they are in Æquilibrio, as *DA* to *DC*.

3. THOSE, who imagine there will not be an Æquilibrium, unleſs the Power be to the Weight, as the whole Breadth of the Back of the Wedge to its Height, ſuppoſe, as in the laſt Caſe, that the Bodies to be ſeparated recede from each other in Directions parallel to the Back of the Wedge; and endeavour to ſupport their Opinion by the following Argument; *viz.* that, when the Wedge is driven into the Situation MNO (*Fig.* 43.) as before, each Part of the Weight having moved through a Space equal to half the Back of the Wedge, the whole Weight has therefore moved through

Dem. The Body *L* is here acted upon in three Directions, *viz.* by the Force of the Weight *W* in the Direction *LF*, by the two Planes *CA* and *EF*, in the Directions *LG* and *LI*, perpendicular to their Surfaces; let *GE* be drawn parallel to *LI*, then will the Triangle *LGE* have all its Sides reſpectively parallel to thoſe Directions; conſequently (Chap. XI. § 14.) if we ſuppoſe *LE* to expreſs the Force of the Weight, *GE* will expreſs that of the ſuppoſed Plane, or which is the equal to it, becauſe they act in contrary Directions to that of the Power: But *GE* is to *EL*, as *DA* to *DC* (for the Triangles *EGL* and *DAC* are ſimilar, the Sides of one being *ex Conſtruct.* reſpectively perpendicular to thoſe in the other; v. g. *LG* to *CA*, *EL* to *DC*, and *GE* to *DA*); conſequently the Power is to the Weight, when they balance each other, as half the Breadth of the Back of the Wedge to its Height. *Q. E. D.*

Coroll. Suppoſe the Body *L* had been drawn againſt the Wedge in the Direction *GL* perpendicular to its Surface, and were to be moved by it in the contrary Direction towards *G*, as in the firſt Caſe; then if *GL* expreſſes the Force with which it is drawn towards the Wedge, *GE* will be that with which it reſiſts the Power; but *GE* is to *GL* as *DA* to *AC*, the Triangles *EGL* and *DAC* being ſimilar; conſequently in this Caſe, the Power will be to the Weight, as half the Breadth of the Back of the Wedge to one of its Sides; as was before demonſtrated.

twice

twice ſo much, or a Space equal to the whole Back: As much as to ſay, the Whole has moved farther than its Parts; which is abſurd.

XX. To determine the Power of this Machine in general, let ABC, in *Fig.* 43. repreſent an Iſoſceles Wedge, whoſe Sides are AC and BC, and its Back AB; draw the Line CD perpendicular to AB; and let R, R, repreſent two equal Bodies to be ſeparated by the Wedge, and ſuppoſe them confined by Planes put under them, or otherwiſe, to move in the Directions *m*T and *s*Z only, that are equally inclined to the Sides AC and BC reſpectively: And let the Wedge be driven in the Direction DC. Let *m* be the Point where the Body R touches the Wedge, and through *n* any Point in the Line *m*T draw the Line *rn* parallel to DC, and parallel to *m*T draw DX meeting CB (produced if neceſſary) in X; then the Triangles *mnr* and CDX will be ſimilar, the Sides of the one being reſpectively parallel to thoſe of the other. And becauſe the Bodies to be ſeparated are equal, and the Wedge Iſoſceles, the Lines *m*T and *s*Z equally inclined to the Sides CB and CA, and the Wedge driven in the Direction DC, the Bodies will be both moved with equal Degrees of Velocity; and while the Wedge moves downwards a Space equal to *rn*, each of the Bodies will be driven in the Directions *m*T and *s*Z over a Space equal to *mn*: If *rn* therefore be taken to meaſure the Velocity of the Wedge,

Wedge, *mn* will meaſure that of both the Bodies; for as to their moving different Ways, that makes no Difference, becauſe they are as free to move one Way as another. Since then the Triangles *mnr* and DCX are ſimilar, and therefore the Proportion of *rn* to *mn* the ſame as that of DC to DX, the Velocity of the Wedge will be to the Velocity with which the Bodies R, R, are driven by it, as DC to DX; and conſequently, whatever Forces, acting in the Directions T*m* Z*s*, oppoſe the Motion of the Wedge, a Force of the ſame Kind, acting upon the Back of the Wedge at D in the Direction DC, and puſhing it forwards, will be in Æquilibrio therewith, when it bears ſuch Proportion to them, as DX does to DC: And therefore, if DC be taken to meaſure the Reſiſtance of the Bodies to be moved by the Wedge, DX will always be the Meaſure of the Power requiſite to make an Æquilibrium therewith.

From this general Theorem the particular Caſes that fall under it are readily deduced. Thus, if the Bodies R, R, are confined to move upwards in Directions parallel to the Sides of the Wedge, *m*T will lie parallel to CB; and therefore DX the Meaſure of the Power neceſſary to make an Æquilibrium, being to be drawn parallel to *m*T, muſt be drawn parallel alſo to CB; it will therefore meet CB produced at no leſs than an infinite Diſtance, and will conſequently itſelf be infinite: That is to ſay,

no

no finite Power will be able to drive the Wedge any farther between the Bodies thus confined. If the Bodies are confined to an horizontal Motion, *m*T becomes parallel to AB the Back of the Wedge, and DX (parallel to *m*T) falls upon DB, and DB half the Back of the Wedge is the Meaſure of the Power requiſite to conſtitute the Æquilibrium. If the Bodies are confined to move off at right Angles to the Sides of the Wedge, DX falls upon DG, and DG meaſures the Power; conſequently, DG being ſhorter that any other Line that can be drawn from D to the Line CB, the Power in this Caſe neceſſary to make an Æquilibrium with the Reſiſtance of the Bodies, is what the Mathematicians call a *Minimum*; that is, it is leſs in this than in any other Suppoſition whatever. And if the Bodies are to be moved perpendicularly downwards, X falls upon C, and DC becomes the Meaſure of the Power: But DC is all along ſuppoſed to meaſure the Reſiſtance of the Bodies to be moved by the Wedge; the Power therefore and the Reſiſtance are now equal to each other; which implies that the Bodies in theſe Circumſtances are capable of being moved either with or without the Wedge with equal Eaſe. If the Bodies are confined to move downwards in Directions parallel to the Sides of the Wedge, DX becomes parallel to BC, the Point X falls upon it infinitely below C, and therefore DX, and conſequently

quently the Power to be meaſured by it, becomes infinite again; that is the Bodies cannot be driven by the Wedge in thoſe Directions by any Power whatever. When the Bodies R, R, are not confined, but are at Liberty to move in any Direction whatever, they will naturally take that in which they may be driven by the Wedge with the greateſt Eaſe; that is, they will take that which is perpendicular to its Sides; and therefore the Perpendicular DG, which was the Meaſure of the Power neceſſary to make the Æquilibrium, when the Bodies were confined to move in that Direction, and was the *Minimum*, will be the Meaſure of it alſo in this Suppoſition.

XXI. The conſidering the Bodies to be ſeparated by the Wedge as confined to move from each other in particular Directions, leads us to an eaſy Solution of a Caſe relating to one of the Uſes of this mechanical Power, I mean the Dividing of Bodies therewith. To determine the Power of it in this, let us conceive the two Pieces LQ and LQ as turning upon a Joint at L, which Joint is to be opened by the Wedge ACB, driven in between the Pieces, and preſſing againſt them only at their Extremities G and H, as repreſented in *Fig.* 45. And let us ſuppoſe that there is a certain Degree of Stiffneſs in the Joint, by which Stiffneſs may aptly be denoted the Tenacity or Coheſion of the Parts of the Body to be divided. Let the Lines GF

and

and HI be drawn perpendicular to the Lines LG and LH, which will therefore be the Directions, which thoſe Extremities will be confined to, when they begin to move: Parallel to one of theſe, as GF, draw DX; then ſince, as we have juſt been ſhewing, if two Bodies, confined to move in the Directions GF and HI only, were to be ſeparated by the Wedge, the Power neceſſary to make an Æquilibrium with the Reſiſtance of thoſe Bodies would be to that Reſiſtance as DX to DC; or that, if DC be taken to meaſure the Reſiſtance they make, DX ſhall meaſure the Power that ſhall cauſe an Æquilibrium between them, it follows, that, in this Caſe, if we meaſure the Force, which, if applied at G and H in the Directions GF and HI, would overcome the Tenacity at L, and rend the Pieces aſunder without a Wedge, DX will be the Meaſure of the Force, which would, with equal Eaſe, perform the ſame Thing by the Help of the Wedge. And when the Point comes infinitely near to L, and DX, parallel to GF, becomes therefore perpendicular to CB, and conſequently the Triangles DCX and DCB become ſimilar (8 *Elem.* 6.) the Power neceſſary to rend the Pieces with the Wedge will be to that which would do the ſame without it, as DX to DC; that is, becauſe of the Similarity of the ſaid Triangles, as DB to BC, or as half the Back of the Wedge to one of its Sides.

BUT, if we have no regard to the Stiffneſs of the Joint L, and conſider only the Quantity of Matter in the Power that drives the Wedge, and in the Bodies LQ and LQ to be turned upon that Joint, neceſſary to make an Æquilibrium between them, as is the uſual Method in Theory, and obſerved in all the other mechanical Powers, and the Thickneſs of the Pieces be the ſame from End to End, and their Subſtance uniform, ſo that their Centers of Gravity may be in the Mid-way between L and G, or L and H, or any where in the Arches *bc* and *de*, whoſe Centers are at L, and which cut the Lines LG and LH in the Middle, the Æquilibrium will be, when the Quantity of Matter in the Power is to the Quantity in the Bodies LQ and LQ, as half DX to DC, and when the Point of the Wedge comes infinitely near to L, and DX becomes perpendicular to BC, and the Triangles DCX and DCB ſimilar (as above) as half DB to BC, or as a Quarter of the Back of the Wedge to one of its Sides: Becauſe, as is evident, in this Suppoſition the Centers of Gravity of the Pieces will have but half the Velocity they would have had, had all Parts of them been to be moved as faſt as the Extremities G and H; half the Power will therefore be ſufficient to move them that would have been required in that Caſe; but in that the Quantity of Matter in the Power muſt have been to the Quantity in the reſiſting Bodies, as DX to DC,

and,

and, when C comes infinitely near to L, as DB to BC; it is therefore, in the preſent ſuppoſition, in the former of theſe Caſes, as half DX to DC, and in the latter, as half DB to BC, or as a fourth Part of the Back of the Wedge to one of its Sides.

Or this Propoſition may be conſidered in another Light, thus: The Centers of Gravity of the two Pieces LQ and LQ being in the Arches *bc* and *de*, that cut LG and LH in the Middle, they are carried upon the Levers LG and LH in the Mid-way between their common Fulcrum L and their Extremities G and H; the Wedge therefore has not only a mechanical Advantage over them, in Conſequence of its own Form, in the Proportion of DX to DC, but in Conſequence alſo of their being carried upon the ſaid Levers, in the Proportion of two to one; the Wedge therefore has twice the Power over them it would otherwiſe have; and conſequently half the Quantity of Matter in the Power that drives the Wedge is ſufficient to conſtitute an Æquilibrium that is requiſite where no ſuch Levers are concerned.

XXII. The Form of the *Inclined Plane* being no other than that of half a Wedge, as is manifeſt from the Repreſentation of it (*Fig.* 46.) it follows, that what has been demonſtrated of the one, may be applied to the other, and the Properties of both will be found the ſame. For Inſtance, if the Weight W is to be drawn

up

up the Plane CB, by the Power P, in a Direction parallel to the Plane, inſtead of that, we may ſuppoſe the Weight prevented from running off the Plane by the String W H, and the inclined Plane driven under it like a Wedge in the Direction DC: Then will the Weight riſe towards G in a Direction perpendicular to CB, for we muſt always ſuppoſe the String WB parallel to the Plane (as it would have been, if the Weight had been drawn up by it;) then will the Action of the Plane upon the Weight be ſimilar to that of the Wedge in the firſt Caſe, and conſequently the Power will bear ſuch Proportion to the Weight, as DB to BC; that is, as the Height of the Plane to its Length. Again, ſuppoſe the Weight was to have been drawn up the Plane by a String in the Direction WF parallel to CD the Baſe of the inclined Plane; then if the Plane be driven under the Weight as before, it muſt riſe in a Direction perpendicular to CD, that is, parallel to DB: Then the Caſe will be analogous to the ſecond of the Wedge; conſequently, the Power will be to the Reſiſtance of the Weight, when there is an Æquilibrium, in the Proportion of DB to DC, as there demonſtrated.

XXIII. THESE are the Powers or Machines which, under different Forms, conſtitute all others how complicated ſoever; and as the Æquilibrium, in any one of theſe, is, when the Power

Power and Weight are inverſely as their Velocities; ſo in a Machine, however compounded, the Power and the Weight will exactly balance each other, when they are in this Proportion; for by §. 7. their Moments will then be equal, and the Machine, if at Reſt, will continue in that State; and, if put into Motion by an external Force, will gradually loſe it when that Force ceaſes to act; on Account of the unavoidable Friction of the Machine, and the Reſiſtance of the Air, which it muſt neceſſarily meet with, unleſs its Motion could be performed in a perfect Vacuum. From hence we ſee the Impoſſibility of contriving an Engine, whoſe Motion ſhould be *perpetual*, that is, ſuch as does not owe its Continuance to the Application of ſome external Force; a Problem that has given Birth to an almoſt infinite Number of Schemes and Contrivances. For unleſs ſome Method could be found out of gaining a Force, by the artful Diſpoſition and Combination of the Mechanical Powers, equivalent to that which is continually deſtroyed by Friction, and the Reſiſtance of the Air, the Motion which was at firſt given to the Machine, muſt at length be neceſſarily loſt. But we ſee, that thoſe Inſtruments are only different Means, whereby one Body communicates its Motion to another, and not deſigned to produce a Force which had no Exiſtence before. 'Tis for Want of a due

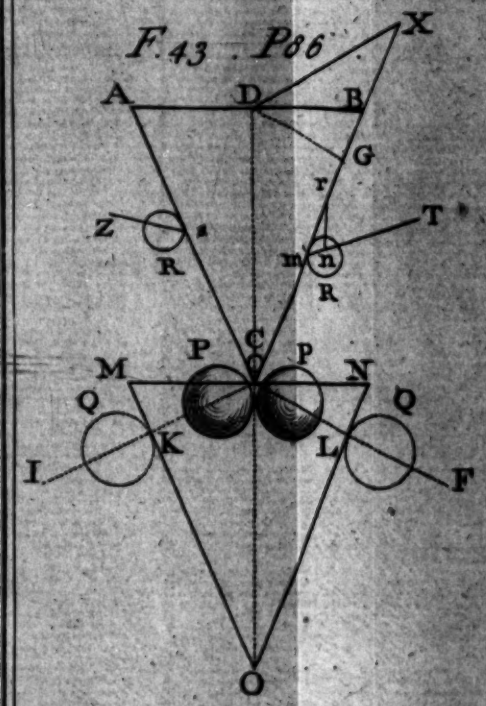
due Conſideration of this, that ſo many Mechanical Deſigns have proved abortive, ſo many Engines unequal to the Performance for which they were deſigned, and ſo many Impoſſibilities attempted.

"IF it were poſſible, ſays Biſhop WILKINS, "to contrive ſuch an Invention, whereby "any conceivable Weight may be moved by "any conceivable Power, both with equal "Velocity (as it is in thoſe Things which are "immediately ſtirred by the Hand, without "the Help of any other Inſtrument) the "Works of Nature would be then too much "ſubjected to the Power of Art; and Men "might be thereby encouraged (with the "Builders of *Babel*, or the Rebel Giants) to "ſuch bold Deſigns as would not become "a created Being. And therefore the Wiſdom "of Providence has ſo confined theſe Human "Arts, that what an Invention hath, in the "*Strength* of its Motion, is abated in the *Slowneſs* of it; and what it has, in the extraordinary *Quickneſs* of its Motion, muſt be allowed for in the great *Strength* requiſite in "the Power which is to move it *."

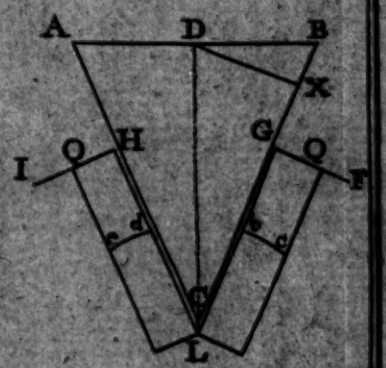
* *Wilkins*'s Mathem. Magick, p. 104.

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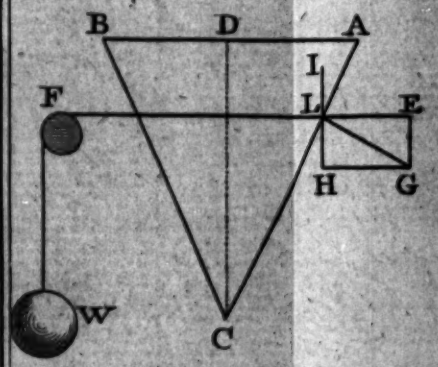
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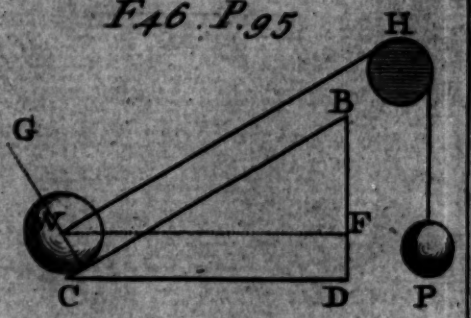
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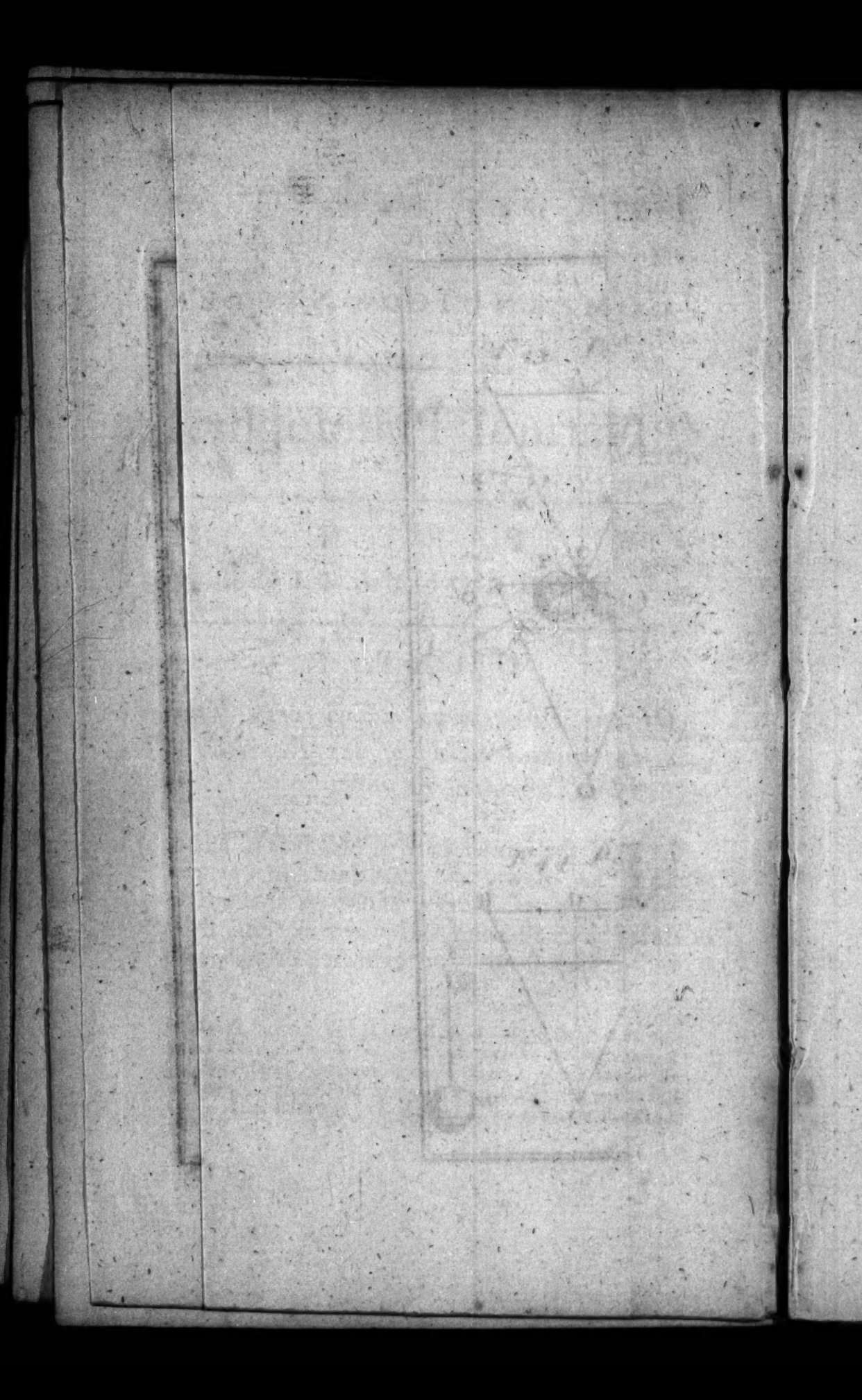


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A COMPENDIOUS SYSTEM OF Natural Philoſophy.

PART II. HYDROSTATICS.

CHAP. I.

Of the Phænomena which ariſe from the mutual Action of the Particles of Fluids upon one another.

I. IN the former Part of this Eſſay, I have laid down and explained the general *Laws of Nature*, and from thence deduced thoſe Phænomena, which are in a ſtrict and proper Senſe * denominated *Mechanical*: I pro-

* In a larger Senſe, all the Effects and Operations of natural Bodies upon one another may be called Mechanical; as being all ſubject to the general Laws of Motion. In Hydroſtatics Fluids are governed by the Laws of Mechaniſm, as much as the Mechanical Powers themſelves are; the ſame holds of the

I proceed now to an Explanation of ſuch, as Philoſophers have comprehended under the Name of *Hydroſtatics*; the Intention of which is to explain the Nature of *Fluids*, and the Manner wherein they act upon one another, and upon Solids.

II. The Nature of a *Fluid* *, as diſtinguiſhed from that of a *Solid*, or hard Body, conſiſts in this, *viz.* that its Particles are ſo looſely connected together, that they readily move out of their Places, when preſſed with the leaſt Force one Way more than another †. From whence Philoſophers conclude, that they

Rays of Light, as will be ſeen when we come to Optics; and in the larger Bodies of the Planetary Syſtem, Mechaniſm equally prevails, as has been demonſtrated by Sir *Iſaac Newton*; which we ſhall endeavour to make out when we treat of the Phyſical Cauſe of the Motion of the Heavenly Bodies.

* Some Philoſophers make the following Diſtinction in Fluids; thoſe which flow or ſpread themſelves till their Surfaces become level or horizontal, they call *Liquid*; in Contradiſtinction to Flame, Smoak, Vapour, &c. which are alſo Fluids, but do not acquire ſuch a Surface. Thoſe, which are capable of exciting in us the Idea of Moiſtneſs, as Water, &c. they call *Humid*, diſtinguiſhing them thereby from Air, Quickſilver, and melted Metals. But thoſe Diſtinctions are quite unneceſſary in a Philoſophical Senſe; the Surfaces of all Fluids being level, when not prevented by the Bodies about them; and Humidity is only a *relative* Quality; for tho' Quickſilver will not moiſten, or ſtick to a Man's Hand, it will to Silver or Gold.

† The common Definition, *Fluidum eſt cujus partes impreſſioni cuicunque cedunt, & cedendo facillimè moventur inter ſe*, though it expreſſes very well the Nature of a *compreſſible* Fluid, as Air, yet does not correſpond to that of Water, whoſe Parts have been found to yield to no Force, with which they have been compreſſed,

they are exceedingly minute, ſmooth, and round *; it being otherwiſe impoſſible they ſhould

compreſſed, unleſs it was greater on one Side than on the other. The Definition therefore ſeems imperfect, as not expreſſing that *Inequality* of Preſſure, which is requiſite to move the Parts of ſome Fluids one among another.

* It is commonly obſerved that the Roundneſs of the Particles conduces very much to Fluidity, not only as it diſpoſes them to move one among another with greater Facility, but becauſe round Bodies touching one another in few Points only, the Force with which they mutually attract each other, is the weaker. But, upon this Suppoſition, the Particles of a Fluid ought to move with leſs Freedom one among another, by how much the greater the Weight is with which they are compreſſed, (for it is the ſame Thing in this Reſpect, whether they preſs againſt each other by Virtue of their own Attraction, or by ſome external Force) but of this we have no Experience. A Diver, upon plunging out of his Bell at the Bottom of the Sea, never finds the Water leſs fluid, notwithſtanding the great Preſſure from above. Mr. *Boyle* having cauſed a Tadpole to be put into a Veſſel of Water, and to be preſſed with a very great Force, tells us, that in Appearance it found no Inconvenience from thence, but ſwam about with the ſame Freedom and Briſkneſs as ever.

Quere, Whether the Particles of which Fluids conſiſt, are in Contact with each other, or not? Perhaps, they are prevented from approaching nearer than to a certain Diſtance, by a repelling Power, diffuſed around each ſingle Particle. The Obſervation, that Water is not render'd leſs fluid by Preſſure, ſeems to favour this Opinion; and the Property, which the Air has of expanding or contracting itſelf, according to the Weight which it ſuſtains, (as ſhall be ſhewn, Chap. 3.) proves beyond Contradiction, that its Particles are endued with ſuch a Power. But then if the Particles of all Fluids have this Power, it will follow that they ought to be in ſome Meaſure capable of being reduced into leſs Space by Preſſure, as Air is; which they have not as yet been ſhewn to be. Further, ſince it has been proved (Part I. Chap. 3.) that if the Parts of Fluids are placed juſt beyond their natural Diſtances from each other, they will approach and run together, and if placed farther aſunder ſtill, will repel each other; it follows, upon the foregoing Suppoſition, that each

ſhould move with ſuch Freedom one among another, upon the leaſt Inequality of Preſſure.

III. Thoſe Particles, conſidered ſeparately, are endued with all the common Properties of

Particle of a Fluid muſt be ſurrounded with three Spheres of Attraction and Repulſion one within another: the innermoſt of which is a Sphere of Repulſion, which keeps them from approaching into Contact; the next, a Sphere of Attraction diffuſed around this of Repulſion, and beginning where this ends, by which the Particles are diſpoſed to run together into Drops; the outermoſt of all, a Sphere of Repulſion, whereby they repel each other, when removed out of that Attraction.

Now, if this *Hypotheſis* ſhould be found to be true; and we might, agreeably to the above-mentioned Conſequence of it, ſuppoſe, that the Particles of all Bodies attract and repel each other *alternately* at different Diſtances, perhaps we might be able to ſolve a great many Phænomena relating to ſmall Bodies, which now lie beyond the Reach of our Philoſophy. However, upon the Suppoſition of the three Spheres of Attraction and Repulſion juſt mentioned, nothing is more eaſy, than to ſee how Solids may be converted into Fluids, and Fluids into Solids (as is done in Liquefaction and Freezing;) for allowing, that the firſt or innermoſt Sphere of Repulſion is capable, like that of the Particles of Air, of being augmented by Heat, and diminiſhed or totally ſuſpended by Cold, it follows, that Bodies muſt be more or leſs fluid, in Proportion to the Degree in which they are affected by Heat or Cold: for when the Action of the firſt Sphere of Repulſion is diminiſhed or deſtroyed by Cold, the Particles of the Fluid muſt neceſſarily be brought into cloſer Contact with each other by the Force of the circumambient Attractions, and by that Means conſtitute an harder Body than before. But, we muſt not dwell too much upon an Hypotheſis which wants Proof; I ſhall only add, that altho' ſome Fluids, as Water, have not been *as yet* contracted in their Dimenſions, or made to take up leſs Space than they naturally do, by any Force with which they have been compreſſed by *Art*; yet there are none but are *naturally* contracted by Cold; from whence it ſeems reaſonable to infer, that their Particles are at leaſt *capable* of being brought into cloſer Contact, which is ſome Confirmation of this Doctrine.

It

of Matter, and ſubject to the ſame Laws of Motion and Gravitation with larger Bodies. To enquire therefore into the Nature of Fluids, is to conſider, what Appearances a Collection of very ſmall round Bodies, ſubject to thoſe Laws, will exhibit under different Circumſtances. In order to which, it is uſual with Hydroſtatical Writers to conſider a Fluid, as divided into ſeveral perpendicular Columns contiguous to each other. Sometimes it is convenient to conceive it divided into thin Plates or *Strata* lying upon one another. In ſome Caſes, the ſame Fluid is conſidered, as diſtinguiſhed both theſe Ways, *viz.* into perpendicular Columns, and alſo into thin Strata or Plates. *Figure* 1. repreſents a Veſſel filled with a Fluid to the Height EF, and divided into the Columns GH, IK, LM, *&c.* and alſo into the Strata RS, TV, XY, *&c.*

IV. From this Obſervation, concerning the Properties of the Particles conſidered ſeparately, there reſults immediately the following Propoſition, *viz.* that in a Veſſel, whoſe Form is ſuch as repreſented by ABCD, (*Fig.* 1.) the Quantity of Preſſure, which each Stratum ſuſtains from the Weight of the incumbent Fluid, is in Proportion to the Number of thoſe

It is an obvious Objection to this, that Water by freezing is augmented in its Bulk; but this may be owing to thoſe Bubbles or Vacuities, obſervable in the Water after it is frozen, which were not in it before: and not to any general and uniform Removal of the Particles of the Fluid from each other, which the Objection, if it is of any Force againſt what has been advanced, muſt ſuppoſe.

Strata, which reſt upon it, that is, as the Height of the Surface of the Fluid above it; for if we ſuppoſe the Strata of equal Thickneſs, the Quantity ſuſtained is proportional to the Number of Strata of which it conſiſts.

V. When the Surface of a Fluid is horizontal or level, each Particle thereof is diſpoſed to continue in its Place, being ſuſtained therein by the contiguous ones.

Let the Fluid be ſuppoſed to be divided into Strata, each of the Thickneſs of a Particle of the Fluid; and if the Truth of this Propoſition be denied, let the Particle *m n* be one of thoſe which is not ſuſtained in its Place by the contiguous ones, but is moving from thence towards ſome other Part of the Veſſel, *v. g.* towards D. Now, ſince all the other Particles of that Stratum are at an equal Depth below the Surface of the Fluid with this, they alſo ſuſtain an equal Degree of Preſſure (by the laſt Propoſition,) conſequently for the ſame Reaſon that one of them is moving towards D, the reſt may all be ſaid to be moving in the ſame Direction: but this cannot be true of the whole Stratum while the Veſſel is entire, and therefore of none of its Parts. Now, the like Reaſoning will hold againſt the Motion of the Particle *m n* towards any other Part of the Veſſel; from whence it follows, that each Particle of the Fluid is ſuſtained in its Place by the contiguous ones,

and

and therefore diſpoſed to continue at Reſt *.

VI. From hence is derived a fundamental Propoſition in Hydroſtatics, *viz.* That when the Surface of a Fluid is level, whatever Preſſure any ſingle Particles or ſmall Portion of it ſuſtains from the contiguous ones on one Part, it ſuſtains the ſame on all the reſt; that is, it is preſſed by them with an equal Degree of Force on all Sides †.

For, by the Definition of the Fluid (§. 2.) each Particle is diſpoſed to give Way, and move out of its Place, when the Preſſure is not equal on all Sides; and (§. 5.) each Particle is preſſed by the contiguous ones in ſuch a Manner, that it is ſuſtained in its Place thereby; it is therefore preſſed with an equal Degree of Force on all Sides.

Corol. From hence it follows, that each Particle, or ſmall Portion of a Fluid, preſſes with the ſame Degree of Force in all Directions on thoſe which are contiguous to it. For, by the third Law of Nature, every Par-

* This ſhews us the Abſurdity of ſome Philoſophers, who make the Nature of a Fluid, as contradiſtinguiſhed from that of a Solid, to conſiſt ſolely in the inteſtine Motion of its Particles.

† This Propoſition, with its Corollary, is not ſtrictly ſpeaking true, unleſs the Particle or Portion of Fluid we ſpeak of is ſuppoſed void of Gravity, for it preſſes downwards with a Force equal to the Weight of thoſe Particles which reſt upon it, added to its own; whereas the Force with which it preſſes upwards is only equal to the Weight it ſuſtains, *viz.* that of the incumbent Fluid. But the Particles of Fluids are ſo exceedingly minute, and the Gravity of each ſo very ſmall, that the Error ariſing from hence is inſenſible.

ticle

ticle preſſes upon the contiguous ones with the ſame Degree of Force, with which it is preſſed upon by them.

VII. The Surface of a Fluid becomes level by its own Gravity, when no external Force prevents it being ſo.

For, the Particles of Fluids preſs in all Directions with Forces proportional to the Height of their Surfaces (Cor. §. 6. and §. 4.) If then the Surface be not level, the different Parts of the ſame inferior Stratum will be preſſed not only downwards, but ſideways againſt each other with unequal Forces; the greater Preſſure therefore overcoming the weaker, the Particles which ſuſtain the leaſt Preſſure will be driven out of their Places, and raiſed up till the Surface becomes level; the Surface being level, each Particle will be equally preſſed in every Direction, (§. 8.) all therefore will remain at Reſt, and the Surface continue in that State *.

VIII.

* This Demonſtration, and alſo both the foregoing, are founded upon a Suppoſition, that Bodies tend downwards by their Gravity in Lines parallel to each other, which, though *phyſically* true, is not ſtrictly ſo, their Tendency being towards the Center of the Earth, and conſequently in Lines which meet in a Point: and therefore, if we would be accurate, the Fluid contained in a Veſſel ſhould be conſidered as divided into Columns and Strata, as repreſented *Figure* the ſecond, where A B D is the Earth, C its Center, E F G H a Fluid contained in a Veſſel, and divided into Columns, which if continued down to the Center of the Earth, would there terminate in a Point C; and into the concentric Strata, *a b*, *c d*, &c. having the Center of the Earth for the Center of their Convexity. And then we ſhould

VIII. Fluids gravitate in Fluids of the ſame Kind.

This Propoſition is a neceſſary Conſequence of what has been obſerved about the Nature of the Particles of which Fluids conſiſt, *viz.* that they are ſolid, and endued with the ſame Properties with other Bodies. The Reaſon why their Gravity is not ſenſible in the Fluid, is becauſe the lower Parts ſuſtain the upper, and hinder them from deſcending. But, it does not follow from thence, that the Gravity of the uppermoſt is entirely taken away, as ſome * Philoſophers have imagined; for, as

ſhould find that the Particles of the Fluid will not be in Æquilibrio with each other, till all the Parts of its Surface are at equal Diſtances from the Center of the Earth, forming thereby the Surface EF, concentric to that of the Earth. For, ſuppoſing the Fluid E F G H continued down to C, ſo as to fill the Space E C F; it is evident the Columns, into which the Fluid is divided, cannot be of equal Lengths, and therefore cannot be of an exact Counterpoiſe to each other, unleſs the Surface EF is a Portion of a Sphere, whoſe Center is C: but the Action of the Parts of Fluids upon each other at the Surface, is the ſame, whether the lower Part GCH be a Fluid, or not. Conſequently the Surfaces of Fluids are not level or plain, but convex, having the Center of the Earth for the Center of their Convexity.

This Convexity, by reaſon of the great Diſtance of the Earth's Center, approaches ſo near to a Plane, that in ſmall Portions of it, the Difference is not ſenſible, and therefore may be neglected: but at Sea, 'tis evident to Senſe; for when they Mariners put to Sea, the Shore firſt diſappears, then the lower Buildings, afterwards the Towers, Mountains, *&c.* in like Manner, when they approach a diſtant Ship, the Top of its Maſt and Sails appear firſt, while the Ship itſelf is intercepted from their View, by the Convexity of the Water between them.

* This was the Notion of the *Carteſians*, who held, that when a Fluid is mixed with another of the ſame Kind, it loſes its own Weight thereby.

much

much as the lower Parts preſs againſt the upper in ſuſtaining them; juſt ſo much do the upper by their Re-action preſs upon the lower. Thus, the Weight of the whole Fluid is communicated to the Veſſel, which therefore weighs according to the Quantity it contains, notwithſtanding the Preſſure of each Portion of the Fluid, taken ſeparately, ſeems not to affect it.

IX. The Preſſure of a Fluid is in Proportion to its perpendicular Height, and the Quantity of Surface againſt which it preſſes.

This Propoſition admits of four Caſes.

1. When the Fluid is contained in a Veſſel of the ſame Dimenſions, from Top to Bottom, and held in an erect Poſition, as that repreſented Fig. 1. it is evident, the Preſſure of the Fluid upon the Bottom will be in Proportion to its Magnitude, and the perpendicular Height of the Surface of the Fluid above it. For, conceiving it divided into Columns, the Preſſure upon the Bottom, by the fourth Propoſition, will be as the Length or Height of the Columns: and it will alſo be as the Number of them, becauſe the Quantity of Fluid, which preſſes upon the Bottom, is in that Proportion, that is, as the Magnitude of the Bottom preſſed upon. But when the Veſſel is inclined or irregular, the Truth of this Propoſition is ſo far from being evident, that it has been commonly looked upon as a Paradox.

2. Let the Veſſel ABCD (*Fig.* 3.) be filled with a Fluid to the Height EF, and held in an inclined

inclined Poſition, as there repreſented; I ſay, the Preſſure of the Fluid is proportional to the Magnitude of the Baſe C D, and F G or H D the perpendicular Height of the Surface of the Fluid above it.

For ſuppoſing the Fluid divided into the Strata E I, K M, L O, *&c.* ſo far as the firſt Stratum E I is prevented from preſſing upon K I the Surface of the next inferior Stratum, by being in ſome Meaſure ſupported by the Side of the Veſſel F I, ſo far is its Preſſure augmented by the Re-action of the oppoſite Side E K upon it, which is exactly equal to the Action of the former, becauſe the Fluid preſſing every Way alike, at the ſame Depths below the Surface, exerts an equal Force againſt both theſe Sides. The ſurface therefore of the ſecond Stratum is preſſed with the ſame Degree of Force with which it would be, if the Quantity of Fluid contained in the former Stratum was included within the Space H K Q I, which is exactly equal to it, as having the ſame Baſe K I, and the ſame perpendicular Height Q I *. Now this being true of each Stratum, their Preſſure upon C D the Baſe of the Veſſel is the ſame, as if they were all placed perpendicularly over it, and filled the Space R H C D; which they would do, ſince the Sum of the perpendicular Heights Q I, K S, L T and N V, are equal to H D the perpendicular Height of this Space, and each of their Baſes K I, L M, *&c.* is equal to C D its Baſe †. But, by the foregoing Caſe, if the Space R H C D

* 31 *El.* 11. † 31 *El.* 11.

was

was filled with a Fluid, the Preſſure of it would be proportional to the Dimenſions of the Baſe C D, and the perpendicular Height D H, therefore it is the ſame in the inclined Tube A B C D ‖.

3. Let the Veſſel A B C be irregular, as repreſented *Figure* the fourth, and filled with Fluid to the Height D, I ſay, the Preſſure of the Fluid upon the Baſe C, is proportional to the Magnitude of the Baſe, and C D the perpendicular Height of the Surface of the Fluid above.

‖ Otherwiſe thus: Through any Part of the Tube ABCD draw the horizontal Line XZ, and let *a* and *b* repreſent two Particles of the Fluid in the horizontal Plane *b a*; then, ſince all the Particles in this Plane are preſſed every Way alike by the incumbent Fluid (for if they were not, they would not continue at Reſt) it follows, that the Force with which *b* is made to preſs towards X, is equal to that with which *a* is made to preſs towards Z. Let the former of theſe Forces be denoted by *b f*, and the latter by *a g*; whence we have *b f* equal to *a g*. Reſolve the former of theſe into *f d* and *d b*; the latter into *g c* and *c a*; (*viz. f d* and *g c* perpendicular to the Sides of the Tube, and the other *d b* and *c a* parallel to them) then the Lines *b f* and *a g* being equal to each other, and the Triangles *b f d* and *a g c* ſimilar, *d b* is equal to *c a*, and *d f* to *c g*: But *d b* and *c a* act in Directions oppoſite to each other, they therefore deſtroy each other; the other Forces *d f* and *c g* act alſo in contrary Directions, and being equal, they alſo deſtroy each other. Thus the Action of the Fluid upon the Tube, as to Preſſure upon the Baſe, is totally deſtroyed; and conſequently Action and Re-action being always equal, that of the Tube upon the Fluid is ſo too: And therefore the Fluid ABCD is in the ſame State, as to its Preſſure upon the Baſe, as if the Tube was away, and the Fluid retained its Form. Suppoſe it ſo; then it is obvious, the Preſſure of the Fluid will fall wholly upon the Baſe DC: But that Preſſure will be entirely as the Quantity of Matter the Fluid contains; which Quantity is as its Baſe C D, and perpendicular Height D H; its Preſſure therefore upon the Baſe is ſo too: And the Preſſure of the Fluid HRCD is alſo as the ſame Baſe CD, and the perpendicular Height D H. The Preſſure therefore in each Tube is the ſame. *Q. E. D.*

In

In order that the Proof of this Propoſition may be the better underſtood, we muſt premiſe the following *Lemma*, viz.

That when a Fluid paſſes through a Pipe, as AB, (*Fig.* 5.) which in ſome Parts is larger than in others, the Moment, or Force with which it moves, is every where the ſame. For while the Fluid is paſſing through the Tube, its Velocity in every Part will be reciprocally as the Quantity of Matter; for Inſtance, it will be as much greater at C, than it is at D, as the Quantity paſſing through C at any Inſtant of Time, is leſs than at D, and ſo of the reſt; becauſe a leſs Quantity would be conveyed through the ſmaller Parts of the Tube in the ſame Time, unleſs it ſhould move faſter there in Proportion to the Smallneſs of them. Now the Momentum of Bodies is partly owing to the Quantity of Matter, and partly to the Velocity; (as explained Part I. Chap. 11. §. 1.) conſequently what the Fluid, which is actually paſſing through the narrower Parts of the Tube, wants in Quantity, is compenſated by its Velocity in thoſe Parts, and what it wants in Point of Velocity in the other Parts, is made up by the Quantity paſſing through them; ſo that the Moment is the ſame in every Part of the Tube, whether larger or narrower *. The ſame is true, whatever be the Poſition which the Tube is held in. Let

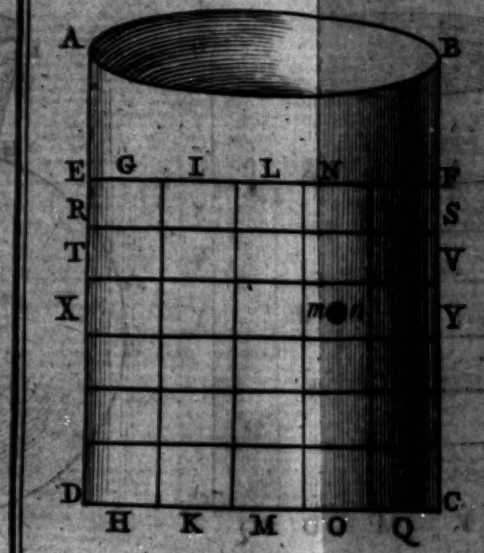
* Thus, we may obſerve in a River or Canal, that by how much the Breadth or Depth is leſs in any Part, ſo much the more

Let us now conceive the Fluid in the Veſſel A B C (*Fig.* 4.) to be diſtinguiſhed into the Strata EF, G H, I K, *&c.* Let us alſo imagine the Bottom of the Veſſel C to be moveable, that is, capable of ſliding up and down the narrow Part of the Veſſel, *v. g.* from C to G H, (without letting any of the Fluid run out.) Let it further be ſuppoſed that this moveable Bottom is drawn up or let down with a given Velocity, while the Veſſel itſelf is fixed and immoveable; it is evident the lowermoſt Stratum, which is contiguous to the Bottom, will be raiſed or let down with the ſame Velocity, and will thereby have a Moment proportional to that Velocity, and the Quantity of Matter it contains: But by the Lemma, all the reſt of the Strata will have the ſame Moment, conſequently the Moment of all taken together, (that is, of the whole Fluid,) is the ſame, as if the Veſſel had been no larger in any one Part, than it is at the Bottom, (for then the Moment of each Stratum would alſo have been as great as that of the lowermoſt;) the Preſſure therefore, or Action of the Fluid, with which it endeavours to force the Bottom out of its Place, is as the Number of Strata, that is, the perpendicular Height of the

more rapid is the Stream in that Part; and on the contrary, where it is wider and deeper, the Motion of the Water is more gentle and languid. So that the Moment, with which it flows, is the ſame in every Part.

Fluid,

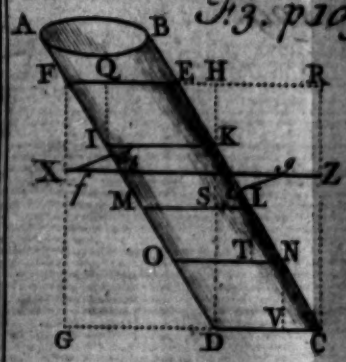
F. 1. p.103.



F. 2. p.106.



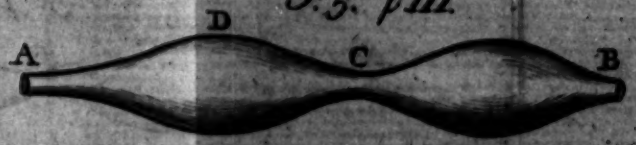
F. 3. p.109.

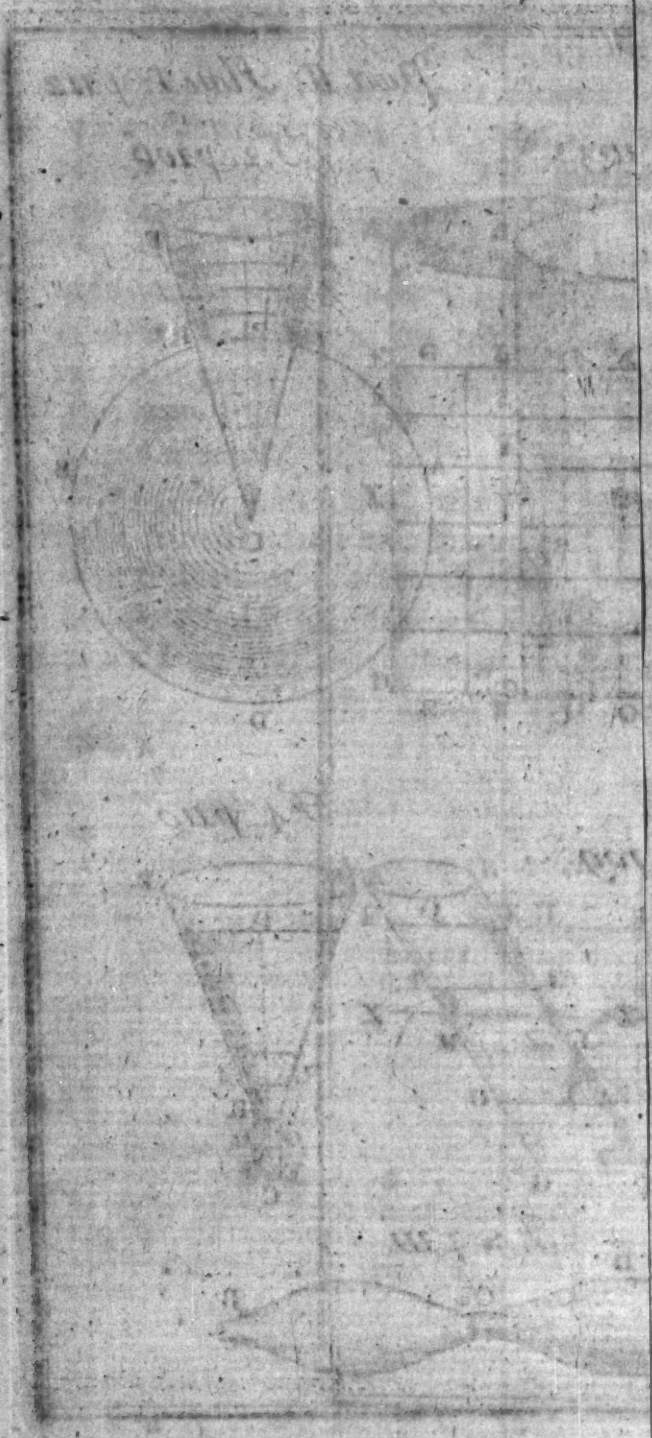


F. 4. p.110.



F. 5. p.111.





Fluid, and the Magnitude of the lowermoſt Stratum, that is, the Bottom.

Again, ſuppoſe the Veſſel ABC (*Fig.* 6.) fill'd with a Fluid to D; I ſay the Preſſure upon the Bottom BC, is proportional to the Dimenſions of the Bottom, and to DE the perpendicular Height of the Fluid.

For, if we ſuppoſe the Bottom moveable as before, and raiſed up or let down with a given Velocity, the Moment of every Stratum will be the ſame with that of the lowermoſt, by the Lemma; therefore the Moment of all taken together, is the ſame, as if the Veſſel had been no leſs in any one Part, than it is at the Bottom; conſequently the Preſſure is proportional to the perpendicular Height, and the Magnitude of the Bottom. This Caſe is the Converſe of the former *.

From

* Upon this is founded the Practice of conveying Water through Pipes from Place to Place, *&c.* For from hence it follows, that if one End of a Pipe is laid in a Reſervoir of Water, the Fluid will run into the Pipe, till it riſes to a Level at the other End with its Surface in the Reſervoir. Thus let ABC (*Fig.* 8.) repreſent a Reſervoir, or Baſon of Water, DGE a Pipe laid from thence to E. If E, the End of the Pipe, is placed above the Line ABF, the Level of the Water in the Reſervoir, the Water will run into the Pipe, till it riſes in the other End to F, the Level with AB; at which Time the Water in the Pipe will be in Æquilibrio with that in the Reſervoir, and remain at Reſt. But if the End of the Pipe is below the Surface of the Water in the Reſervoir, it will continue to run out, till they are reduced to a Level. For, let GH be the loweſt Part of the Pipe, then ſince F, the perpendicular Height of the Fluid on one Side, is equal to B, the perpendicular Height of the Fluid on the other, and GH, which (being the Place

From hence it follows, that, if a Veſſel is made of ſuch a Form, as is repreſented (*Fig.* 7.) by ABCDEFG, and filled with a Fluid to the Height C, the Weight which the Bottom ſuſtains, is as great as it would be, had the Veſſel been IKFG (which is every where of the ſame Dimenſions that the other is of at the Bottom) and filled to the Top IK. Becauſe the Preſſure, by the Propoſition, is proportional to the Bottom and perpendicular Height, which, in both Caſes, are the ſame *.

X. The

where the Fluids preſs one againſt another) may be conſider'd as a Baſe to both, is common; it follows from this Propoſition, that the Preſſures on each Side are exactly equal; and therefore being in contrary Directions will neceſſarily deſtroy each other, and the Fluid will remain in Æquilibrio. But while the End E is below the Level, this Equilibrium cannot be obtained; and therefore the Fluid will continue to run out.

For the ſame Reaſon, when two or more Tubes communicate with each other, the Surface of the Fluid they contain will ſtand at the ſame Level in all.

* The Reaſon why the Veſſel ABCDEFG, with the Fluid contained in it, does not weigh ſo much, as the Veſſel IKFG, when full to the ſame Height, notwithſtanding the Preſſure upon the Bottom is the ſame in both, is, becauſe ABDE, the upper Part, or Cover of the former Veſſel, is preſſed upwards by the Fluid below it, with a Force equal to the Endeavour the Fluid in the ſmall Tube BCD has to deſcend. Which Endeavour is the ſame that it would be, if the Tube BCD comprehended alſo the two Spaces ICBA and CKED, its Moment being the ſame in both Caſes by the Demonſtration; the Cover therefore is preſſed upwards with a Force equal to the Weight of as much Fluid as would fill the two Spaces ICBA and CKED; conſequently the Veſſel, whoſe Form is ABCDEFG, is ſo much lighter than the other, that is, as much as the Fluid it contains is leſs.

From

X. The Velocity with which a Fluid ſpouts out at an Hole in the Bottom of a Veſſel, is equal to that which a Body would acquire by falling freely from the Level of the Surface of the Fluid to the Hole.

Let there be a large cylindrical Tube A B C D (*Fig.* 9.) in the upper Part of which let us imagine a Cylinder of Ice F G H I exactly fitting it; let it further be ſuppoſed, that H I, the lower Surface of the Ice, is continually melting, ſo as to afford a Stream of Water running down the Middle of the Tube. Now the Form of this Stream of Water will neceſſarily be ſuch as is repreſented in the *Figure* by H L I; for the Water falling freely, will deſcend faſter and faſter like other Bodies, cauſing thereby the Stream to become narrower and narrower. Now let it be ſuppoſed that the Tube has a Bottom, as C D, with an Hole in it at K, juſt ſufficient to let the Stream paſs freely; it is evident there will be no Obſtruction on this Account, but that the

From hence ariſes this Paradox, that the leaſt Quantity of Fluid may be made to raiſe any Weight, how great ſoever it be.

For ſince the Cover A B D E is preſſed upwards with a Force equal to the Weight of as much Fluid as would fill the two Spaces I C B A and C K E D, and thoſe Spaces may be enlarged at Pleaſure in Height, by lengthening the Tube B C D (which at the ſame Time muſt be made proportionally ſmaller, otherwiſe the ſame Quantity of Fluid will not fill it;) it follows, that the ſame Quantity of Fluid may be made to preſs the Cover upwards with a given Force; if that Cover then is made moveable, any Weight that is laid upon it may be ſupported thereby.

Fluid will paſs through the Hole with ſuch Velocity, as it naturally acquires by falling from H I, the lower Surface of the Ice. And if we ſuppoſe M and N, the empty Parts of the Tube, to be filled with Water, the Water will preſs equally upon the Sides of the Stream in every Direction (§. 6.) and therefore will be no Impediment to its Motion on that Account. Laſtly, let us ſuppoſe the Ice taken away, and the Stream ſupplied from the Water at the Sides, as is the Caſe when a Fluid runs out through the Bottom of a Veſſel; then will the Velocity, with which the Water flows through the Hole, continue the ſame; for ſo far as the Water coming from the Sides, endeavours to deſcend itſelf, ſo far it obſtructs the Deſcent of the Stream, and no farther; and conſequently cauſes no Alteration in the Velocity, or Quantity of Fluid running out. The Velocity, therefore, with which the Fluid paſſes through the Hole, is equal to that, which a Body would acquire by falling freely from the Level of the Surface of the Fluid to that Place.

If the Hole is made in the Side of the Veſſel at the ſame Diſtance below the Surface, the Velocity will be the ſame, on Account of that equal Tendency Fluids have to move every Way alike *.

XI. The

* Upon this Principle is founded the Practice of making artificial Fountains. For if to a Veſſel or Reſervoir A B C D (*Fig.* 10.) filled with a Fluid to the Height E F, be fixed the Pipe

XI. The Velocity with which a Fluid ſpouts out from the Bottom or Side of a Veſſel, is as the Square Root of the Height of its Surface above the Hole. *

The Cauſe why a Fluid ſpouts out through an Hole made in the Bottom or Side of a Veſſel, is the Preſſure or Weight of the Fluid incumbent upon the Hole; from whence it ſhould ſeem, that the Velocity ought to be as the Preſſure; but if ſo, then the Quantity

Pipe CH, with a ſmall Aperture at K, the Fluid will ſpout up from thence to FL, the Level of the Surface of the Fluid in the Veſſel. For, by this Propoſition, it will ſpout from K with ſuch a Velocity, as a Body would acquire by falling from FL, the Level of the Surface, to the Aperture at K; that is, ſuch as will carry it from the Aperture to the Level; becauſe that Velocity which a Body acquires by falling from a certain Height, is ſufficient to carry it back to the ſame Height from whence it fell.

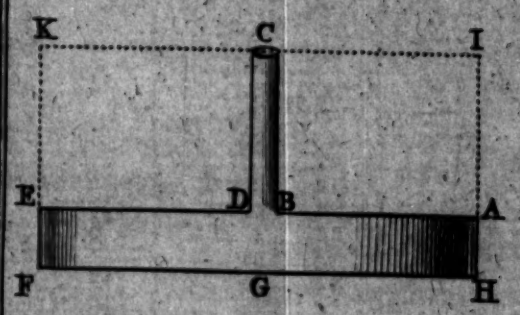
But in Practice, the Height which the Fluid riſes to, is leſs than that of the Level of its Surface in the Reſervoir: This is owing to the Reſiſtance it meets with from the Air, its Friction againſt the Sides of the Pipe, *&c.* It is found impoſſible to make it much exceed the Height of an hundred Feet: For, when it ſpouts out of the Aperture with a Velocity neceſſary to carry it higher, the Stream is immediately daſhed to Pieces by the Reſiſtance of the Air; whereby it loſes its Force, and is prevented from riſing to any conſiderable Height.

* This Propoſition may be otherwiſe demonſtrated from the laſt, in the following Manner: For, ſince the Velocity with which a Fluid ſpouts out through an Hole in the Bottom or Side of a Veſſel, is equal to that which a Body would acquire by falling from the Level of the Surface of the Fluid to the Hole, and the Velocities which Bodies acquire by falling, are as the Square Roots of the Heights which they fall from (Part I. Chap. 5. §. 5.) it follows, that the Velocity with which a Fluid ſpouts out from an Hole in the Bottom or Side of a Veſſel, is as the Square Root of the Height of the Level of the Surface of the Fluid above the Hole.

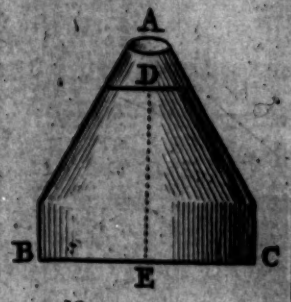
run out would alſo be as the Preſſure, (for, the faſter the Liquor flows, the greater is the Quantity thrown out in a given Time, and *vice verſâ*) conſequently, upon this Suppoſition we ſhould have two Effects, each depending on the ſame Cauſe, and equal to it, which is abſurd. 'Tis not then the Quantity of Fluid run out, nor the Velocity with which it flows, but its Moment, or both theſe multiplied together, (Part I. Ch. 11. §. 1.) that is the true and adequate Effect of the Preſſure. Now theſe being ever in the ſame Ratio with each other, 'tis neceſſary, in order that the Effect may be proportional to its Cauſe, that each of them be only as the ſquare Root of the Preſſure: For then, being multiplied together, their Product, or the Moment of the ſpouting Fluid, is adequately as the Preſſure which occaſions it; but the Preſſure is as the perpendicular Height (§. 4.) therefore the Velocity, and alſo the Quantity of Fluid ſpouting out, is only as the ſquare Root of the Height of its Surface above the Hole.

To give an Inſtance or two: Suppoſe two Holes made in the Side of a Veſſel, the one an Inch below the Surface of the Fluid it contains, the other four Inches; the Velocity with which the Liquor flows out of the lower Hole, will not be four Times as great as that with which it flows through the upper, notwithſtanding the Preſſure is four Times greater: For, if it ſhould, the Quantity run out in a given Time would

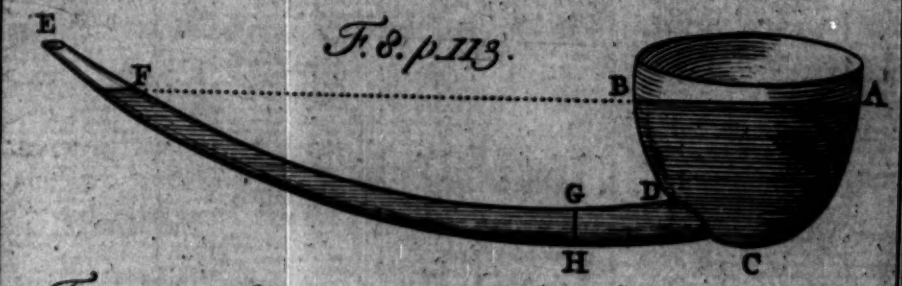
F. 7. p. 114.



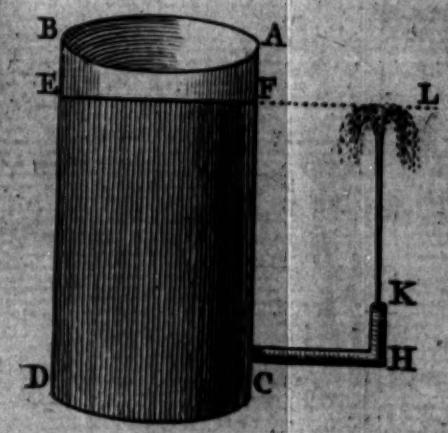
F 6. p. 113.



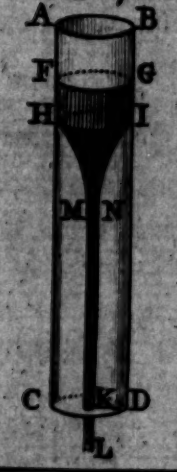
F. 8. p. 113.



F. 10. p. 116.



F. 9. p. 115.



would alſo be four Times greater, conſequently the Effect produced would be ſixteen Times greater than it is at the upper Hole, that is, four Times greater than the Cauſe; which is abſurd. Whereas the Velocity and Quantity of Matter will each be only twice as great as they are above, producing thereby a Force, or Moment, only four Times as great, which is proportional to the Cauſe. So, if an Hole were made ſixteen Times lower than the firſt, the Velocity and Quantity of Matter will not be each ſixteen Times greater than at the other, but only four Times greater a-piece, and ſo the Moment ſixteen Times greater, as the Preſſure is †.

XII. When

† From hence we may ſee the Error ſome of the foreign Mathematicians have fallen into, with Regard to the Forces of moving Bodies, who contend that they are as the Squares of the Velocities multiplied by the Quantities of Matter. For, from this Propoſition it is, that one of the principal Arguments brought in Favour of this Opinion is derived. They argue thus: *Effects* are ever proportional to their *Cauſes*, the *Preſſure* of the incumbent Fluid is the *Cauſe* of its ſpouting out, the *Force* with which it ſpouts out, is the *Effect*; but by this Propoſition the *Preſſure* is as the *Square* of the *Velocity* it flows with, therefore the *Force* is likewiſe as the *Square* of the *Velocity*. True, it is ſo, but let us ſee the Conſequence. The Force, with which the Fluid ſpouts out, is not only owing to the Velocity, but alſo to the Quantity run out in a given Time. They have each their Share in producing the Force, conſequently the Force is in a Ratio compounded of both, or as the Product of one multiplied by the other, or, which comes to the ſame Thing (ſince as was obſerved before, they are in the ſame Ratio with each other) as the Square of either of them. From hence it is that the Forces of Fluids in Motion are ſaid to be as the Squares of their Velocities; not that they are ſo in Virtue of thoſe

XII. When a Current of Water, or other Fluid, falls perpendicularly upon the Surface of a Plane, or flows againſt it (as the Wind againſt the Sail of a Ship, or the like) the Force which the Fluid exerts upon it, is equal to the Weight of a Column of the ſame Fluid, whoſe Baſe is equal to the Plane, and its Height ſuch, that a Body falling freely through it, would acquire a Degree of Velocity equal to that with which the Fluid moves *.

In order to demonſtrate this Propoſition, let us ſuppoſe the Veſſel A B C D (*Fig.* 11.) filled with a Fluid, and having a large Hole E F in the Bottom, then will the Preſſure of the Fluid cauſe a Stream to flow out, which in the Hole itſelf will have ſuch a Degree of Velocity, as a Body would acquire by falling freely from the Surface of the Fluid in the Veſſel to the Hole (as demonſtrated §. 10.)

Velocities, as ſuch, but in Virtue of them, and the Quantities of Matter taken together, or becauſe the Squares of the Velocities is the ſame Thing with the ſimple Velocities multiplied by the Quantities of Matter. Therefore when it is ſaid, the Forces of Fluids are as the Squares of the Velocities, that Part of the Force which ariſes from the Quantity of Matter is really taken into Conſideration. How ridiculous then muſt it be in thoſe Gentlemen, to fetch an Argument from hence to prove, that the Forces of Bodies in Motion are as the Squares of the Velocities and Quantities of Matter too, when they are as the Squares of the Velocities, only becauſe the Quantities of Matter are implied in them.

* From this Propoſition is deduced the Method of computing the Power of a Machine, which is to be moved by Wind or Water, *&c.* See an Inſtance of ſuch a Calculation in the Memoirs of the Royal Academy of Sciences, for the Year 1725.

In

In the Midſt of this Hole, and conſequently in the Stream, let us ſuppoſe a Plane as P Q ſuſpended, but ſomewhat leſs than is ſufficient to fill the Hole, let it ſtop the Current of the Water. Now, 'tis certain, this Plane ſupports a Column of the Fluid, equal to that which preſſes upon any other Part of the Bottom of the Veſſel of equal Dimenſions with itſelf (for being thus placed it may be looked upon as a Part of the Bottom) but every Part bears a Column, whoſe Baſe is equal to its own Dimenſions, and its Height the ſame with that of the Surface of the Fluid in the Veſſel: Conſequently this Plane ſupports ſuch a Column, that is, it is reſiſted by the Stream with a Force equal to the Weight of a Column, whoſe Baſe has the ſame Dimenſions with itſelf, and whoſe Height is equal to that of the Surface of the Fluid in the Veſſel; that is, ſuch an Height as a Body by falling freely from, would acquire a Velocity equal to that with which the Fluid moves.

XIII. The Preſſure of a Fluid againſt a perpendicular Bank or Sluice, *&c.* is equal to the Weight of a Column of the ſame Fluid, whoſe Baſe is equal to ſo much of the Bank as is below the Surface of the Fluid, and whoſe Height is equal to Half the Depth of the Fluid †.

If

† From hence we ſee the Reaſon, why the Water of the Sea or great Lakes is as eaſily kept within their Banks (ſetting aſide the Force which ariſes from the Motion of the Waves, *&c.*)

If the Preſſure upon every Part of the Bank from the Surface to the Bottom, was as great as it is at the Bottom, the Preſſure againſt it would be equal to the Weight of a Column, whoſe Baſe is equal to ſo much of the Bank as is under the Surface of the Fluid, and which has the *whole* Depth of the Fluid for its Height; for the Preſſure upon every Part of the Bank at the Bottom is equal to the Weight of a Column, whoſe Baſe correſponds to the Part preſſed upon, and whoſe Height is that of the Depth of the Fluid; conſequently if the Preſſure was the ſame every where from Top to Bottom, it would be equal to the Weight of as many ſuch Columns as would anſwer to all the Parts of the Bank: But the Preſſure every where diminiſhes in Proportion as we approach the Top, where it is nothing; it is therefore but *Half* † what it would be in the other Caſe; from whence the Propoſition is clear.

&c.) as that of the narroweſt Canal, *viz.* becauſe the Preſſure of Fluids is not in Proportion to their Surfaces, but their Depths, and the Surfaces they preſs againſt.

† Becauſe the Sum of a Number of Terms in Arithmetical Progreſſion, beginning from Nothing, is Half the Sum of an equal Number of Terms, each of which is equal to the laſt in the Progreſſion.

CHAP.

CHAP. II.

Of the Effects which Fluids have on Solids immerſed therein.

I. THE *Specific* Gravity of a Body is that by which it is ſaid to be heavier, or lighter, than another of a different Kind: Thus Lead is ſaid to be ſpecifically heavier than Cork; becauſe ſuppoſing an equal Bulk of each, the one would be heavier than the other. From hence it follows, that a Body, ſpecifically heavier than another, is alſo more denſe, that is, contains a greater Quantity of Matter under the ſame Bulk, becauſe Bodies weigh in Proportion to the Quantities of Matter they contain (Part I. Chap. 3. §. 7.)

II. If a Solid be immerſed in a Fluid of the ſame ſpecific Gravity with itſelf, it will remain ſuſpended therein, in whatever Part of the Fluid it is put.

Let the Body FGHI (*Fig.* 12.) be immerſed in the Fluid ABCD to the Depth MN, or any other whatever; I ſay, it will continue in the ſame Part of the Fluid, when left to itſelf, without either riſing towards the Surface, or ſinking towards the Bottom.

For, the Body being (by the Suppoſition) of equal Gravity with the Fluid, the Weight of the Column KLHI, which conſiſts partly of Fluid,

Fluid, and partly of the Body, is the ſame as if it had been all Fluid; conſequently HI, that Part of the Surface of the Stratum MN, which lies immediately under the Body, is preſſed with the ſame Degree of Force that any other Part of the ſame Dimenſions is, and therefore the whole Column KLHI will be ſupported in its Place. Now, the ſame being true of the Column KLHI, whatever be its Length, 'tis evident the Body will be ſuſpended in its Place at any Depth.

III. But if the Body is ſpecifically heavier than the Fluid in which it is immerſed, it will ſubſide to the Bottom: For then, in whatever Part of the Fluid it is put, the Column KLHI will always be heavier than an equal Column, that conſiſts all of Fluid; conſequently HI, that Part of the Stratum MN which lies immediately under the Body, will ſuffer a greater Preſſure than any other Part of the ſame Dimenſions; and therefore will give way, and permit the Body to ſubſide continually, till it reaches the Bottom.

IV. On the contrary, if the Body is ſpecifically lighter than the Fluid, it will riſe to the Top, in what Part of the Fluid ſoever it is put. For then, the Column KLHI will always be lighter than an equal Column which is all Fluid; conſequently HI will be leſs preſſed downwards, than any other Part of the ſame Stratum of equal Dimenſions, and will

will therefore continually riſe up, carrying the Body with it, till it arrives at the Top.

V. A Body, being laid on the Surface of a Fluid ſpecifically heavier than itſelf, ſinks into it, till the immerſed Part takes up the Place of a Quantity of Fluid, whoſe Weight is equal to that of the whole Body.

Let EFGH (*Fig.* 13.) be a Body, floating on a Liquor ſpecifically heavier than itſelf, it will ſink into it, till the immerſed Part IKGH takes up the Place of ſo much Fluid as is equal to it in Weight. For, in that Caſe, GH, that Part of the Surface of the Stratum upon which the Body reſts, is preſſed with the ſame Degree of Force, as it would be, was the Space IKGH full of the Fluid; that is, all the Parts of that Stratum are preſſed alike, and therefore the Body, after having ſunk ſo far into the Fluid, is in *Æquilibro* with it, and will remain at Reſt.

From hence it follows, that a Body is as much ſpecifically lighter than the Fluid on which it floats, as the immerſed Part is leſs than the Whole. For, by how much the leſs the immerſed Part is, ſo much the leſs Fluid is equal in Weight to the whole Body; that is, the Body is ſo much the lighter in Reſpect of the Fluid. And, if the ſame Body be made to float ſucceſſively in Fluids, whoſe ſpecific Gravities differ among themſelves (but all exceed that of the Body) the lighter the Fluids

are,

are, ſo much greater will be the Part immerſed †.

VI. A Body, ſuſpended in a Fluid ſpecifically lighter than itſelf, loſes a Part of its Weight (or rather communicates it to the Fluid) equal to that of a Quantity of Fluid of the ſame Bulk.

Let us, inſtead of ſuppoſing the Body ſuſpended in the Fluid, imagine it to be away, and its Place filled with the Fluid; now 'tis evident, this being of the ſame ſpecific Gravity with the circumjacent Fluid, will be entirely ſupported by it; or, if we ſuppoſe the Body to be of the ſame ſpecific Gravity with the Fluid, it will be wholly ſuſpended by it; we ſee therefore the Preſſure of the circumambient Fluid, whereby it endeavours to buoy up the Body, is equivalent to the Weight of ſo much Fluid as would fill the Place the Body takes up. But, ſince the Fluid preſſes only on the Surface of the Body, that Preſſure is the ſame, whatever be the ſpecific Gravity of the Body; the Body therefore loſes ſo much of its Weight, as the Fluid would naturally buoy up; that is, ſo much as is the Weight of a Quantity of Fluid of the ſame Bulk.

† This Phænomenon is what gave Riſe to the Hydrometer, an Inſtrument of great Uſe in aſcertaining the Genuineneſs of Liquors; for it rarely happens, that the adulterated and the genuine Liquor (however they may agree in Appearance) are of the ſame ſpecific Gravity.

This

This Propoſition affords us a Method of determining the Relation which the ſpecific Gravities of Bodies, whether Fluid or Solid, bear to each other. For whereas by weighing a Solid in a Fluid, ſpecifically lighter than itſelf, we find the abſolute Weight of a Quantity of the Fluid equal to it in Bulk (*viz.* the Weight the Solid loſes) the Relation that Weight bears to the Weight of the Solid, is the Relation of their ſpecific Gravities; becauſe the Weights of Bodies, whoſe Bulks are equal, are as their ſpecific Gravities: Conſequently, if the ſame Solid is weighed ſucceſſively in different Fluids (all lighter than itſelf) we gain the Relation which the ſpecific Gravity of each bears to that of the Solid, and therefore to one another. Again, if different Solids are weighed in the ſame Fluid, the Relation which the ſpecific Gravity of that Fluid bears to each Solid, is had, and therefore alſo the ſpecific Gravities of the Solids among themſelves †.

CHAP.

† Upon this is founded the Uſe of the Hydroſtatical Balance for determining the ſpecific Gravities both of Solids and Fluids. The Practice is thus: Firſt, let the Solid be weighed in Air, that is, out of the Fluid; afterwards in it; (this ought to be done by ſuſpending it at one End of the Balance by a String, that is as nearly of the ſame ſpecific Gravity with the Fluid made uſe of as poſſible, and letting it ſink into the Fluid till it is wholly immerſed below the Surface; if the Fluid is Water, an Horſe-hair is moſt convenient to hang the Body at the End of the Balance by) then ſubtract its Weight in the Fluid from its Weight in Air, the Difference is what it loſes in the

CHAP. III.

Of the Air.

I. THAT Part of Natural Philoſophy which treats of the Properties of the *Air*, and the Effects of its *Preſſure* and *Elaſticity*, is called *Pneumatics*.

II. Air is a thin tranſparent elaſtic Fluid, ſurrounding the Earth to a certain Height, and, taken all together, is called the *Atmoſphere*.

III. That Air is a Fluid, is evident from the eaſy Paſſage it affords to Bodies moving in it: For this ſhews it to be a Body, whoſe Parts eaſily yield to a Preſſure that is greater on one Side than on the other, which is the Definition of a Fluid.

IV. Air gravitates towards the Earth, or is heavy like other Bodies.

To prove this we have Abundance of Arguments, both from Senſe and Experiment. Thus, when the Hand is applied to the Orifice of a Veſſel, it readily perceives the Weight of the incumbent Atmoſphere, as ſoon as the

the Fluid. This done, ſay, by the Rule of Proportion, as the Weight loſt in the Fluid is to its Weight in Air, ſo is Unity, or any Number taken at Pleaſure, to a Fourth, which by its Relation to the former, will expreſs the Relation, or ſpecific Gravity of the Solid to that of the Fluid. Thus, the Relation which the ſpecific Gravity of the ſame Fluid bears to that of various Solids, or of the ſame Solid to that of various Fluids, and conſequently the Relation of the ſpecific Gravities of all among themſelves, may be obtained.

Air

Air included in the Veſſel begins to be drawn out. Thus Glaſs Veſſels exhauſted of their Air (if not ſtrong enough to ſuſtain the Preſſure of the incumbent Atmoſphere) are cruſhed to Pieces by the Weight of the Air without. When the Air is exhauſted out of a Veſſel, the Veſſel weighs leſs than before. With a great many more Experiments generally mentioned by Authors on this Subject †.

V. The exact Weight of the incumbent Air is determined by filling a Tube with Mercury, and immerging the open End in a Veſſel of the ſame Fluid: For then the Mercury will run down the Tube, till its Surface is fallen to the perpendicular Height of about twenty-nine or thirty Inches above the Surface of the Mercury in the Veſſel, and no further: If the ſame Experiment is made with Water, the Surface of it will ſtand at about the Height of thirty-two Feet above the Surface of that in the Veſſel; the Column of Mercury in one Caſe, and the Column of Water in the other, exactly balancing the Weight of a Column of Air, which reaches to the Top of the Atmoſphere, and preſſes upon the Surface of the Fluid in the Veſſels. This is what is called the Torricellian Experiment, from TORRICELLI the Inventor, and is the ſame with the common Barometer.

† See *Boyle's* Tracts, or *Graveſande*, Lib. II. P. III.

From hence it follows (Chap. I. §. 9.) that all Bodies, at the Surface of the Earth, ſuſtain as great a Weight from the Preſſure of the Air, as is that of a Column of Water, whoſe Height is thirty-two Feet, and its Baſe equal to the Surface of the Body preſſed upon †.

VI. That the Suſpenſion of the Mercury in the Barometer depends on the Preſſure of the external Air, is beyond all doubt; for if the Barometer is included in the Air-Pump, the Mercury falls in the Tube, in Proportion as the Air is exhauſted out of the Receiver; and if the Air is let in again gradually, the Mercury re-aſcends proportionally, till it reaches its former Height.

VII. That the Atmoſphere is extended to a determined Height, appears from hence; *viz.* that when the Torricellian Tube is removed to a more elevated Place, the ſuſpended Column of Mercury becomes ſhorter; which is, becauſe a ſhorter Column of Air

† The Preſſure of the Atmoſphere upon every ſquare Inch, near the Surface of the Earth, is about fifteen Pounds, being equal to the Weight of a Column of Mercury, whoſe Height is thirty Inches, and its Baſe one ſquare Inch: For ſuch a Column of Mercury would weigh about fifteen Pounds. The Weight of the Atmoſphere therefore, which preſſes upon a Man's Body, is equal to ſo many Times fifteen Pounds, as the Surface of his Body contains ſquare Inches.

The Reaſon why a Perſon ſuffers no Inconvenience from ſo great a Preſſure, is owing to the Air included within the Pores and Fluids of the Body, which by its Reaction is a Counterpoiſe to the Preſſure of the external Air.

preſſes

preſſes upon it; or, that the Tube, in this Situation, is nearer the Top of the Atmoſphere.

VIII. The Elaſticity of the Air is that Property by which it contracts itſelf into leſs Space, when an additional Preſſure is laid upon it; and recovers its former Dimenſions, when the Preſſure is taken off. This is accounted its diſtinguiſhing Property, all the reſt being common to it with other Fluids.

Of this we have numerous Proofs. Thus, a Bladder full of Air being compreſſed by the Hand, the included Air gives way; but when the Preſſure is taken off, the Air expands itſelf, and readily fills up the Cavity, or Impreſſion made in the Surface of the Bladder. And, if a larger Quantity of Air, than is naturally preſſed into a Veſſel by the Weight of the incumbent Atmoſphere, is forced into it by the *Condenſer* (an Engine for that Purpoſe) and if that Air is afterwards let out by opening the Veſſel, the Remainder is found to be of the ſame Weight as at firſt; from whence it follows, that the Air, by means of its Elaſticity, or Spring, drives out all that which was forced in by the Condenſer, recovers its former Dimenſions, and fills the Veſſel as before.

IX. From hence, together with what has been obſerved about the Preſſure of the Atmoſphere, it follows, that the Air near the Surface of the Earth, is compreſſed into a much narrower Space by the Weight of the

Air above, than that which it would naturally take up, was it free from that Preſſure; accordingly it is found by Experiment, that, when the Preſſure of the Atmoſphere is taken off from any Portion of the Air, it immediately expands itſelf into a vaſt Extent. Hence it is, that thin Glaſs Bubbles or Bladders filled with Air, being included in the Receiver of the Air-Pump, are broke in Pieces by the Spring of the Air, which they contain within them, when the Preſſure of the external Air is taken off. Thus, a Bladder quite flaccid, containing only a ſmall Quantity of Air in it, ſwells upon the Removal of the external Air, and appears diſtended, as if it contained as great a Quantity as poſſible. The ſame Effect is found in carrying a Bladder ſomewhat flaccid to a more elevated Place: for there the external Preſſure being leſs, the Air included in the Bladder is in ſome Meaſure free from the Preſſure of the Atmoſphere; it therefore dilates itſelf, and diſtends the Bladder as in the former Caſe.

X. It is found by Experiment, that the Quantity of Space into which Air may be contracted by Preſſure, is reciprocally proportional to the compreſſing Force. From whence it follows, that the Denſity of the Air is proportional to the Preſſure which it ſuſtains; becauſe the leſs the Space is, into which a given Quantity of it is contracted, the denſer it is. As to the utmoſt Degree of Expanſion

ſion and Contraction, which the Air is capable of, they are as yet unknown. In ſeveral Experiments made by Mr. BOYLE, Air in its natural State, that is, preſſed only with the Weight of the incumbent Atmoſphere, dilated itſelf, when that Preſſure was taken off, into more than thirteen thouſand Times the Space it took up before. And he was able ſo far to compreſs it, that it ſhould take up more than five hundred and twenty thouſand Times leſs Space than that, into which it would dilate itſelf, when free from it Preſſure †.

XI. From this Property it follows, that the Air in the inferior Parts of the Atmoſphere is more denſe than that which is at great Heights in the ſame; or, that the Denſity of the Air decreaſes continually, as we approach the Top of the Atmoſphere. For the Denſity of the Air is proportional to the Force

† See *Boyle*'s Tracts and Experiments on the Spring and Preſſure of the Air.

Various have been the Opinions of Philoſophers concerning the Cauſe of this prodigious Spring in the Particles of Air; ſome holding it to depend on their Figure, which they ſuppoſe to reſemble in ſome Manner little Bundles of Twigs, or the Branches of Trees; ſome think them like Fleeces of Wool, others conceive them as rolled up like Hoops, or the Springs of Watches, and endeavouring to expand themſelves by Virtue of their Texture. But Sir *Iſaac Newton* is of Opinion, that ſuch a Texture is by no Means ſufficient to account for that vaſt Power of Expanſion obſerved above; but that each Particle is endued with a repelling Force which increaſes as they approach one another, and accordingly keeps them aſunder at Diſtances reciprocally proportional to the Preſſure they ſuſtain.

See *Hales*'s Statical Eſſays, Vol. I. Chap. 6.

with which it is compreſſed, and that Force continually decreaſes, as we approach the Top.

Was the Denſity of the Atmoſphere every where the ſame as it is near the Surface of the Earth, its Height (as is computed from the Quantity of Preſſure it exerts in raiſing the Mercury in the Barometer) would be about five Miles. But whereas its Denſity continually decreaſes as we approach the Top, and it is uncertain how far the Particles may expand themſelves, where there is little or no Preſſure, the true Height cannot be obtained. It is computed to continue of a ſenſible Denſity to the Height of about forty-five, or fifty Miles.

XII. The Elaſticity of the Air produces the ſame Effects with its Preſſure.

For, Action being equal to Reaction, the Force which the Spring of the Air exerts in endeavouring to expand itſelf, is equal to the Force with which it is compreſſed; juſt as it is in the Spring of a Watch, which exerts no Force, but in Proportion as it is wound up; conſequently a Quantity of Air in ſuch a State of Contraction, as it would be compreſſed into by the Weight of the incumbent Atmoſphere, exerts a Force equal to that Weight. If a Quantity of Air, therefore, is included in a Veſſel, and is of the ſame Denſity with the circumambient Air, its Preſſure againſt the Sides of the Veſſel is equal to the Weight of the Atmoſphere. Thus, Mercury is ſuſtained to the ſame Height by the elaſtic Force of Air included

included in a Glaſs Veſſel, no Way communicating with the external Air, as by the Weight of the Atmoſphere itſelf.

XIII. The Elaſticity of the Air is augmented by Heat, and diminiſhed by Cold*. For if a Bladder, which is about half filled with Air, is laid before the Fire, it will, when it is ſufficiently heated, be diſtended, and burſt. Thus, Glaſs Bubbles being laid upon the Fire, immediately burſt with great Violence by the augmented Spring of the included Air.

XIV. The Denſity of the Air thus continually varying, according to the different Degrees of Heat and Cold to which it is expoſed, makes it difficult to aſcertain its true ſpecific Gravity. RICCIOLUS eſtimates it to be to that of Water, as one to a thouſand: MERSENNUS, as one to one thouſand three hundred: Mr.

* This Property is found in all Bodies both ſolid and fluid, but in a much leſs Degree than it is in Air. Thus, if a Flaſk be filled with Water, only to the lower Part of the Neck, and is then ſet upon the Fire; the Water, when it begins to grow warm, will riſe into the Neck, and continue to aſcend, as the Heat is increaſed. And when a Wire or Bar of Iron is heated, it is augmented both in Length and Diameter.

Upon this Property depends the Phænomena of the Thermometer, which is a Glaſs Bubble, with a ſmall hollow Stem ariſing from it. This Bubble and Part of the Stem is uſually filled with Mercury, or Spirit of Wine, which will riſe or fall in the Stem, as they are affected by the Heat or Cold of the external Air. If a ſufficient Degree of Heat is ſuddenly applied to this Inſtrument, the Liquor is obſerved to deſcend a little before it riſes, becauſe the Glaſs diſtending itſelf, the Capacity of the Bubble is augmented, before the included Liquor is affected by the Heat.

Boyle, by more accurate Experiments, found it to be, as one to nine hundred and thirty-eight; and thinks, that all Things conſider'd, the Proportion of one to a thouſand may be taken as a Medium; for there is no fixing any preciſe Proportion, becauſe not only the ſpecific Gravity of Air, but that of Water alſo, is continually varying. However, by ſome Experiments made ſince with more Accuracy before the *Royal Society*, the Proportion has been fixed at about one to eight hundred and eighty.

XV. Air is neceſſary for the Preſervation of Animal and Vegetable Life: neither will Fire ſubſiſt without it. The Reaſon of this is as yet unknown to Philoſophers. Mr. Hales, by ſeveral curious Experiments in his Statical Eſſays, makes it probable, that 'tis owing to its Elaſticity †.

CHAP. IV.

Of the Reſiſtance of Fluids.

THE Reſiſtance a Body meets with in moving through a Fluid, is of three Kinds. The firſt ariſes from the Friction of the Body againſt the Particles of the Fluid; the ſecond from their Coheſion or Tenacity

† See his *Analyſis* of it. Statical Eſſays, Vol. I. Cap. 6.

among

among themſelves; the third, from their Inactivity, or the Tendency they have in common with other Bodies, to keep the Places they poſſeſs.

The firſt, *viz.* that which ariſes from the Friction of the Body againſt the Particles of the Fluid, is very inconſiderable; for whatever the Weight is which preſſes the Particles of a Fluid together, the Freedom with which a Body moves through it, is not ſenſibly diminiſhed thereby; as was obſerved, Chap. I. §. 2. in the Notes.

The ſecond, or that which ariſes from the Tenacity of the Particles of the Fluid, is as the Time that the Body continues paſſing in it *;

Thus,

* We have a very curious Argument in Confirmation of this, and which at the ſame Time illuſtrates the Manner in which a Body makes its Way through a tenacious Fluid, by Sir *Iſaac Newton* himſelf, in a Poſtſcript to a Letter in the Philoſophical Tranſactions, N° 371. It is as follows: 'Suppoſe Pieces of 'fine Silk, or the like thin Subſtance, extended in parallel 'Planes, and fixed at ſmall Diſtances from each other. Suppoſe 'then a Globe to ſtrike perpendicularly againſt the Middle of 'the outermoſt of the Silks, and by breaking through them to 'loſe Part of its Motion. If the Pieces of Silk be of equal 'Strength, the ſame Degree of Force will be required to break 'each of them; but the Time, in which each Piece of Silk re'ſiſts, will be ſo much ſhorter as the Globe is ſwifter; and the 'Loſs of Motion in the Globe conſequent upon its breaking 'through each Silk, and ſurmounting the Reſiſtance thereof, 'will be proportional to the Time in which the Silk oppoſes it'ſelf to the Globe's Motion; inſomuch that the Globe, by the 'Reſiſtance of any one Piece of Silk, will loſe ſo much leſs of 'its Motion as it is ſwifter. But on the other Hand, by how 'much ſwifter the Globe moves, ſo many more of the Silks it 'will break through in a given Space of Time; whence the 'Number

Thus, ſuppoſe a Body continues to move within a Fluid for the Space of one Hour; if its Motion be very quick during that Time, it then moves a great Way in that Fluid, and conſequently meets with a great deal of the Fluid to reſiſt it; but is reſiſted the leſs in each Part thereof in Proportion to the Stay it makes there: whereas, if its Motion be ſlow, it then moves but a little Way in that Time, but is reſiſted ſo much the longer in each Part thereof. So that whether the Body moves faſt or ſlow, the Reſiſtance it ſuffers is the ſame; and depends therefore on nothing but the Time we ſuppoſe it to continue moving within the Fluid. This Species of Reſiſtance is alſo very ſmall, except in glutinous and viſcid Fluids, whoſe Parts are not eaſily ſeparated.

'Number of the Silks, which oppoſe themſelves to the Motion 'of the Globe in a given Time, being reciprocally proportional to the Effect of each Silk upon the Globe, the Reſiſtance made to the Globe by theſe Silks, or the Loſs of Motion the Globe undergoes by them in a given Time, will be always the ſame.

'Now, if the Tenacity of the Parts of Fluids obſerves the 'ſame Rule, as the Coheſion of the Parts of theſe Silks; namely, that a certain Degree of Force is required to ſeparate and diſunite the adhering Particles, the Reſiſtance ariſing from the Tenacity of Fluids muſt obſerve the ſame Rule, as the Reſiſtance of the Silks; and therefore in a given Time, the Loſs of Motion a Body undergoes in a Fluid by the Tenacity of its Parts, will in all Degrees of Velocity be the ſame; or in fewer Words, that Part of the Reſiſtance of Fluids, which ariſes from the Coheſion of their Parts, will be uniform.'

The

The third Species is the principal Reſiſtance that Fluids give to Bodies, and ariſes from their Inactivity, or the Tendency the Particles, of which they conſiſt, have to continue at Reſt. The Quantity of this Reſiſtance depends on the Velocity the Body moves with on a double Account: For, in the firſt Place, the Number of Particles put into Motion by the moving Body, in any determinate Space of Time, is proportional to the Velocity wherewith the Body moves; and in the next Place, the Velocity with which each of them is moved, is alſo proportional to the Velocity of the Body; this Species therefore of Reſiſtance is in a duplicate Proportion, or as the Square of the Velocity with which the Body moves through the Fluid *.

II. Farther, the Reſiſtance which a Body, moving in a Fluid, meet with from thence, may be conſidered with Regard to the Fluid; and then it will be found to be more or leſs, ac-

* This may be otherwiſe demonſtrated from the twelfth Section of the firſt Chapter; for from thence it follows, that the Reſiſtance a Fluid gives to a Solid againſt which it moves, is proportional to the Height a Body muſt fall from, to acquire ſuch a Degree of Velocity as the Fluid moves with: But the Heights which Bodies fall from are as the Squares of the Velocities they acquire by falling; conſequently the Reſiſtance which a Fluid gives to a Solid, againſt which it moves, is alſo as the Square of its Velocity. Now it matters not, as to the Reſiſtance, whether the Fluid moves againſt the Solid, or whether it be at Reſt, and the Solid moves in it; the Reſiſtance therefore which a Fluid gives to a Solid moving in it, is as the Square of the Velocity with which it moves.

cording

cording to the Denſity of the Fluid. For by how much denſer the Fluid is, ſo much the greater Number of Particles are to be put into Motion by the Body, in order to make its Way through it.

III. The next Thing to be conſidered, is the Effects of the Reſiſtance of Fluids upon Bodies moving in them; that is, the Retardation which they cauſe in the Motion of a Body by their Reſiſtance, or the Quantity of Motion they deſtroy.

And this, in ſimiliar Bodies of equal Magnitudes, is inverſely as their Denſities, or the Quantity of Matter they contain: for by how much the greater the Quantity of Matter in any Body is, ſo much the more eaſily does it overcome the Reſiſtance it meets with from the Fluid. Thus we ſee the Reſiſtance of the Air has a much leſs Effect in deſtroying the Motion of an heavy Body, than of a light one which has the ſame Dimenſions.

IV. In ſimilar Bodies of equal Denſities, but different Magnitudes, the Retardation is inverſely as their homologous Sides. For the Reſiſtance which Bodies meet with in a Fluid, is inverſely as the Quantities of Matter they contain (by the laſt) that is inverſely as the Cubes of their homologous Sides; and it is alſo directly as their Surfaces, becauſe 'tis by them that they move the Fluid out of its Place; that is, directly as the Squares of their homologous Sides;

con-

conſequently the Retardation is inverſely as their homologous Sides †.

Having given the fundamental Principles of *Hydroſtatics*, and ſhewn how Fluids, both compreſſible and incompreſſible, are diſpoſed to act upon each other, and upon Solids by their Preſſure, Motion, Elaſticity, and Reſiſtance; I proceed now to account for ſome of the more remarkable Phænomena of Nature, in which they are in Part, or altogether concerned: And this I deſign for the Subject of the following Diſſertations.

† Becauſe the inverſe Ratio of the Cubes of any Numbers being compounded with the direct Ratio of the Squares of the ſame, gives the inverſe Ratio of the Numbers themſelves.

DIS-

DISSERTATION I.

Of Sound.

WHEN the Parts of an elaſtic Body are put into a tremulous Motion, by Percuſſion, or the like; ſo long as the Tremors continue, ſo long is the Air included in the Pores of that Body, and likewiſe that which preſſes upon its Surface, affected with the like Tremors and Agitations: Now, the Particles of Air being ſo far compreſſed together by the Weight of the incumbent Atmoſphere, as their repulſive Forces permit, (as has been explained, Chap. 3.) it follows, that thoſe, which are immediately agitated by the reciprocal Motions of the Particles of the elaſtic Body, will, in their Approach towards thoſe which lie next them, impel them alſo towards each other; and thereby cauſe them to be more condenſed, than they were by the Weight of the incumbent Atmoſphere, and in their Return will ſuffer them to expand themſelves again; whereby the like Tremors and Agitations will be propagated to them; and ſo on, till having arrived at a certain Diſtance from the Body, the Vibrations ceaſe, being gradually deſtroyed by a continual ſucceſſive Propagation of Motion to freſh Particles of Air throughout their Progreſs.

Thus

Thus it is that Sound is communicated from a tremulous Body to the Organ of Hearing. Each Vibration of the Particles of the ſounding Body is ſucceſſively propagated to the Particles of the Air, till it reaches thoſe which are contiguous to the *Tympanum* of the Ear, (a fine Membrane diſtended acroſs it,) and theſe Particles, in performing their Vibrations, impinge upon the Tympanum, which agitates the Air included within it; and that being put into a like tremulous Motion, affects the auditory Nerve, and thus excites in the Mind the Senſation or Idea of what we call *Sound*.

Now ſince the repulſive Force of each Particle of Air is equally diffuſed around it every Way; it follows, that when any one approaches a Number of others, it not only repels thoſe which lie before it, in a right Line; but the reſt alſo, laterally, according to their reſpective Situations: that is, it makes them recede every Way from itſelf, as from a Center: and this being true of every Particle, it follows, that the aforeſaid Tremors will be propagated from the ſounding Body in all Directions, as from a Center: And further, if they are confined for ſome Time from ſpreading themſelves by paſſing through a Tube or the like, will, when they have paſſed through it, ſpread themſelves from the End in every Direction. In like Manner, thoſe which paſs through an Hole in an Obſtacle they meet with in their Way, will afterwards ſpread themſelves

themſelves from thence, as if that was the Place where they began; ſo that the Sound, which paſſes through an Hole in a Wall, or the like, is heard in any Situation whatever, that is not at too great a Diſtance from it. Something analogous to this we may obſerve in the Motion of Waves upon the Surface of a Fluid, which are propagated equally through all Parts of the Surface in a Circle, though not occaſioned by a circular, but reciprocal Motion and Agitation of the Finger, or any other Body in a ſtrait Line.

Since the Repulſive Force with which the Particles of Air act upon each other, is reciprocally as their Diſtances; (Chap. 3. §. 10.) it follows, that when any Particle is removed out of its Place by the Tremors of a ſounding Body, or the Vibrations of thoſe which are contiguous to it, it will be driven back again by the repulſive Force of thoſe towards which it is impelled, with a Velocity proportional to the Diſtance from its proper Place; becauſe the Velocity will be as the repelling Force. The Conſequence of this is, that let the Diſtance be great or ſmall, it will return to its Place in the ſame Time; (for the Time a Body takes up in moving from Place to Place will always be the ſame, ſo long as the Velocity it moves with is proportional to the Diſtance between the Places.) The Time therefore in which each Vibration of the Air is performed, depends on the Degree of Repul-

ſion

sion in its Particle: And so long as that is not altered, will be the same at all Distances from the tremulous Body; consequently, as the Motion of Sound is owing to the successive Propagation of the Tremors of a sounding Body through the Air, and as that Propagation depends on the Time each Tremor is performed in, it follows, that the Velocity of Sound varies as the Elasticity of the Air, but continues the same at all Distances from the sounding Body.

And as the Velocity with which Sound is propagated, depends on the Elasticity of the Air, so it does also on its Density; for when the Density of the Air is augmented, while its Elasticity remains the same †, a greater Number of Particles will move forwards and backwards in each Vibration; now, since we suppose the Cause by which they put each other

† Perhaps it will not appear to every one, how the Density of the Air can be augmented without a proportional Increase of its Elasticity; because, *cæteris paribus*, the nearer the Particles approach each other, the stronger is the Action of their repulsive Force.

But it is to be consider'd, that when the Air becomes colder, its Elasticity is diminished, and then the Particles are brought closer together by the Pressure of the Atmosphere, till they acquire an Elasticity equal to what they had before; *viz.* such as answers to the Pressure they sustain (Chap. 3. §. 12.) From hence we may infer, that the Propagation of Sound is slower in Winter than in Summer, when the Mercury in the Barometer is at the same Height; for the Pressure of the Air being the same, its Elasticity, which depends upon it, is so too; but the Air is denser by Reason of the Cold, and therefore its Vibrations slower.

into Motion (*viz.* their Elaſticity,) the ſame, they will each receive a leſs Degree of Velocity; and ſo the Vibrations will be performed in a longer Time, whence the Succeſſion of them will be ſlower, and the Progreſs of the Sound proportionally retarded *.

Whereas the undulatory Motion of the Air, which conſtitutes Sound, is propagated in all Directions from the ſounding Body; it will frequently happen, that the Air, in performing its Vibrations, will impinge againſt various Objects, which will reflect it back, and ſo cauſe new Vibrations the contrary Way; now, if the Objects are ſo ſituated, as to reflect a ſufficient Number of Vibrations back (*viz.* ſuch as proceed different Ways) to the

* The Method of determining the Velocity with which Sound is propagated, is (by the Help of a ſhort Pendulum) to eſtimate the Time which paſſes between ſeeing a Fire of a Gun at a Diſtance, and hearing the Report. Its great Velocity makes it difficult to be determined exactly; accordingly Authors differ much in their Accounts. The moſt accurate Obſervers, Dr. *Halley* and Dr. *Derham*, have found it to be about one thouſand one hundred and forty-two Feet, which is almoſt a Quarter of a Mile, in a Second.

The uſual Experiments to prove that the Air is neceſſary for the Propagation of Sound, are ſuch as theſe. A ſmall Bell being put into the Receiver of the Air-Pump, may be heard at a conſiderable Diſtance before the Air is exhauſted out of it; but when the Air is much rarefied by exhauſting, can ſcarcely be heard at all. When the Air is condenſed, the Sound is augmented in Proportion to the Condenſation. Theſe Experiments do not only ſucceed in forced Rarefactions and Condenſations, but in ſuch alſo as are natural; Sound being obſerved to be much weaker on the Tops of high Mountains, where the Air is leſs condenſed by the Weight of the incumbent Atmoſphere, than in the Valleys below.

ſame Place, the Sound will be there repeated, and is called an *Echo* *. And, the greater the Diſtance of the Objects is, the longer will be the Time before the Repetition is heard. And when the Sound in its Progreſs meets with Objects, at different Diſtances, ſufficient to produce an Echo, the ſame Sound will be repeated ſeveral Times ſucceſſively, according to the different Diſtances of thoſe Objects from the ſounding Body; which makes what is called a *repeated* Echo.

If the Vibrations of the tremulous Body are propagated through a long Tube, they will be continually reverberated from the Sides of the Tube into its Axis, and by that Means prevented from ſpreading, till they get out of it; whereby they will be exceedingly increaſed, and the Sound rendered much louder than it would otherwiſe be †.

The Difference of *Muſical* Tones depend on the different Number of Vibrations communicated to the Air, in a given Time, by the Tremors

* In *Woodſtock* Park, near *Oxford*, there is an Echo, which repeats diſtinctly ſeventeen Syllables by Day, and twenty by Night. See *Plot*'s Natural Hiſtory of *Oxfordſhire*.

The Reaſon why it repeats more Syllables by Night, than by Day, is becauſe the Air, being colder at that Time, is more denſe; and therefore the Return of the firſt Vibrations being ſlower, gives Time for the Repetition of more Syllables.

† This is the Caſe in the *Stentorophonic* Tube or Speaking-Trumpet. See *Kircher* de Re Muſica, Lib. 9. Par. 3. *Ode* Philoſ. Nat. Princip. p. 293.

Tremors of the ſounding Body; and the quicker the Succeſſion of the Vibrations is, the acuter is the Tone, and *è contra*.

A muſical Chord performs all its Vibrations, whether great or ſmall, in the ſame Time. For, if a String is ſtretched betweeen two Pins, and a Force is applied to the middle Point, to draw it out of its rectilineal Situation; it is found by Experiment, that the Diſtance (if it be ſmall) to which it is drawn, is as the Force applied; conſequently the Velocity, with which it returns, when left to itſelf, will be as the Space it has to move over; it will therefore perform all its Vibrations in the ſame Time: This is the Reaſon why the ſame Chord, however ſtruck, produces the ſame Note. It is alſo found by Experiment, that when Strings of equal Diameters, but different Lengths, are equally ſtretched, the longer they are, ſo much the leſs Weights will draw them from their rectilineal Situation to the ſame Diſtance;

Upon this Principle it is, that Sound is conveyed from one Side of a *Whiſpering* Gallery to the oppoſite one, without being perceived by thoſe who ſtand in the Middle. The Form of a *Whiſpering* Gallery is that of a Segment of a Sphere, or the like arched Figure; and the Progreſs of the Sound through it may be illuſtrated in the following Manner.

Let A B C (*Fig.* 14.) repreſent the Segment of a Sphere; and ſuppoſe a low Voice uttered at D, the Vibrations expanding themſelves every Way, ſome will impinge upon the Points E, E, &c. and from thence be reflected to the Point F, from thence to G, and ſo on, till they all meet in C; and by their Union there cauſe a much ſtronger Sound, than in any other Part of the Segment whatever, even at D, the Point from whence they came.

the

the Forces therefore by which they return are leſs, and the Times of their Vibrations longer.

When two Chords perform their Vibrations in equal Times, the Tone produced is called an *Uniſon*. If one performs two, while the other one; 'tis an *Octave*. If one three, while the other two; 'tis a *Fifth*. If one three, while the other four; 'tis called a *Fourth*, &c.

To make an Uniſon Sound, it is not neceſſary that the Vibrations of the two Strings ſhould actually concur, but only that they ſhould be performed in equal Times; ſo that they would always concur, if they began at the ſame Inſtant. For the Ear perceives not the ſingle Vibrations diſtinctly, but only finds that Difference, which proceeds from the Intervals of Time that paſs between them *.

* Upon theſe Principles we may account for that remarkable Phænomenon in *Muſic*, that an intenſe Sound being raiſed, either with the Voice, or a ſonorous Body, another ſonorous Body near it, and in Uniſon with it, will thereby be made to ſound. For the Vibrations of the Air, which correſpond to the Tremors of the firſt ſounding Body, agreeing exactly, in Point of Time, with thoſe which are capable of being given to the other Body at Uniſon with it; when they have by their firſt Impulſe communicated a ſmall Degree of Motion to it, will, by conſpiring with it, as it moves forwards and backwards, continually increaſe its Motion, till it becomes ſenſible. The contrary happens in Strings which are in Diſcord with each other; for in this Caſe, though the firſt Vibration of one may give Motion to the other, yet their Vibrations not being performed in equal Times, the ſecond will come *unſeaſonably*, *i. e.* when the other is moving the contrary Way, and obſtruct its Motion. It is farther obſervable, that in two Strings, one of which vi-

brates twice, while the other once; if the firſt be ſounded, the two Extremes of the other will each ſound an Uniſon with it, while the middle Point remains at Reſt. So if one vibrates thrice, while the other once, the laſt will be divided into three Parts, each of which will ſound an Uniſon with it, and the two Points between thoſe Parts will remain at Reſt. For otherwiſe, that which vibrates twice, while the other once, muſt neceſſarily interfere with it at every ſecond Vibration; and that which vibrates thrice, while the other once, would interfere with it at every third; ſo that it would not be put into a ſufficient Motion to produce a Sound. But, when it is divided by the quieſcent Points, it becomes ſo many Strings at Uniſon with the former, each of which eaſily receives its Vibrations from thence.

From hence likewiſe it is, that if we take two or three Drinking-Glaſſes, and put ſome Water or other Fluid into each of them, and place them near to each other, taking Care to fill them to ſuch Heights, that (when ſtruck) their Tones ſhall be in Uniſon; and then, if we ſlide the Finger along the Brim of one of the Glaſſes, preſſing pretty ſtrongly upon it, (which will cauſe it to ſound) we ſhall ſee the Surface of the Fluids in the other Glaſſes begin to tremble; which ſhews, that the Vibrations of the firſt Glaſs cauſe the like in the other at Uniſon with it; though not perhaps in a Degree ſufficient to produce a Sound ſtrong enough to be heard diſtinctly from the former.

Thus it is that ſome Perſons are able to break a Drinking-Glaſs by a Tone of their Voice at Uniſon with it. They firſt try the Tone of the Glaſs by ſtriking it, then applying their Mouth near to the Brim of it, ſound the ſame Note with their Voice; this ſets the Glaſs a trembling; they then continually raiſe their Voice, ſounding ſtill the ſame Note; this increaſes the Tremors of the Glaſs, which by that Means (if it is not too ſtrong) is broke in Pieces.

The Effect of *Muſic* upon Perſons bit with a *Tarantula*, (if the Accounts we have of it from Abroad may be credited) is very ſurpriſing. A Perſon bit with the *Tarantula*, after ſome Time loſes both Senſe and Motion, and dies if deſtitute of Help. The moſt effectual Remedy is *Muſic*. The Muſician tries Variety of Airs, till he hits upon one that affects the Patient, who upon that begins to move by Degrees; firſt keeps Time with his Fingers, Arms, and Legs, afterwards is violently agitated in every Part of his Body; and then leaps up, begins to dance, and increaſes in Activity every Moment; till after five or ſix Hours, being much fatigued, he is put to

Bed,

Bed, and left to ſweat. The next Day the ſame Air brings him out of Bed for a new Dance. Which Exerciſe being thus continued, the Diſtemper is abated in the Space of four or five Days, the Effects of the Bite being in ſome Meaſure carried off by Sweat, and the Patient begins then to recover his Senſe and Knowledge by little and little.

The Reaſon why the Patient is thus affected by the *Muſic*, is, becauſe the Nerves of his Body are ſo diſpoſed in that Diſtemper, as eaſily to be agitated by the Vibrations which are occaſioned by the principal and ſtronger Notes of what is played.

See, on the Subject of this Diſſertation, Philoſoph. Tranſact. N° 134, 234, 302, 313, 319, 337. Hiſt. de l'Acad. 1702, 1708. *Grew*'s Coſmolog. Sacr. Book I. Chap. 5. *Mead* upon Poiſons, p. 59. *Keil*'s Anatomy, p. 214. *Baglivi* Prax. Medic. Diſſert. I. *Bellini* de Urinis & Pulſibus, &c, ſub fine Capitis de Mania.

DISSERTATION II.

Of Capillary Tubes.

BY a *Capillary Tube* is generally underſtood a Glaſs Pipe, the Diameter of whoſe Bore is at moſt but about one tenth of an Inch; though any Tube, whoſe Cavity does not exceed that Magnitude, may be ſo called.

The Phænomena of Capillary Tubes being ſuch as contradict a known Law in *Hydroſtatics*, viz. *that a Fluid riſes in a Tube to the ſame Height with the Level of its Source* †; and likewiſe of Affinity with the Aſcent of the Sap through the Stems of Plants for the Nouriſhment of their Fruit, and with divers other Operations of Nature: it has been thought of no ſmall Moment in Philoſophy to find out and eſtabliſh their true Cauſe; which, after numerous Experiments, and ſeveral Conjectures about it, is found to be no other than the Attraction of Coheſion; by which ſmall Particles of Matter mutually adhere together, and form larger Bodies ‖. I ſhall lay down

† See Chap. I. §. 9. Caſe 4. in the Notes.

‖ See *Hauksbee*'s and *Power*'s Experiments. *Muſſchenbroeck*, 4to Edit. Philoſoph. Tranſact. N° 355. Mem. de l'Acad. 1705, 1714, 1722, 1724, with others referr'd to in Quæſtiones Philoſoph.

the

the ſeveral Phænomena, as ſo many Matters of Fact; and ſubjoin to each a Solution from that Cauſe. In order to which, it may not be improper to premiſe the following Conſideration by Way of *Lemma*.

Let us ſuppoſe the Veſſel A B C D (*Fig.* 15.) filled with a Fluid to the Height L M, and let it be conceived as divided into the equal Portions E F G H, G H I K, I K L M, *&c.* Farther, let it be ſuppoſed, that each Particle of Matter in the inner Surface of the Veſſel, has a Sphere of Attraction, whoſe Semidiameter is equal to the Breadth of three of thoſe Portions; that is to ſay, that the Attraction of the Particle M reaches upwards as far as F, and downwards as far as S; and that of the Particle O, upwards as far as H, and downwards as far as U; and ſo of all the reſt quite round the Tube. From hence it will follow, that every Particle of the inner Surface of the Veſſel, which lies between E F and R S, conſpires in endeavouring to raiſe the Fluid towards A B the Top of the Veſſel; and that the Fluid is not affected by any other. For Inſtance, the Particle S, and all below it, will attract downwards three Strata of the Fluid (ſuch as are contained in three equal Diviſions of the Veſſel) from above, and as many upwards from below; and therefore will have no Effect at all in raiſing or depreſſing the Fluid. But the Particle Q will attract only two Strata downwards; becauſe there are no more above it,

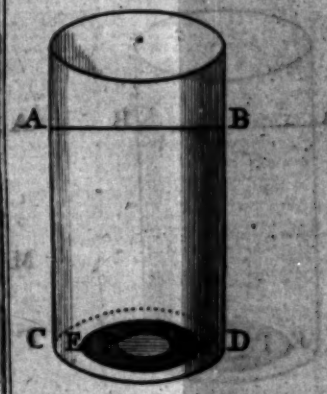
it, and three upwards, and therefore will in ſome Meaſure tend to raiſe the Fluid; ſo the Particle O will attract but one downwards, and three upwards; the Particle M none downwards, and three upwards; the Particle K two upwards, and H only one: all which may clearly be ſeen by their Situations in the Figure with Reſpect to the Surface of the Fluid. Therefore in every Veſſel, where there is a mutual Attraction between the Fluid it contains and the Particles of which it is compoſed, there will be a certain Number of Particles diſpoſed around it in Form of a broad Periphery or Zone, as repreſented by AB (*Fig.* 16.) half of which lies above the Surface of the Fluid, and half below it, and will tend to make it riſe towards the Top. This being underſtood, the following Phænomena will not be difficult *.

* I have been the more particular in explaining this *Lemma*, becauſe it is not a bare *Periphery* of no Breadth, to which the Aſcent of the Fluid is owing, but a *Zone* or *Cingulum* of Particles diſtended equally in Breadth both Ways from the Surface of the Fluid; and becauſe it is upon the Breadth of this, that ſome of the following Solutions depend. As to the Thickneſs of it, that undoubtedly is equal to the Semidiameter of the Sphere of Attraction in the Particles of the Veſſel; and therefore Veſſels, whoſe Sides are of different Thickneſſes (provided thoſe Thickneſſes be leſs than that Semidiameter) muſt have different Effects upon the ſame Fluid, though no one has as yet been ſo accurate as to obſerve it. The Reaſon why a Fluid will not riſe in a large Veſſel, as well as in one that is Capillary, is, becauſe the Attraction of its Particles does not reach far enough into the Middle of the Veſſel; and therefore it only riſes about the Sides ſtanding higher than in the Middle: as may be ſeen in a Drinking-Glaſs, when a Quantity of Water is put into it, ſomewhat leſs than is ſufficient to fill it.

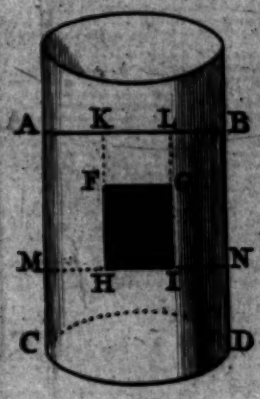
I. Let

Part II. Plate III. p154

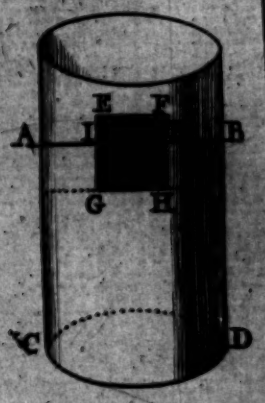
F.11. p.120.



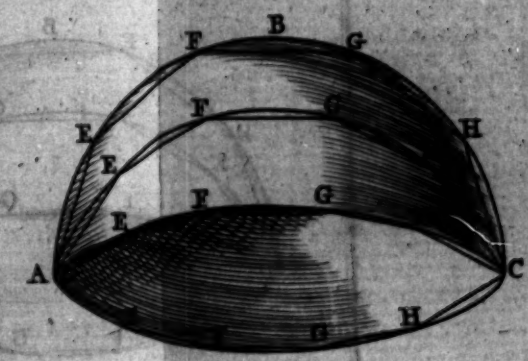
F.12. p123.



F.13. p125.



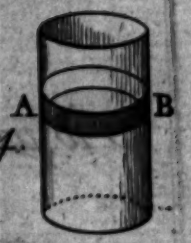
F.14. p.148.

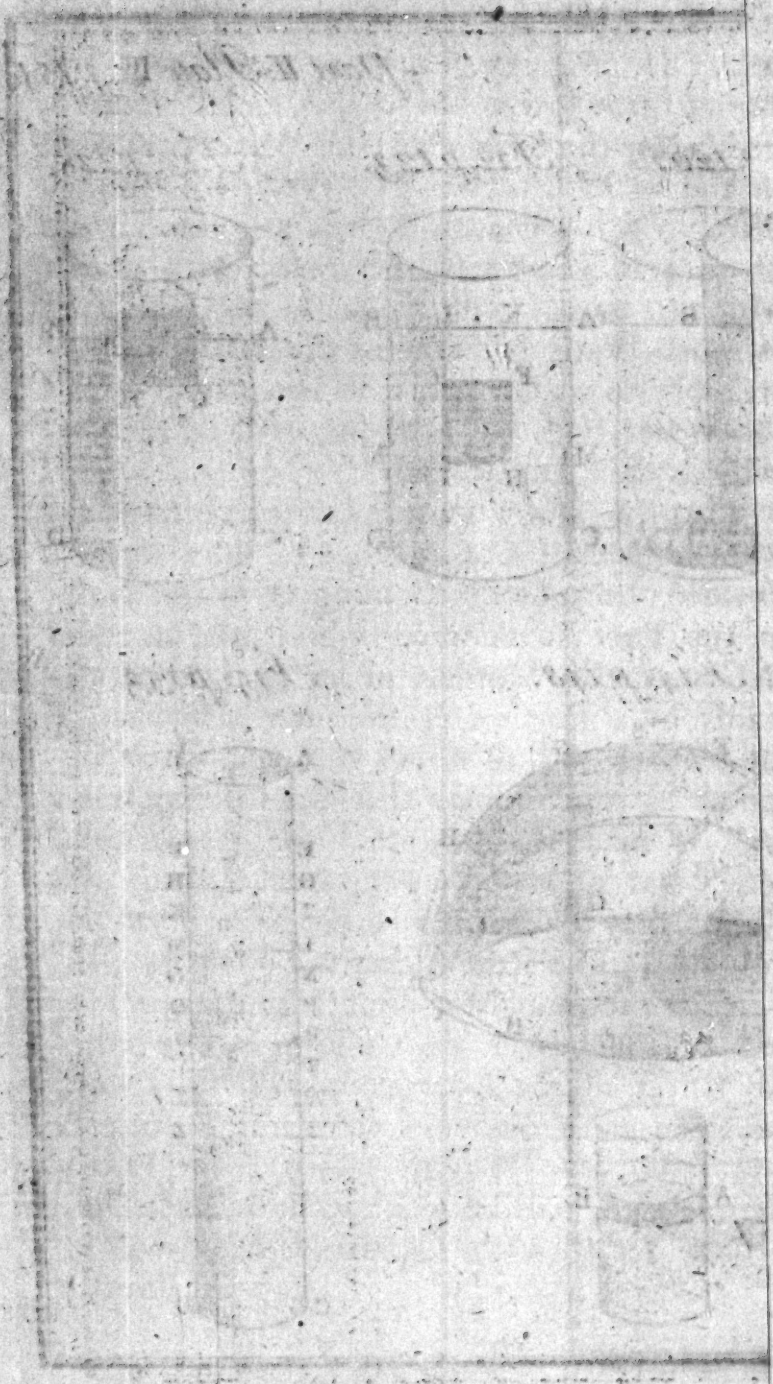


F.15. p.153.



F.16. p154.





I. Let there be two Capillary Tubes, AB and CD (*Fig.* 17.) open at both Ends; and having their lower Orifices, A and C, immerged below the Surface of the Water contained in the Veſſel FGHI: The Water will immediately riſe up in each Tube above the Surface of that in the Veſſel, beginning with a ſwift Motion, which will gradually decreaſe; till as much Water has entered the Tubes, as they are able to raiſe: And the Heights to which the Water will riſe in them, will be reciprocally as their Diameters.

That the Water ought to riſe in both Tubes is an immediate Conſequence of the foregoing *Lemma*; becauſe the Column of Water within the Tube is rendered lighter than an equal Column on the Outſide, as being attracted upwards by a Portion of the interior Surface of the Veſſel; and therefore will riſe till it becomes as much longer than the external ones, as it is made lighter; that the *Æquilibrium*, which was deſtroyed by the Attraction of the Tube, may be reſtored by the Weight of the Column. The Reaſon that the Velocity with which it riſes, ought conſtantly to decreaſe, is, becauſe the heavier the Column is, the leſs is the Effect of the Attraction, which is always the ſame in a Tube of the ſame Diameter. And laſtly, the Heights to which the Water riſes in them, will be reciprocally as their Diameters; for then the Quantities raiſed will be directly

directly as the Diameters †; but the Peripheries that raiſe them, (being always of the ſame Breadth, and having their Lengths equal to the Circumferences of the Tubes) are as thoſe Diameters; the Quantities of Water, therefore, being in the ſame Ratio, are as the Peripheries, *i. e.* as the cauſes by which they are raiſed.

II. If the Tubes, before they are immerged in the Water, are filled to greater Heights than thoſe to which it would naturally riſe in them, and then have their lower Orifices immerged in Water, the Water will ſubſide till it ſtands in each at the ſame Height to which it would have riſen; but, if they are held in a perpendicular Poſition, and the Water be ſuffered to run out at their lower Orifices, without being immerged, the Water will not ſubſide in the Tubes *quite* ſo far.

The Reaſon why the Water in the Tube, when its lower Orifice is immerged, ſubſides to the ſame Height it would have riſen to, had the Tube been immerged when empty, is becauſe the Column is ſuſpended in one Caſe by the ſame Cauſe, by which it is raiſed in the other; but when the Tube, full of Water, is

† The Heights to which the Water riſes, being in a reciprocal Ratio of the Diameters; and the Contents of Cylindrical Tubes being in a direct Ratio of their Heights, and of the Squares of their Diameters; the Quantities of Water raiſed in this Caſe will be in a reciprocal Ratio of the Diameters, and a direct one of the Squares of the ſame. Now theſe two Ratios being compounded together, give the direct one of the Diameters themſelves; becauſe the ſimple reciprocal Ratio deſtroys one of thoſe, which are contained in the direct one of the Squares.

held

held erect, without being immerged, it will not ſubſide *quite* ſo far; becauſe the lower End of the Tube, which the Water leaves behind it as it drops out, attracts it the contrary Way; ſo that the Column in this Caſe is ſuſpended, not only by the inner Surface of the Tube at the Top, but alſo by its lower End; and therefore a greater Quantity of Water is ſuſpended than in the former Caſe.

III. If a Tube, having its lower Orifice immerged in Water, be held obliquely, it will raiſe the Water to the ſame perpendicular Height, as when held erect.

For ſince Fluids preſs according to their perpendicular Heights, the Weight of the Column raiſed will not be proportional to the attractive Force of the Tube, till it has arrived at the ſame perpendicular Height, to which it would have roſe, if held erect.

IV. If a Tube, when the Water is riſen into it to its wonted Height, is laid in an horizontal Situation, the Water will move towards the Middle of the Tube, leaving the End which was immerſed a little behind.

The Solution of this Phænomenon depends on what was obſerved in the *Lemma* about the Breadth of the attracting Periphery, and its being equally ſituated on each Side the Surface of the Water; for from thence it follows, that if the Water ſhould not run from the full End of the Tube, after it has laid in an horizontal Situation, but remains contiguous

to

to it; that End of the Column of Water would be attracted only by ſuch a Portion of a Periphery as lies within its Surface at that End; becauſe the End of the Tube coinciding with the Surface, the other Half of the Periphery is wanting. Whereas, at the other End of the Column, there is a Periphery whoſe Breadth is entire; which, overpowering the other, cauſes the Water to move towards the Middle of the Tube, till as much Periphery is left behind, as can at once affect the Water by the Attraction of its Particles; after which the Water being attracted equally each Way, remains at Reſt.

V. Let there be a Tube (*Fig.* 18.) conſiſting of two Parts D R and R C K, of different Diameters; it follows from what has been ſaid, that D R the ſmaller Part of the Tube, is able to raiſe Water higher than the other: Let then the Height to which the larger would raiſe it, be T F, and that to which it would riſe in the leſſer (was it continued down to the Surface of the Fluid) be X L: If this compound Tube be filled with Water, and the larger Orifice C K be immerſed in the ſame Fluid, the Surface of the Water will ſink no farther than X L, the Height to which the leſſer Part of the Tube would have raiſed it.

But if the Tube be inverted, (as in *Fig.* 19.) and the ſmaller Orifice X L be immerſed, the Water will run out till the Surface falls to T F; the Height to which the larger Part of the Tube

Tube would have raiſed it. The Size of the lower Part making no Alteration in the Height, at which the Fluid is ſuſpended in either Caſe.

In order to account for theſe Phænomena, it muſt be conſidered, that when a Body is ſo diſpoſed, that its different Parts ſhall move with different Degrees of Velocity, the greater Proportion the Velocity of that Part to which a moving Power is applied, bears to that of the Reſt, ſo much the more *effectual* is the Power in moving that Body: As is the known Caſe of the *Lever*, and all the other *Mechanical* Powers.

Now let us conceive the Tube D R (*Fig.* 18.) continued down to H I, and let it be ſuppoſed at preſent that the Fluids contained in the Tube X L H I, and the compound one X L K C, are not ſuſpended by the Periphery at L, but that they preſs upon their reſpective Baſes, H I and C K. Let it farther be ſuppoſed that theſe Baſes are each of them moveable, and that they are raiſed up or let down with equal Velocities; then will the Velocity with which X L the uppermoſt Stratum of the Fluid X L C K moves exceed that of the ſame Stratum conſidered as the uppermoſt of the Fluid in the Tube X L H I, as much as the Tube R C K is wider than D R (by the *Lemma*, Chap. I. §. 9.) that is, as much as the Space M N K C exceeds X L I H; conſequently, by what was obſerved in the foregoing Paragraph, the

Effect

Effect of the attracting Periphery X L, as it acts upon the Fluid contained in the Veſſel X L C K, exceeds its Effect, as it acts upon that in X L H I, in the ſame Proportion. Since therefore it is able (*ex Hypoth.*) to ſuſtain the Weight of the Fluid X L H I by its natural Power, it is able under this *Mechanical* Advantage, to ſuſtain the Weight of as much as would fill the Space M N K C: But the Preſſure of the Fluid X L C K is equal to that Weight, as having the ſame Baſe and an equal Height (Chap. I. §. 9.) its Preſſure therefore, or the Tendency it has to deſcend in the Tube, is equivalent to the Power of the attracting Periphery X L, for which Reaſon it ought to be ſuſpended by it.

Again, the Height (*Fig.* 19.) at which the attracting Periphery in the larger Part of the Tube is able to ſuſtain the Fluid, is no greater than N F, that to which it would have raiſed it, had the Tube been continued down to M N. For here the Power of the attracting Periphery acts under a like *Mechanical* Diſadvantage, and is thereby diminiſhed in Proportion to the Capacity of the Tube T F N M to that of H I X L; becauſe, if the Baſes of theſe Tubes are ſuppoſed to be moved with equal Velocities, the Riſe or Fall of the Surface of the Fluid T F X L, would be ſo much leſs than that of T F M N. And, whereas the attracting Periphery T F is able by its natural Power to ſuſpend the Fluid only to the Height

Height N F in the Tube T F M N; it is in this Caſe able to ſuſtain no greater Preſſure than what is equal to the Weight of the Fluid in the Space H I X L: but the Preſſure of the Fluid T F X L, which has equal Height, and the ſame Baſe with it, is equal to that Weight; and therefore is a juſt *Æquipondium* to the attracting Power.

VI. From hence we may clearly ſee the Reaſon, why a ſmall Quantity of Water put into a Capillary Tube, which is of a Conical Form, and laid in an Horizontal Situation, will run towards the narrower End. For let A B (*Fig.* 20.) be the Tube, C D a Column of Water contained within it; when the Fluid moves, the Velocity of the End D will be to that of the End C reciprocally as the Cavity of the Tube at D, to that at C (by the *Lemma*, Chap. 1. §. 9,) that is, reciprocally as the Square of the Diameter at D, to the Square of the Diameter at C †; but the attracting Periphery at D is to that at C in the ſimple Ratio of the Diameter at D, to the Diameter at C. Now, ſince the Effect of the Attraction depends, as much upon the Velocity of that Part of the Fluid where it acts, as upon its natural Force, its Effect at D will be greater than at C; for though the Attraction at D be really leſs in itſelf than at C, yet its Loſs of Force upon that Account is more than

† 12 *El.* 2.

compenſated by the *mechanical* Advantage it has ariſing from hence, *viz.* that the Velocity of the Fluid in that Part is greater than it is at C, in a greater Proportion than the Force itſelf is leſs: The Fluid will therefore move towards B.

VII. From hence likewiſe it follows, that if a Veſſel, as ABC (repreſented *Fig.* 21.) of any Form whatever, have its upper Part drawn out into a Capillary Tube, as B; and if this Veſſel is filled with Water, and have its lower Orifice placed on FD, the Surface of the ſame Fluid; then the Water will remain ſuſpended in the Veſſel, provided the Capillary at the Top be ſmall enough (was it continued down to the Bottom) to raiſe the Fluid to the Height B. Becauſe, by Propoſition the fifth, the lower Part of the Tube makes no Alteration in the Height, at which the Capillary B is able to ſuſtain the Fluid.

VIII. And if the ſame Veſſel be filled only to the Height DE (*Fig.* 22.) and a Drop of Water be put into the Capillary at B (the intermediate Part BDE being full of Air) the Water will continue ſuſpended at the Height DE.

For, although the Fluid ACDE is not in Contact with the Drop of Water in the Capillary Tube, and therefore not immediately ſupported by it; yet the Preſſure of the Atmoſphere upon the Surface FG, and againſt the upper Part of the Drop in the Capillary B, keeps

keeps the Fluid ADEC, and the Drop, and the intermediate Air from ſeparating, juſt as in the former Caſe the Attraction of Coheſion in the Particles of the Water prevented a Separation between that in the Veſſel and that in the Capillary. Conſequently, as the Water in the Capillary was able in the former Caſe to ſuſtain as much Fluid as the Veſſel could contain, it is now ſufficient to ſuſtain the Fluid A D E C †.

IX. Let there be a Capillary Siphon, as that repreſented *Fig.* 23, 24, or 25, and let EF be the Height to which Water might be raiſed by a Periphery equal to that at A. Now, ſince (as was obſerved, §. 2.) the lower End of a Tube when it is not immerged, cauſes a longer Column to be ſuſpended than otherwiſe would be; that is, it ſupports a ſhort Column beſides that which is ſuſtained by the attracting Periphery; let H I be the Height of ſuch a Column as might be ſuſpended by the End C: Then, if any of thoſe Tubes are filled with Water, and held as in the *Figure* (neither End being immerged) the Fluid will run out of the Tube at C, if C D, the Difference of the

† This happens quite otherwiſe in Vacuo, becauſe the Preſſure of the Air, which, as it were, connects the Drop with the Water A D E C, being wanting, it immediately falls for Want of a Support. Whereas the former Phænomenon equally ſucceeds in Vacuo; which ſhews that the Parts of the Fluid in the Veſſel are connected with each other, and with that in the Capillary, by their own mutual Attraction of Coheſion, there being nothing elſe whereby they can be ſupported.

Legs, exceeds E F and H I added together, otherwiſe not.

For the Column A B is a Counterpoiſe to B D, being of the ſame perpendicular Height; and therefore it is only the Weight of the Column C D which determines the Fluid to move; unleſs that Weight therefore exceeds the Force of the attracting Periphery at A, (which the Fluid A B muſt leave in riſing up the Tube) and alſo what may be ſupported by the End C, that is (*ex Hypoth.*) two Columns whoſe Heights are F E and H I, it cannot run out at C; otherwiſe it will, as being deſtitute of a ſufficient Support.

X. If the End A is immerged in Water, and the other not, (ſuppoſing the Tube full as before) it will run out at C, though C D, the Difference of the Legs, only exceeds H I. For then the Attraction at A ceaſes, and there is nothing to ſupport the Column C D, but the Power which the End C has to prevent Drops from falling off from it.

XI. Again, if the End C is immerged in Water (and the other not) it will run out at C, if C D exceeds E F, otherwiſe not.

For in this Caſe, there is nothing to ſupport the Column C D but the attracting Periphery at A, whoſe Power is ſuppoſed able to raiſe a Column as E F, and no more.

XII. And if both Ends are immerſed (the Tube being ſuppoſed full as before) the Water will run out at the lower, which ever it is.

For

For then the Attraction of both Ends ceaſes; and the longer Column over-balancing the ſhorter, the Fluid is determined thereby to run out at the lower End.

XIII. If either of the Tubes (*Fig.* 23 or 24.) are ſmall enough to raiſe the Water from A to B, and if the Orifice A is immerged, the Fluid will riſe to B, paſſing on to C, where it will run out or be ſuſpended, according to the foregoing Caſes: But if the Periphery at G (*Fig.* 25.) is ſuch as would ſupport a Column of the Length of AM only; and if the Tube be filled from A to G (the Extremity A being ſtill immerged as before) the Liquor will proceed no farther, but remain in the Poſition in which it is put.

For that Fluid which has paſſed B, aſſiſts by its Weight the attracting Periphery, in raiſing the Column AB, and therefore it runs down to C. But if, when the Tube is filled to G, the Periphery there is not able to ſupport more than AM, the Difference of the Legs AB and BG, the Fluid muſt neceſſarily ſtand there; ſince BG is no more than a Counterpoiſe to MB, and AM is ſuppoſed to be as much as the Periphery at G can ſuſtain.

XIV. Tho' a Capillary Tube be ſhorter than the Height to which its attracting Periphery is able to raiſe a Fluid, *v. g.* tho' the Tube AB (*Fig.* 26.) be ſmall enough (was it of ſufficient Length) to raiſe Water as high as C; yet when the End A is immerged, the Fluid will

not run out at B, but only be ſuſpended at that Height.

For when the Fluid is riſen as high as B, it has then no more Periphery above it, to attract it any farther; and if it was forced up a little above the End, it would be attracted back again by it †.

XV. The Aſcent of different Fluids in the ſame Tube is various. *Muſſchenbroek* has found, that in a Tube in which Water will riſe to the Height of twenty-ſix Lines, Oil of Wormwood will riſe but eighteen or nineteen, whereas Urine will riſe thirty-three or thirty-four.

† Hence we ſee the Abſurdity of ſuppoſing that a Fluid may be made continually to flow from a lower Place to an higher by a Capillary Tube, as ſuch; for whether the Tube be of ſuch Form, as is repreſented, *Fig.* 23, 24, 25, or 26, the Fluid will always ſtop when it comes at the higher End; becauſe the Attraction is then in a Direction contrary to its Motion; and the Weight of the whole Fluid, if the Tube is ſtrait, or of ſo much as is contained in D C, the Difference of the Legs, if it be crooked, is likewiſe an Impediment in it.

Hence alſo the Abſurdity of ſuppoſing a Fluid to *circulate* by Virtue of a Capillary Tube, as ſuch. For ſuppoſe the Ends of the Capillary Tube joined together, ſo that the Tube ſhall return into itſelf; if the Tube be not full of Liquor, the Liquor will place itſelf as near as poſſible to the ſmalleſt Part of the Tube, as may eaſily be inferred from what was obſerved in §. 6. and will reſt there: If it be full, in which Caſe the Extremities of the Fluid join, then there is no Periphery or Zone of Particles lying Part out of the Fluid, and Part in it, and conſequently all Ground or Cauſe of Motion, ſo far as it ariſes from the Tube, being *capillary*, is entirely taken away. And if Part of the Tube be capillary, and Part not, the Caſe is ſtill the ſame. How ridiculous then is it to affirm, as ſome have done, that the Circulation of the Animal Fluids is promoted by the Paſſages through which they flow, being of that Kind.

The

The Reaſon of which is, becauſe ſome Fluids are attracted more ſtrongly by Glaſs than others are. Mercury exhibits Phænomena juſt the Reverſe of the former; for if the End of a Capillary Tube be immerſed below the Surface of that Fluid, it will not riſe in the Tube to a Level with that on the Outſide. This is becauſe the Particles of Mercury attract each other more forcibly than they are attracted by thoſe of Glaſs †.

† See *Jurin*'s Diſſert. Philoſoph. Tranſ. No 363.

According to *Muſſchenbroek*, the Length of the upper Part of a Tube, which is above the Height to which it is able to raiſe a Fluid, conduces ſomething towards the raiſing it; and therefore in a longer Tube a Fluid riſes higher than in one of the ſame Dimenſions that is ſhorter; and if a Tube, with ſo much Fluid contained in it as it is able to raiſe, be laid in an Horizontal Situation, the Fluid will run to the Middle of it. But of this I have had no Experience: 'Tis poſſible that ingenious Profeſſor, tho' very accurate in making Experiments, might herein be deceived. He acknowledges (Experiment the Fifteenth) that it ſometimes happens otherwiſe.

Other Authors, beſides thoſe already referred to, that have treated on this Subject, are, *Boyle* Exper. Phyſ. Mech. Exp. 9. *Sturmius* Colleg. Cur. Tentam. 8. *Bernoulli* Gravit. Æth. *Hooke* Microg. Obſer. 6. *Leeuwenhoek* Continuat. Arcan. Nat. Epiſt. 131. *Sinclaire* Art. Gravit.

DISSERTATION III.

Of the Origin of Fountains.

MANY have been the Conjectures of Philoſophers concerning the Origin of Fountains; and great Pains have been taken both by the Members of the Royal Society, and thoſe of the Academy of Sciences at *Paris*, in order to aſcertain the true Cauſe of it. It was ARISTOTLE's Opinion, and held by moſt of the antient Philoſophers after him, that the Air contained in the Caverns of the Earth, being condenſed by Cold near its Surface, was thereby changed into Water; and that it made its Way through, where it could find a Paſſage, But we have no Experience of any ſuch Tranſmutation of Air into Water.

Thoſe who imagine that Fountains owe their Origin to Waters brought from the Sea by ſubterraneous Ducts, give a tolerable Account how they loſe their Saltneſs by Percolation, as they paſs through the Earth; but find great Difficulty in explaining by what Power the Water riſes above the Level of the Sea, to near the Tops of Mountains, where Springs generally abound; it being contrary to the Laws of *Hydroſtatics*, that a Fluid ſhould

riſe

riſe in a Tube above the Level of its Source. However, they have found two Ways, whereby they endeavour to extricate themſelves from this Difficulty. The one is that of DES CARTES, who imagines, that after the Water is become freſh by Percolation, it is raiſed out of the Caverns of the Earth in Vapour towards its ſurface; where meeting with Rocks near the Tops of Mountains, in the Form of Arches or Vaults, it ſticks to them, and runs down their Sides (like Water in an Alembic) till it meets with proper Receptacles, from which it ſupplies the Fountains. Now this is a mere *Hypotheſis*, without Foundation or Probability; for, in the firſt Place, we know of no internal Heat of the Earth to cauſe ſuch an Evaporation; or if that were allowed, yet 'tis quite incredible that there ſhould be any Caverns ſo ſmooth, and void of Protuberances, as to anſwer the Ends of an Alembic, in collecting and condenſing the Vapours together, in every Place where Fountains ariſe. There are others (as VARENIUS, *&c.*) who ſuppoſe that the Water may riſe through the Pores of the Earth, as through Capillary Tubes, by Attraction; but hereby they ſhew, that they are quite unacquainted with what relates to the Motion of a Fluid through ſuch Tubes. For when a Capillary Tube opens into a Cavity at its upper End, or grows larger

and

and larger, ſo as to ceaſe to be Capillary at that End; the Water will not aſcend through that Tube into the Cavity, or beyond where the Tube is Capillary; becauſe that Part of the Periphery of the Cavity, which is partly above the Surface of the Water, and partly below it, is not of the Capillary Kind. Nay, if the Cavity is continually ſupplied with Water, it will be attracted into the Capillary Tube, and run down it, as through a Funnel, if the lower End is immerged in the ſame Fluid, as in this Caſe it is ſuppoſed to be *.

It has been a generally received Opinion, and much eſpouſed by MARRIOTTE (a diligent Obſerver of Nature) that the Riſe of Springs is owing to the Rains and melted Snow. According to him, the Rain Water, which falls upon the Hills and Mountains, penetrating the Surface, meets with Clay or Rocks contiguous to each other, along which it runs, without being able to penetrate them, till, being got to the Bottom of the Mountain, or to a conſiderable Diſtance from the Top, it breaks out of the Ground, and forms Springs.

In order to examine this Opinion, Mr. PERRAULT, DE LA HIRE, and D. SIDELEAU, endeavoured to make an Eſtimate of the Quantity of Rain and Snow that falls in

* See the foregoing Diſſertation, Sect. 14.

the

the Space of a Year, to ſee whether it would be ſufficient to afford a Quantity of Water equal to that which is annually diſcharged into the Sea by the Rivers. The Reſult of whoſe Inquiries was, that the Quantity of Rain and Snow which fell in a Year into a Cylindrical Veſſel, would fill it (if ſecured from evaporating) to the Height of about nineteen Inches. Which Quantity D. SIDELEAU † ſhewed, was not ſufficient to ſupply the Rivers; for that thoſe of *England*, *Ireland*, and *Spain*, diſcharge a greater Quantity of Water annually, than the Rain, according to that Experiment, is able to ſupply. Beſides which, another Obſervation was made by them at the ſame Time, *viz.* that the Quantity of Water raiſed in Vapour, one Year with another, amounted to about thirty-two Inches, which is thirteen more than falls in Rain: A plain Indication, that the Water of Fountains is not ſupplied by Rains and melted Snow.

Thus the true Cauſe of the Origin of Fountains remained undiſcovered, till Dr. HALLEY, in making his Celeſtial Oſervations upon the Tops of the Mountains at St. *Helena*, about eight hundred Yards above the Level of the Sea, found that the Quantity of Vapour which fell there (even when the Sky was clear) was

† Memoirs of the Royal Academy of Sciences for the Year 1693.

ſo

ſo great that it very much impeded his Obſervations, by covering his Glaſſes with Water every Half-quarter of an Hour; and upon that attempted to determine, by Experiment, the Quantity of Vapour exhaled from the Surface of the Sea, as far as it riſes from Heat; in order to try, whether that might be a ſufficient Supply for the Water continually diſcharged by Fountains. The Proceſs of his Experiment was as follows: he took a Veſſel of Water ſalted to the ſame Degree with that of Sea-Water, in which he placed a Thermometer; and by Means of a Pan of Coals, brought the Water to the ſame Degree of Heat, which is obſerved to be that of the Air in our hotteſt Summer; this done, he affixed the Veſſel of Water, with the Thermometer in it, to one End of a Pair of Scales, and exactly counterpoiſed it with Weights on the other: Then, at the End of two Hours, he found by the Alteration made in the Weight of the Veſſel, that about a ſixtieth Part of an Inch of the Depth of the Water was gone off in Vapour; and therefore, in twelve Hours, one tenth of an Inch would have gone off. Now this accurate Obſerver allows the *Mediterranean Sea* to be forty Degrees long, and four broad; (the broader Parts compenſating for the narrower; ſo that its whole Surface is one hundred and ſixty ſquare Degrees;) which, according to the Experiment, muſt yield at leaſt five thouſand two hundred and eighty Millions

of

of Tuns of Water. In which Account, no Regard is had to the Wind, and the Agitation of the Surface of the Sea; both which undoubtedly promote the Evaporation.

It remained now, to compare this Quantity of Water with that which is daily conveyed into the ſame Sea by the Rivers. The only Way to do which was, to compare them with ſome known River; and accordingly he takes his Computation from the River *Thames*, and, to avoid all Objection, makes Allowances, probably greater than what were abſolutely neceſſary.

The *Mediterranean* receives the following conſiderable Rivers, *viz.* the *Iberus*, the *Rhone*, the *Tyber*, the *Po*, the *Danube*, the *Neiſter*, the *Boryſthenes*, the *Tanais*, and the *Nile*. Each of theſe he ſuppoſes to bring down ten Times as much Water as the *Thames*; whereby he allows for ſmaller Rivers, which fall into the ſame Sea. The *Thames* then he finds by Menſuration to diſcharge about 20,300,000 Tuns of Water a Day. If therefore the above-ſaid nine Rivers yield ten Times as much Water as the *Thames* doth, it will follow, that all of them together yield but 1827 Millions of Tuns in a Day; which is but little more than One-third of what is proved to be raiſed in Vapour out of the *Mediterranean* in the ſame Time. We have therefore from hence a Source abundantly ſufficient for the Supply of Fountains.

Now,

Now, having found that the Vapour exhaled from the Sea, is a ſufficient Supply for the Fountains; he proceeds, in the next Place, to conſider the Manner in which they are raiſed, and how they are condenſed into Water again, and conveyed to the Sources of Springs.

In order to this he conſiders, that, if an Atom of Water was expanded into a Shell, or Bubble, ſo as to be ten Times as big in Diameter as when it was Water, that Atom would become ſpecifically lighter than Air; and therefore would riſe ſo long as the Warmth which firſt ſeparated it from the Surface of the Water, ſhould continue to diſtend it to the ſame Degree; and conſequently, that Vapours may be raiſed from the Surface of the Sea in that Manner †, till they arrive at a certain Height in the Atmoſphere, at which they find the Air of equal ſpecific Gravity with themſelves. Here they will float, till being condenſed by Cold, they become ſpecifically heavier than the Air, and fall down in Dew, or being driven by the Winds againſt the Sides of Mountains (many of which far ſurpaſs the uſual Height to which the Vapours would of themſelves aſcend) are compelled by the Stream of the Air to mount up with it to the Tops of them; where, being condenſed into Water, they preſently precipitate, and

† See this confuted in Diſſertation 6.

gleeting

gleeting down by the Crannies of the Stone, Part of them enters into the Caverns of the Hills; which being once filled, all the Overplus of Water that comes thither, runs over by the loweſt Place, and breaking out by the Sides of the Hills, forms ſingle Springs. Many of theſe running down by the Vallies between the Ridges of the Hills, and coming to unite, form little Rivulets or Brooks: Many of theſe again meeting in one common Valley, and gaining the plain Ground, being grown leſs rapid, become a River; and many of theſe being united in one common Channel, make ſuch Streams as the *Rhine* and the *Danube*; which latter, he obſerves, one would hardly think to be a Collection of Water condenſed out of Vapour, unleſs we conſider how vaſt a Tract of Ground that River drains, and that it is the Sum of all thoſe Springs which break out on the South Side of the *Carpathian* Mountains, and on the North Side of the immenſe Ridge of the *Alps*; which is one continued Chain of Mountains from *Switzerland* to the *Black Sea*.

Thus one Part of the Vapours, which are blown on to the Land, is returned by the Rivers into the Sea, from whence it came; another Part falls into the Sea before it reaches the Land; and this is the Reaſon why the Rivers do not return ſo much Water into the *Mediterranean* as is raiſed in Vapour. A third Part falls on the Low-Lands, and is the

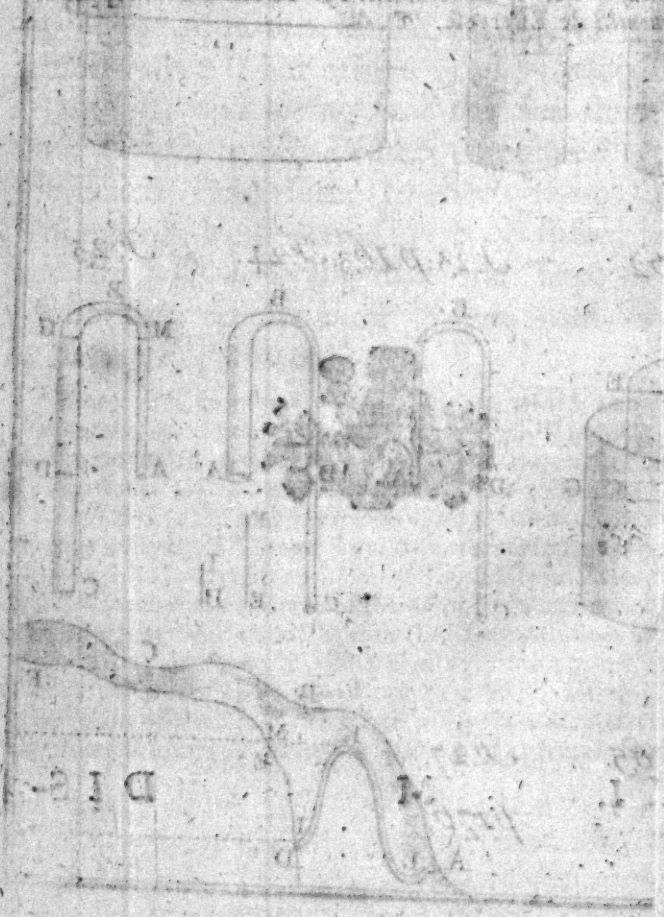
the *Pabulum* of Plants, where it yet does not reſt; but is again exhaled in Vapour, by the Action of the Sun, and is either carried by the Winds to the Sea, to fall in Rain or Dew there, or elſe to the Mountains, to become the Sources of Springs.

However, it is not to be ſuppoſed that all Fountains are owing to one and the ſame Cauſe; but that ſome proceed from Rain and melted Snow, which ſubſiding through the Surface of the Earth, makes its Way into certain Cavities, and thence iſſues out in the Form of Springs; becauſe the Waters of ſeveral are found to increaſe and diminiſh in Proportion to the Rain which falls: That others again, eſpecially ſuch as are ſalt, and ſpring near the Sea-ſhore, owe their Origin to Sea-Water percolated through the Earth; and ſome to both theſe Cauſes: Though without doubt moſt of them, and eſpecially ſuch as ſpring near the Tops of high Mountains, receive their Waters from Vapours, as before explained *.

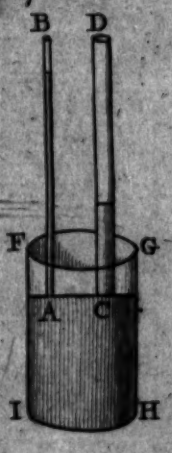
* There is a certain Species of Springs which ebb and flow alternately, and ſome that ceaſe to flow for a Time, which from thence are called reciprocating or intermitting ones. Their Reciprocations may be accounted for in the following Manner. Let A B C (*Fig.* 27.) repreſent one Side of an Hill, in which there is a Cavity D E F, and from this a ſubterraneous Duct I K L. Now as this Cavity fills with Water (ſuppoſe from Vapours percolating through the Surface of the Hill, or in any other Manner whatever) its Surface will riſe in the Duct as it does in the Cavity, till it arrives at M, the Level with the upper Part of the Duct; at which Time it will run over at K, filling K L A the other Part of the Duct. Now if the perpendicular Height of the Point K above L (or rather above A) be greater than its perpendicular

perpendicular Height above I, the Column KLA will overpoiſe the Column KI, and ſo cauſe the Water to run out at A, till its Surface in the Cavity ſinks as far as I (provided the Duct is large enough to convey the Water away faſter than it enters the Cavity) at which Time the Fountain at A will ceaſe to play, till the Surface of the Water in the Cavity riſes again to M, and runs over at K as before. The Reaſon why the Water continues running (when the Duct is once full) till its Surface ſinks to I, is becauſe the Air preſſing againſt it as it runs out at A, and alſo upon its Surface in the Cavity, keeps the Duct full, as long as the Water in the Cavity is high enough to feed its Orifice at I.

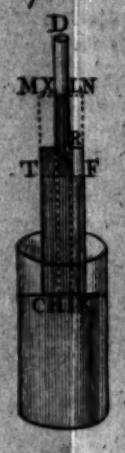
See more on this Subject in Philoſoph. Tranſact. No. 119, 189, 192, 384, 424. Hiſtoire de l'Acad. 1693, 1703, 1713. *Guilielmini* della Natura de Fiumi. *Dale*'s Hiſtory of *Harwich*. *Mariotte*'s Hydroſtatics. *Nieuwentyt* Contempt. 19. *Varenius* Geograph. Cap. 16. *Regnault*, Vol. 2. Converſat. 6. *Hales*'s Statical Eſſays, Vol. 1. Experiment 19. *Micheliettus* in Append. ad *J. Bernouilli* de Effervesc.



F.17. p.155.



F.18. p.158.



F.19. p.158.



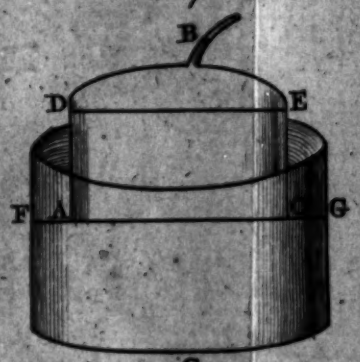
F.20. p.161.



F.21. p.162.



F.22. p.162.

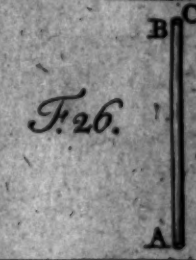


F.23. p.163. F.24.



F.25.

F.26. p.165



F.27. p.176



DISSERTATION IV.

Of the Barometer.

IN treating of the Properties of the Air (Chap. III.) I have already taken Notice of the Conſtruction of the common Barometer; and proved, that the Aſcent and Suſpenſion of the Mercury therein, is owing to the Preſſure * of the Air. I proceed now to a more particular Inquiry into the Original, and Uſe of this Inſtrument; and the different Forms under which it has appeared, ſince the Time of its Inventor *Torricelli*.

In

* To ſay the Aſcent and Suſpenſion of the Mercury is owing to the Preſſure and Elaſticity of the Air, as is commonly done, is inaccurate. The Variation, indeed, in the Height of the Mercury, may be aſcribed to the Elaſticity of the Air, but no otherwiſe, than as to its *remote* Cauſe; *viz.* as it occaſions an Alteration in the Quantity of Air, impending over the Place where the Variation happens; which alters its Weight, and ſo the Mercury is proportionally raiſed or depreſſed. To illuſtrate this, let it be ſuppoſed, that the Air is every where in *Æquilibrio* quite round the Globe, and at perfect Reſt; and then, that its Elaſticity, in ſome one Place near the Surface of the Earth, is augmented by the Heat of the Sun, all the reſt of it remaining as before. The Conſequence of this will be, that the ſuperior Part of the Atmoſphere, over this Place, will be raiſed higher by the Expanſion of the inferior Air; and therefore, being unconfined, will ſpread itſelf every Way over the neighbouring Columns, which we ſuppoſe to retain their former State. The Quantity of Matter therefore in thoſe Columns of Air, in whoſe lower Parts its Elaſticity was increaſed, will be diminiſhed, and

In the Beginning of the laſt Century it was a prevailing Opinion among Philoſophers, that the Univerſe was full of Matter; and that Nature (as they expreſſed it) abhorred a *Vacuum:* Accordingly they imagined, that if a Fluid was ſucked up a Pipe with a ſufficient Force, it would riſe to any Height whatever; ſince Nature would not ſuffer any Part of the Pipe to be empty. *Galilæo*, who flouriſhed about that Time, found, upon Trial, that the common Pump would not raiſe Water, unleſs the Sucker reached within three and thirty Feet of its Surface in the Well †: From hence

that of the neighbouring ones augmented. A Barometer therefore placed in thoſe Regions, where the Air was rarefied, will ſubſide; while one in the neighbouring Countries will aſcend; and they will continue at different Heights, till the denſer Air, ruſhing in upon the rarefied, reſtores the *Æquilibrium*. Thus we ſee the Variation of the Air's Elaſticity is not the *immediate* Cauſe of the Variation in the Barometer; it firſt affects the Weight of the Air, by altering the Quantity incumbent over any Place, and that affects the Barometer. But, if we may have Recourſe to *remote* Cauſes, we may, if we pleaſe, go one Step farther; and ſay, the Aſcent and Suſpenſion of the Mercury is owing to the Heat of the Sun; for, by the foregoing Inſtance, a Variation in the Heat of the Sun may ſometimes be the Occaſion of a Variation in the Height of the Mercury.

Neither is the Suſpenſion of the Mercury, in a Tube, that is kept within Doors, to be aſcribed to the Elaſticity of the Air; for that exerts no Force, but as the internal Air is preſſed by the external, which endeavours to get in, where-ever it can find a Way.

† It is a common Notion, that a ſucking Pump will not raiſe Water above thirty-three Feet, whereas it will raiſe it to any Height whatever, if the Sucker reaches within thirty-three Feet of the Surface of the Water; as will be evident to any one that conſiders the Structure of the Pump: For all the Water, which has

hence he judiciouſly inferred, that a Column of Water thirty-three Feet high was a Counterpoiſe to a Column of Air of an equal Baſe, whoſe Height extended to the Top of the Atmoſphere; and that, for this Reaſon, the Water would not follow the Sucker any farther. *Torricelli*, obſerving this, took the Hint; and conſidered, that, if a Column of Water of about thirty-three Feet was equal in Weight to a Column of Air of the ſame Baſe §; a Co-

has once paſſed through the Valve in the Sucker, is ſupported by that, as the Sucker is drawn up, and as the Sucker is let down it, together with the Water between that and another placed in the Pump below, is ſupported by that other; ſo that the Height of the Water above the Sucker can be no Impediment to the riſing of the Water below the Sucker, whatever the Length of the Column, which it forms, may be. The placing one Pump above another, where Water is to be raiſed from great Depths, is rather for Strength and Conveniency, than out of Neceſſity.

§ Perhaps it may be enquired here, how it comes to paſs, that the Column of Air, which preſſes upon the ſtagnant Mercury in the Baſon, is always ſuppoſed to have an equal Baſe with the ſuſpended Column in the Tube; whereas, in Reality, its Baſe is equal to the Surface of the ſtagnant Mercury? The Reaſon is, that, as the Baſe of the Column of Air increaſes, in the ſame Proportion the Velocity wherewith it deſcends, decreaſes, when it forces down the Surface of the Mercury in the Baſon; conſequently its Moment, or Preſſure upon the Surface of the ſtagnant Mercury (ſo far as it relates to the ſuſpending of it in the Tube) is no greater than it would have been, had its Baſe been equal to that of the ſuſpended Column; and therefore, in conſidering it as ſuſpending a Fluid in a Tube, it is properly enough ſaid to be a Column of ſuch a Baſe.

Neither is this Suppoſition inconſiſtent with the ninth Propoſition of the firſt Chapter, where it is demonſtrated, that the Preſſure of a Fluid is in Proportion to its perpendicular Height, and the Quantity of Surface againſt which it preſſes. For, as the Surface of the Mercury may be conſidered as a Baſe on which

Column of Mercury, no longer than about twenty-nine Inches and a half, would be ſo too; ſuch a Column of Mercury being as heavy, as thirty-three Feet of Water. Accordingly he tried the Experiment in a Glaſs Tube (in the Manner laid down, Chap. III. §. 4.) and found it to ſucceed ‡. The Apparatus he made Uſe

the Column of Air reſts, ſo the Baſe of the Column of Air may be conſider'd as a Surface againſt which the Mercury preſſes. Theſe two being equal, 'tis clear, that only the Relation of the Heights of the Columns are to be conſidered, and not that of their Baſes.

‡ Notwithſtanding this clear Proof of the Preſſure of the Atmoſphere, the Aſſertors of a *Plenum* would by no Means be prevailed upon to allow it to be ſuch; but tried all Ways to account for this *Phænomenon* from ſome other Cauſe. The moſt chimerical Solution, and which at the ſame Time gave the adverſe Party the greateſt Difficulty to overthrow it, was that of *Linus*. He contended, that in the upper Part of the Tube there is a Film, or Rope of Mercury, extended thro' the ſeeming Vacuity, and that the reſt was ſuſpended by it, and kept from falling into the Baſon; and that this Film is able to ſupport about twenty-nine Inches of Mercury. He confirms his *Hypotheſis* by the following Experiment: Take, ſays he, a ſmall Tube, open at both Ends, ſuppoſe about twenty Inches long; fill this Tube with Mercury, ſtopping the lower Orifice with your Thumb: Then cloſing the upper with your Finger, and immerging the lower in ſtagnant Mercury, you ſhall perceive, upon the Removal of your Thumb, a manifeſt Suction of your Finger into the Tube; and the Tube and Mercury will both ſtick ſo cloſe to it, that you may carry them about the Room. Therefore, ſays he, the internal Cylinder of Mercury in the Tube is not held up by the preponderant Air without; for if ſo, whence comes ſo ſtrong a Suction, and ſo firm an Adheſion of the Tube to your Finger?

Or if you fill the ſame Tube almoſt full of Mercury, leaving a little Space of Air within, and then immerge it in the ſtagnant Mercury, you will find, that, notwithſtanding its Surface is at ſome Diſtance from your Finger, there will be a conſiderable

Uſe of, is now the common Barometer, or Weather Glaſs *.

The Mercury ſtanding at a leſs Height, the nearer it is carried to the Top of the Atmoſphere,

able Suction of it, as before. From hence he infers, that the Finger ſupports the Mercury, by Means of the above-mentioned Film, and that the Preſſure of the Atmoſphere is not concerned.

But when it was found that the Mercury would not ſtand ſo high in the Tube, on the Top of a Mountain, as below, and would quite fall, when the circumambient Air was extracted from it by the Pump, all Objections vaniſhed; and *Linus*'s funicular *Hypotheſis* (as it was called) though it ſeemed to ſolve all other *Phænomena* relating to the Suſpenſion of the Mercury, was with Juſtice rejected.

Kircher, when this new Doctrine of a *Vacuum* was firſt advanced at *Rome*, contended, that the Authors of it were eſtabliſhing Principles not only repugnant to thoſe of Nature, but ſuch as would be prejudicial to the Orthodox Faith; as endeavouring to evince by this ſubtle Experiment, that there might be in Nature *locatum ſine loco*, *accidentia ſine ſubjecto*, and therefore made the Experiment with Water in the following Manner. He cauſed a ſmall Bell to be fixed in the upper Part of the Tube, imagining, that, if there ſhould be a *Vacuum*, the Bell would not be made to ſound: But in making the Experiment, ſome Air got into the Tube, (for he tells us, that but ten Feet of Water remained in the Tube, after it was inverted,) the Bell therefore was heard to ſound; and ſo the Notion of a *Vacuum*, till more accurate Experiments evinced the contrary, was exploded with Contempt.

* *Huygens* obſerved, that, if a Tube, ſeventy-five Inches long, was filled with Mercury well purged of its Air, the whole Quantity of Mercury would remain ſuſpended; whereas, according to the *Torricellian* Experiment, the Mercury ought to have ſubſided to the Height of about twenty-nine Inches.

The Cauſe of this *Phænomena* ſeems to be, that, by the great Weight of ſo long a Column of Mercury, it was preſſed into ſo cloſe Contact with the Glaſs in pouring in, that by the mutual Attraction of Coheſion between the Mercury and the Glaſs, the whole Column was ſuſtained, after the Tube was inverted.

(Chap. III. §. 7.) renders it uſeful in determining the Height of Mountains; and finding out the different Elevation of one Place above another. Accordingly Dr. *Halley* has given us a Table for that Purpoſe, in the Philoſophical Tranſactions, N° 181, ſhewing how many Feet each Inch in the Deſcent of the Mercury anſwers to, as it is conveyed to the Top of a Mountain, or other elevated Place. And Dr. *Nettleton* has done the like in the Philoſophical Tranſactions, N° 388, ſhewing what Number of Feet anſwers to each tenth Part of an Inch, from twenty-ſix to thirty-one Inches of Mercury.

But the principle Uſe of it is, to eſtimate the Gravity of the Air at different Times, in Order to foreſee the Alterations of the Weather, which are conſequent thereon. To this End, Dr. *Halley*, in the ſame Tranſaction, has alſo laid down the more remarkable *Phænomena*, relating to the different Heights of the Mercury at different Times, together with the Solution of each; which are ſo juſt, and ſo agreeable to true Philoſophy, that I doubt not but the Reader will excuſe me for giving his Account in his own Words, rather than to render it imperfect, by endeavouring to vary from it, or abridge it.

" 1. In calm Weather, when the Air is in-
" clined to Rain, the Mercury is commonly
" low.

" 2. In

" 2. In ſerene, good, and ſettled Weather,
" the Mercury is generally high.

" 3. Upon very great Winds, though they
" be not accompanied with Rain, the Mercury
" ſinks loweſt of all, with Relation to the
" Point of the Compaſs the Wind blows upon.

" 4. *Cæteris paribus*, the greateſt Heights
" of the Mercury are found upon eaſterly and
" north-eaſterly Winds.

" 5. In calm froſty Weather, the Mercury
" generally ſtands high.

" 6. After very great Storms of Wind,
" when the Mercury has been low, it gene-
" rally riſes again very faſt.

" 7. The more northerly Places have greater
" Alterations of the Barometer, than the more
" ſoutherly.

" 8. Within the Tropics, and near them,
" thoſe Accounts we have had from others,
" and my own Obſervations at St. *Helena*,
" make very little or no Variation of the
" Height of the Mercury in all Weathers.

" Hence I conceive that the principal Cauſe
" of the Riſe and Fall of the Mercury, is from
" the variable Winds, which are found in the
" temperate Zone, and whoſe great Incon-
" ſtancy, here in *England*, is moſt notorious.

" A ſecond Cauſe is the uncertain Exhalation
" and Precipitation of the Vapours lodging in
" the Air, whereby it comes to be at one Time
" much more crouded than at another, and

" con-

" conſequently heavier, but this latter in a great " Meaſure depends upon the former. Now, " from theſe Principles, I ſhall endeavour to " explicate the ſeveral *Phænomena* of the Ba- " rometer, taking them in the ſame Order I " laid them down. Thus,

" 1. The Mercury's being low inclines it " to rain, becauſe, the Air being light, the " Vapours are no longer ſupported thereby, " being become ſpecifically heavier than the " Medium wherein they floated, ſo that they " deſcend towards the Earth, and in their Fall, " meeting with other aqueous Particles, they " incorporate together, and form little Drops " of Rain; but the Mercury's being at one " Time lower than at another, is the Effect of " two contrary Winds blowing from the Place " where the Barometer ſtands; whereby the " Air of that Place is carried both Ways from " it, and conſequently, the incumbent Cylin- " der of Air is diminiſhed, and accordingly the " Mercury ſinks: As for Inſtance, if in the " *German* Ocean it ſhould blow a Gale of " eaſterly Wind, and at the ſame Time a " weſterly Wind in the *Iriſh* Sea; or if in " *France* it ſhould blow a ſoutherly Wind, " and in *Scotland* a northerly; it muſt be " granted, that that Part of the Atmoſphere, " impendant over *England*, would thereby be " exhauſted and attenuated, and the Mercury " would ſubſide, and the Vapours, which be-

" fore

" fore floated in thoſe Parts of the Air of
" equal Gravity with themſelves, would ſink
" to the Earth.

" 2. The greater Height of the Barometer
" is occaſioned by two contrary Winds blow-
" ing towards the Place of Obſervation, where-
" by the Air of other Places is brought thither
" and accumulated; ſo that the incumbent
" Cylinder of Air, being increaſed both in
" Height and Weight, the Mercury preſſed
" thereby muſt needs ſtand high, as long as
" the Winds continue ſo to blow; and then
" the Air being ſpecifically heavier, the Va-
" pours are better kept ſuſpended, ſo that they
" have no Inclination to precipitate and fall
" down in Drops, which is the Reaſon of the
" ſerene good Weather which attends the
" greater Heights of the Mercury.

" 3. The Mercury ſinks the loweſt of all by
" the very rapid Motion of the Air in Storms
" of Wind. For the Tract or Region of the
" Earth's Surface, wherein the Winds rage,
" not extending all round the Globe, that
" ſtagnant Air which is left behind, as like-
" wiſe that on the Sides, cannot come in ſo
" faſt as to ſupply the Evacuation made by ſo
" ſwift a Current, ſo that the Air muſt neceſ-
" ſarily be attenuated, when and where the
" ſaid Winds continue to blow, and that more
" or leſs, according to their Violence: Add
" to which, that the horizontal Motion of the

" Air

" Air being ſo quick as it is, may, in all Pro-
" bability, take off ſome Part of the perpendi-
" cular Preſſure thereof; and the great Agi-
" tation of its Particles is the Reaſon why the
" Vapours are diſſipated, and do not condenſe
" into Drops, ſo as to form Rain, otherwiſe
" the natural Conſequence of the Air's Rare-
" faction †.

" 4. The Mercury ſtands the higheſt upon
" the eaſterly and north-eaſterly Wind, be-
" cauſe in the great *Atlantic* Ocean, on this
" Side the thirty-fifth Degree of North Lati-
" tude, the Winds are almoſt always weſter-
" ly or ſouth-weſterly; ſo that, whenever
" here the Wind comes up at eaſt and north-
" eaſt, 'tis ſure to be checked by a contrary
" Gale as ſoon as it reaches the Ocean, where-
" fore, according to what is made out in our
" ſecond Remark, the Air muſt needs be heap-

† The Reaſon the Doctor aſſigns for the ſinking of the Mercury the loweſt of all in violent Storms of Wind, ſeems not ſufficient. Perhaps it may be better accounted for thus; the Cauſe why the Winds blow at all, is in order to reſtore the *Æquilibrium* of the Atmoſphere, when loſt (as may be inferred from what was ſaid in the firſt Note of this, and will be more largely explained in the following Diſſertation;) it therefore always blows towards that Point, where the Air is moſt rarefied and lighteſt. Now the Air, in its Progreſs to that Point, muſt certainly move faſter and faſter; for the Cauſe, which gave it Motion at firſt, continues to act upon it all the Way. Conſequently, in whatever Place the Wind blows with great Rapidity, that Place is at, or near the Point, where the Air is moſt rarefied, and ligheſt; which is a ſufficient Reaſon for the Mercury's ſtanding low at that Place.

" ed

" ed over this Iſland, and conſequently the " Mercury muſt ſtand high, as often as theſe " Winds blow. This holds true in this Coun- " try, but is not a general Rule for others, " where the Winds are under different Cir- " cumſtances; and I have ſometimes ſeen the " Mercury here as low as twenty-nine Inches " upon an eaſterly Wind, but then it blew ex- " ceeding hard, and ſo comes to be accounted " for, by what was obſerved upon the third " Remark.

" 5. In calm froſty Weather the Mercury " generally ſtands high, becauſe (as I con- " ceive) it ſeldom freezes, but when the Winds " come out of the northern, and north-eaſtern " Quarters; or, at leaſt, unleſs thoſe Winds " blow at no great Diſtance off: For the " north Parts of *Germany*, *Denmark*, *Swe-* " *den*, *Norway*, and all that Tract from " whence north-eaſtern Winds come, are " ſubject to almoſt continual Froſt all the " Winter; and thereby the lower Air is very " much condenſed, and in that State is brought " hitherwards by thoſe Winds, and being ac- " cumulated by the Oppoſition of the weſter- " ly Wind blowing in the *Ocean*, the Mer- " cury muſt needs be preſſed to a more than " ordinary Height; and, as a concurring " Cauſe, the ſhrinking of the lower Parts of " the Air into leſſer Room by Cold, muſt

" needs

" needs cauſe a Deſcent of the upper Parts of " the Atmoſphere, to reduce the Cavity made " by this Contraction to an *Æquilibrium*.

" 6. After great Storms, when the Mer- " cury has been very low, it generally riſes " again very faſt: I once obſerved it to riſe " one Inch and a half in leſs than ſix Hours, " after a long continued Storm of ſouth-weſt " Wind. The Reaſon is, becauſe the Air be- " ing very much rarefied, by the great Eva- " cuations which ſuch continued Storms make " thereof, the neighbouring Air runs in the " more ſwiftly, to bring it to an *Æquilibri-* " *um*; as we ſee Water runs the faſter for " having a greater Declivity.

" 7. The Variations are greater in the more " northerly Places, as at *Stockholm*, greater " than that at *Paris* (compar'd by M. *Paſ-* " *chal*;) becauſe the more northerly Parts " have uſually greater Storms of Wind than " the more ſoutherly, whereby the Mercury " ſhould ſink lower in that Extream; and " then the northerly Winds bringing the more " denſe and ponderous Air from the Neigh- " bourhood of the Pole, and that again being " checked by a ſoutherly Wind at no great " Diſtance, and ſo heaped, muſt of Neceſſity " make the Mercury in ſuch Caſe ſtand higher " in the other Extream.

" 8. Laſtly,

" 8. Laſtly, this Remark, That there is lit-
" tle or no Variation near the *Equinoctial*,
" does above all others confirm the *Hypothe-*
" *ſis* of the variable Winds being the Cauſe of
" theſe Variations of the Height of the Mer-
" cury; for, in the Places above-named, there
" is always an eaſy Gale of Wind blowing
" nearly upon the ſame Point, *viz.* eaſt-north-
" eaſt at *Barbadoes*, and eaſt-ſouth-eaſt at St.
" *Helena**, ſo that, there being no contrary
" Currents of Air to exhauſt or accumulate
" it, the Atmoſphere continues much in the
" ſame State: However, upon Hurricanes (the
" moſt violent of Storms) the Mercury has been
" obſerved very low, but this is but once in
" two or three Years, and it ſoon recovers its
" ſettled State, about 29 ½ Inches."

Monſieur *Leibnitz* accounted for the Deſcent of the Mercury before Rain, upon another Principle †, *viz.* as a Body ſpecifically lighter than a Fluid, while it is ſuſpended by it, adds more Weight to that Fluid, than when, by being reduced in its Bulk, it becomes ſpecifically heavier, and deſcends; ſo the Vapour, after it is reduced into the Form of Clouds, and deſcends, adds leſs Weight to the Air than before; and therefore the Mer-

* See the Cauſe of this aſſigned in the following Diſſertation.

† Memoir. de l'Acad. 171.

cur-

cury falls. To which it is anſwered, 1*ſt*, That, when a Body deſcends in a Fluid, its Motion, in a very little Time, becomes uniform, (or nearly ſo) a farther Acceleration of it being prevented by the Reſiſtance of the Fluid; and then by the third Law of Nature, it preſſes the Fluid downwards, with a Force equal to that whereby it tends to be farther accelerated, that is, with a Force equal to its whole Weight. 2*dly*, The Mercury, by its Deſcent, foretels Rain a much longer Time before it comes, than the Vapour, after it is condenſed into Clouds, can be ſuppoſed to take up in falling. 3*dly*, Suppoſing that as many Vapours as fall in Rain, during the Space of a whole Year, were at once to be condenſed into Clouds, and even quite ceaſe to gravitate upon the Air, its Gravity would ſcarce be diminiſhed thereby, ſo much as is equivalent to the Deſcent of two Inches of Mercury in the Barometer. Farther, in many Places between the Tropics, the Rains fall at certain Seaſons, in very great Quantities *, and yet the Barometer ſhews there very little or no Alteration in the Weight of the Air.

The following are Mr. *Patrick*'s Obſervations on the riſing and falling of the Mercury. They are very juſt, and are to be accounted

* See Diſſertation the IVth.

for

for on the ſame Principles with thoſe of Dr. *Halley*.

" 1. The riſing of the Mercury preſages in " general fair Weather; and its falling, foul " Weather; as Rain, Snow, high Winds and " Storms.

" 2. In very hot Weather, the falling of " the Mercury foreſhews Thunder.

" 3. In Winter the riſing preſages Froſt; " and in froſty Weather, if the Mercury falls " three or four Diviſions, there will *certainly* " follow a Thaw. But in a continued " Froſt, if the Mercury riſes, it will *certainly* " ſnow.

" 4. When foul Weather happens ſoon after " the falling of the Mercury, expect but little " of it. And, on the contrary, expect but " little fair Weather, when it proves fair " ſhortly after the Mercury has riſen.

" 5. In foul Weather, when the Mercury " riſes much and high, and ſo continues for " two or three Days before the foul Weather " is quite over, then expect a Continuance of " fair Weather to follow.

" 6. In fair Weather, when the Mercury " falls much and low, and thus continues for " two or three Days before the Rain comes, " then expect a great deal of Wet, and proba- " bly high Winds.

" 7. The unſettled Motion of the Mercury " denotes uncertain and changeable Weather.

" 8. You are not ſo ſtrictly to obſerve the " Words engraved on the Plates (though, for " the moſt Part, it will agree with them) as " the Mercury's *Riſing* and *Falling:* For if " it ſtands at *Much Rain*, and then riſes up " to *Changeable*, it preſages fair Weather, al- " though not to continue ſo long, as it would " have done, if the Mercury were higher: " And ſo on the contrary, if the Mercury " ſtood at *Fair*, and falls to *Changeable*, it " preſages foul Weather; though not ſo much " of it, as if it had ſunk down lower."

From theſe Obſervations, it appears, That it is not ſo much the Height of the Mercury in the Tube, that indicates the Weather, as the Motion of it up and down; wherefore in order to paſs a right Judgment of what Weather is to be expected, we ought to know, whether the Mercury is actually *Riſing* or *Falling*; to which End the following Rules are of Uſe.

1. If the Surface of the Mercury is convex, ſtanding higher in the Middle of the Tube than at the Sides, it is generally a Sign that the Mercury is then riſing.

2. If the Surface is concave, or hollow in the Middle, it is ſinking. And,

3. If

3. If it is plain, the Mercury is ſtationary, or rather, if it is a little convex; for Mercury being put into a Glaſs Tube, eſpecially a ſmall one, will naturally have its Surface a little convex; becauſe the Particles of Mercury attract each other more forcibly than they are attracted by Glaſs. Further,

4. If the Glaſs is ſmall, ſhake the Tube; and if the Air is growing heavier, the Mercury will riſe about half the tenth of an Inch higher, than it ſtood before; if it is growing lighter, it will ſink as much. This proceeds from the Mercury's ſticking to the Sides of the Tube, which prevents the free Motion of it, till it is diſengaged by the Shock. And therefore, when an Obſervation is to be made with ſuch a Tube, it ought always to be ſhaken firſt; for ſometimes the Mercury will not vary of its own Accord, till the Weather, it ought to have indicated, is preſent.

The Uſefulneſs of knowing whether the Mercury is actually riſing or falling, and the Advantage that would ariſe from perceiving the moſt minute Variations in eſtimating the Heights of Places, have given Occaſion to the Invention of ſeveral Kinds of Barometers different from the *Torricellian*, though founded on the ſame Principle; wherein the Scale of Variation, which in that is not above three Inches,

ſhould be conſiderable larger. Of which I am now to give ſome Account.

1. The firſt is that of *Des Cartes*, which was made in the Form expreſſed, (*Fig.* 28.) where AB is a Tube hermetically * ſealed at A, and having its lower Orifice immerged in ſtagnant Mercury EF, and filled with the ſame Fluid to G, from thence to H with Water, and empty from thence to the Top. Now, when the Mercury riſes in this Tube, ſuppoſe from G to L, the Water will be raiſed in the ſmall Tube, perhaps from H to M, *viz.* as many Times further, as the Tube CA is ſmaller than CD; by which Means the Variations become much more ſenſible, than they are in the common Barometer. The Inconvenience of this was, that the Air, included in the Water, getting looſe by Degrees, filled the void Space at the Top, and ſo ſpoiled the Machine.

2. He then contrived it thus, ABC (*Fig.* 29.) is a bent Tube hermetically ſealed at A, filled with Water from F to D (tinged with *Aqua Regia* to prevent its freezing) from D to E with Mercury, and empty from thence to the Top. Then, upon the Mercury's riſing, ſuppoſe from

* A Tube is ſaid to be *hermetically* ſealed, when the End is ſo cloſed, that nothing can poſſibly evaporate through it. And, becauſe this is beſt done, when it is cloſed up with its own Subſtance; or when its Bore does not reach quite to the End of it, it is then ſaid to be *hermetically* ſealed.

E to

E to M, and falling as much at D, the Surface of the Water at F would ſink ſo many Times farther than the Surface of the Mercury at D, as the Tube C G was ſmaller than G H. The Water here is liable to evaporate, though that may, in ſome Meaſure, be prevented, by pouring a few Drops of Oil of Sweet Almonds upon it. Others have contrived,

3. The Horizontal or Rectangular Barometer (Fig. 30.) hermetically ſealed at A, and filled with Mercury from D to E; then, as the upper Surface of it riſes in the Tube, ſuppoſe from E to F, the lower will be driven from D to G, as many Times farther, as this Part of the Tube is leſs than that at E. But it often happens, that ſome Parts of the Mercury break off from the reſt in the Leg B C, and are left behind. This Inconvenience is remedied in,

4. The Diagonal Barometer A B C (*Fig.* 31.) wherein the Mercury, inſtead of riſing from B to D (ſuppoſe that Space to correſpond to the Scale of Variation in a ſtrait Tube) will riſe from B to A; for it will always ſtand at the ſame perpendicular Height, whatever be the Inclination of the Tube; becauſe Fluids preſs only according to their perpendicular Altitude*. But A B the upper Part of the Tube muſt not be too much inclined, leſt the Mercury break in it, as in the former.

* Chapter I. §. 9.

5. AB (*Fig.* 32.) is Dr. *Rook*'s Wheel-Barometer, wherein A B D is a Tube filled with Mercury from *a* to E; *a* is an Iron Ball, ſwimming on the Surface of the Mercury; this, as it ſubſides with the Surface of the Mercury, draws the little Wheel *mn* round, to whoſe Circumference it is fixed by Means of the String *ac* *: This Wheel carries the Index P Q, which points to the graduated Edge of the Circle K L, and by its Motion ſhews the moſt minute Variations of the Mercury. When the Ball *a* is raiſed by the Mercury on which it ſwims, the Index is drawn the contrary Way by a leſſer Ball *b*, which hangs on the other Side the Wheel. The Friction in this Machine, unleſs it be made with great Accuracy indeed, renders it uſeleſs.

6. The pendant Barometer is another Contrivance to render the Variations more ſenſible. It conſiſts of a ſmall conical Tube, (repreſented *Fig.* 33.) hermetically ſealed at A, and filled with Mercury from C to D, and empty from thence to A. Now, ſuppoſing the Gravity of the Air increaſed, it will raiſe the Mercury higher in the Tube, and ſo force it into a narrower part; by which Means the Column becoming longer, its perpendicular Preſſure upon the Air below will be proportionally increaſed.

* The Tube is ſmaller at *a* than at E, that the greateſt Variation may be at that Surface of the Mercury on which the Ball reſts.

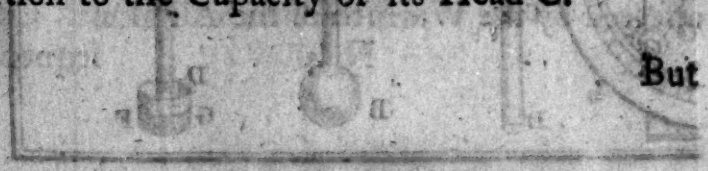
On

On the contrary, when the Air becomes lighter, the Mercury deſcends into a larger Part of the Tube, and by that Means has the Length of its Column proportionally contracted. The Inconvenience that attends this Barometer, is that the Tube muſt be very ſmall, otherwiſe the Mercury will fall out; or the Air will be apt to get into it, and divide the Column in ſeveral Places; and when the Tube is very ſmall, the Friction of the Mercury againſt the Sides of it will hinder it from riſing and falling freely.

7. Dr. *Hook*, obſerving how unfit the common Barometer was to be uſed on board of a Ship, by Reaſon its Poſition ought to be ſteady, contrived the following one, called, from its Uſe, a *Marine* Barometer, conſiſting of two Parts, the one A B (*Fig.* 34.) the common Spirit Thermometer, the other C D, a Tube filled with Air from C to E, and from thence to the End D with tinged Water. This End is immerged in the ſame Fluid contained in the Veſſel G F; and having its Surface expoſed to the Preſſure of the external Air. Now, the laſt of theſe Machines will be affected both by the Warmth of the external Air, and alſo by its Preſſure: The former dilating the Air included in C E, and by that Means driving the Water downwards; the latter preſſing it up higher in the Tube: Whereas the other, *viz.* A B, is affected by the Warmth of the Air alone. Conſequently

ſequently, were theſe Inſtruments graduated in ſuch a Manner, that, if the Gravity of the external Air ſhould always remain the ſame it was when the Inſtruments were made, their Variations (then only depending on its Warmth) ſhould exactly correſpond with each other; that is, when the Spirit in the Tube AB ſhould aſcend to 1, the Water in CD ſhould aſcend to 1, *&c.* Then, whenever their Variations ſhould be obſerved to differ from each other, the Difference could only be aſcribed to ſome Alteration in the Preſſure of the Air upon the Surface of the Water in the Veſſel GF. In Proportion therefore as the Difference is greater, or leſs, ſo is the Alteration in the Gravity of the Air, from what it was when the Inſtruments were adjuſted. For Inſtance, when the Water ſtands above the Diviſion, which correſponds to that, which the Spirit points to in the other Machine, it is an Indication, that the Preſſure of the Air is greater at that Time, than when the Inſtruments were graduated, and *vice verſâ*.

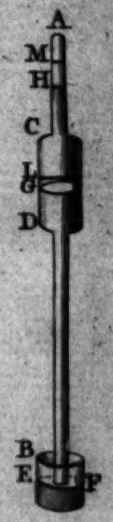
This Machine is not only more uſeful at Sea, than the common one, as not requiring a ſteady Poſition; but may have its Scale of Variation conſiderably enlarged, by making the Bore of the Tube CD very ſmall, in Proportion to the Capacity of its Head C.



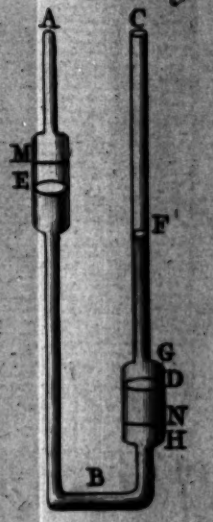
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Part II. Plate V. p. 200.

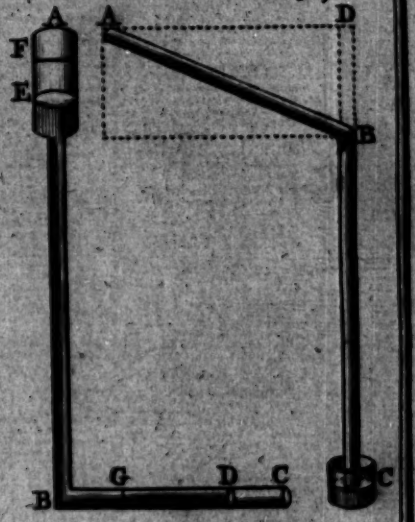
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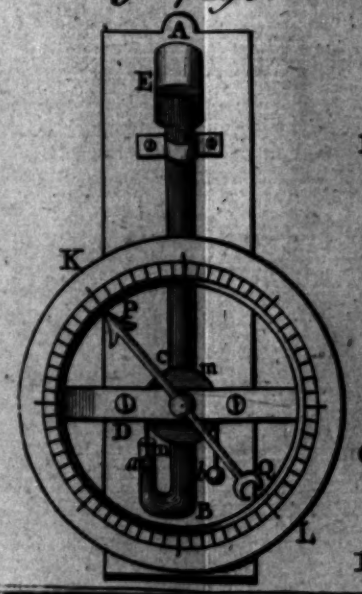
F. 29. p. 196.



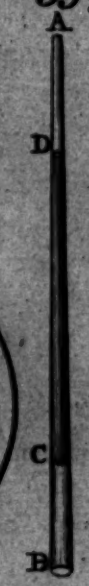
F. 30. p. 197. F. 31. p. 197.



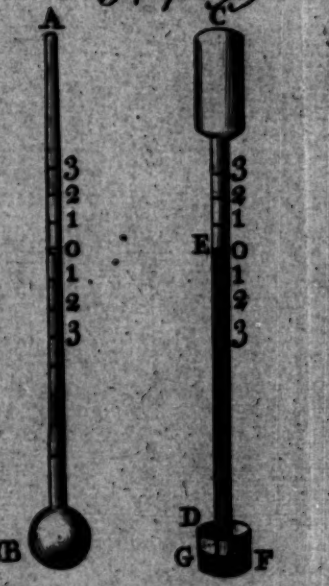
F. 32. p. 198.

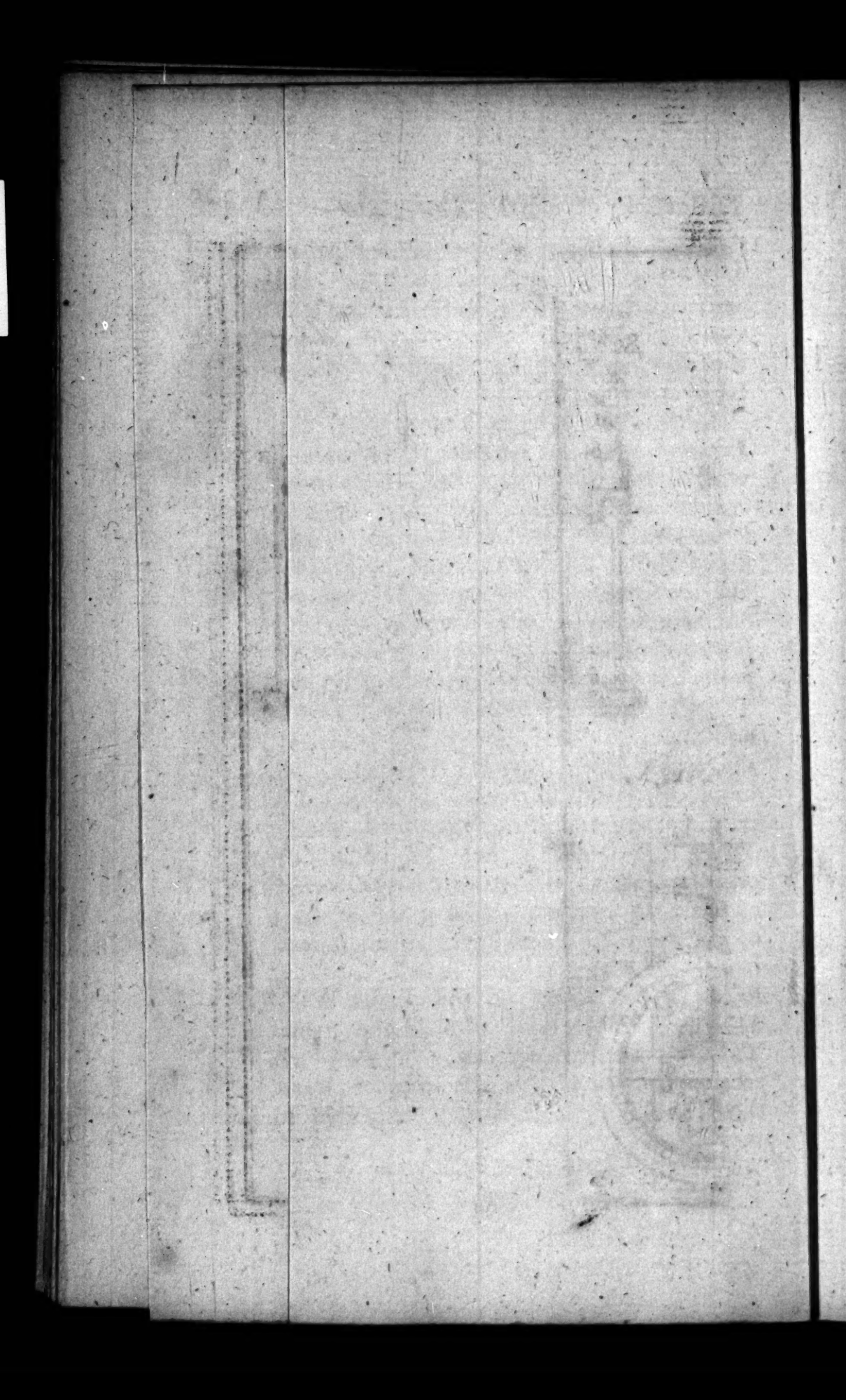


F. 33. p. 198.



F. 34. p. 199.





But it is obſerved, that, in long keeping the Inſtrument, the included Air loſes ſomewhat of its Elaſticity; whereby, in Proceſs of Time, the Water ſtands higher than it ought, and therefore indicates the Gravity of the Air to be greater than what it is.

In the Philoſophical Tranſactions, N° 427, I have given an Account of a Barometer, wherein the Scale of Variation may be increaſed *ad Infinitum*. The Deſcription of it is as follows: ABCD (*Fig.* 35.) is a cylindrical Veſſel, filled with a Fluid to the Height W, in which is immerged the Barometer SV, conſiſting of the following Parts: The principal of which is a Glaſs Tube TP, (repreſented ſeparately at *tp*) whoſe upper End T is hermetically ſealed: This End does not appear to the Eye, being received into the lower End of a Tin Pipe GH, which in its other End G receives a cylindrical Rod, or Tube ST, and thereby fixes it to the Tube TP. This Rod ST may be taken off, in order to put in its Stead a larger, or leſſer, as Occaſion requires. S is a Star at the Top of the Rod ST, and ſerves as an Index, by pointing to the graduated Scale LA, which is fixed to the Cover of the Veſſel ABCD. MN is a large cylindrical Tube made of Tin (repreſented ſeparately at *mn*) which receives in its Cavity the ſmaller Part of the Tube TP, and is well cemented to it at

at both Ends, that none of the Fluid may get in.

The Tube TP, with this Apparatus, being filled with Mercury, and plunged into the Baſon V, which hangs by two, or more Wires, upon the lower End of the Tube MN, muſt be ſo poized, as to float in the Liquor contained in the Veſſel ABCD, and then the whole Machine will riſe, when the Atmoſphere becomes lighter, and *vice verſâ*.

I ſhall here add a Computation, in order to ſhew the Poſſibility of the Variation being infinite, upon a given finite Variation of the Weight of the Atmoſphere, and withal the Reaſon why it may be ſo. Thoſe who would ſee a Mathematical Proof of it, may conſult the Note below *.

Let

* Let the ſpecific Gravity of Quickſilver be to that of Water, or to the Liquor the Barometer floats in, as s to 1; and if it be propoſed, that the Variations in this compound Barometer ſhall be to the contemporary Variations of the common Barometer in the given Ratio of n to 1, this Effect will be obtained, by making the Diameter of the Rod ST to the Diameter of the Cavity of the Tube HI, as $\sqrt{\frac{n+s}{ns}}$ to 1, which may be thus demonſtrated.

Let us ſuppoſe, that the Variation in the Height of the Quickſilver in the common Barometer, which we will call v, is ſuch, that a cubic Inch of Quickſilver ſhall riſe into the *Vacuum* XT; in order to which, a cubic Inch of Quickſilver muſt riſe from the Veſſel V; that is, the Surface P muſt ſubſide ſo far, that a cubic Inch of Water, (if that be the Fluid made Uſe of) ſhall enter the Veſſel V, by which Means the Barometer with the Parts annexed will be heavier by a cubic Inch of the Fluid.

Now

Let it be ſuppoſed that the Fluid made Uſe of is Water, and that the given Variation in the Weight of the Atmoſphere is ſuch, that, by

Now this additional Weight of a cubic Inch of Fluid, will make the whole Barometer ſubſide (according to the Laws of Hydroſtatics) till a cubic Inch of the Rod HS, immediately extant above the Surface at W, ſhall come under it; but the Length of ſuch a Magnitude of HS will exceed the Length of an equal Magnitude of Quickſilver in the larger Tube X, as many Times as the Square of the Diameter at X exceeds the Square of the Diameter at H (the Length of equal Cylinders being reciprocal to their Baſes.) That is, the perpendicular Deſcent of the compound Barometer will be to v, the perpendicular Aſcent of the Mercury in the common Barometer, as d to 1, (ſuppoſing this the Ratio of their Baſes) and conſequently will be equal to dv.

But, by this Deſcent, the Diſtance PW, between the Surface of the ſtagnant Quickſilver and the Top of the Fluid, will be augmented by a Column, whoſe Height is dv, the Deſcent of the compound Barometer; and conſequently the Weight of the whole Column of the Fluid preſſing on the lower Surface of the Quickſilver (to which the Height X is partly owing) will be increaſed by a Column of that Length; and this Increaſe would produce a ſecond Aſcent of the Mercury at X equal to itſelf, namely, dv, were the Fluid as heavy as Quickſilver; but ſince it is ſuppoſed to be lighter in the Ratio of s to 1, the Aſcent of the Quickſilver, on this Account, will only be $\frac{dv}{s}$.

But now, as in the former Caſe, when the Aſcent of the Mercury was v, the Deſcent of the compound Barometer was ſhewn to be dv; ſo here, the Aſcent of the Mercury being $\frac{dv}{s}$, the Deſcent of the compound Barometer will be $\frac{ddv}{s}$, and the next Deſcent $\frac{dddv}{ss}$, and the next $\frac{d^4v}{s^3}$, and ſo on to Infinity. Therefore the whole Deſcent of the compound Barometer is to the Aſcent of the Mercury in the common Barometer, that is, n is to 1 as $d + \frac{dd}{s} + \frac{ddd}{ss} + \frac{d^4}{s^3} +$, &c. to 1, or as $\frac{ds}{s-d}$ to 1; becauſe, the Terms of the Series being in geometrical Proportion, the

by preſſing upon the Surface of it at W, the Surface of the Mercury at X may be raiſed an Inch higher (meaſuring from its Surface at P) than before; and that the Breadth of the Cavity of the Tube at X, and of the Baſon at P are ſuch, that, by this Aſcent of the Mercury, there may be a cubic Inch of it in the Cavity X more than before, and conſequently in the Baſon a cubic Inch leſs. Now, upon this Suppoſition, there will be a cubic Inch of Water in the Baſon more than there was before; becauſe the Water will ſucceed the Mercury to fill up its Place. Upon this Account the whole Machine will be render'd heavier than it was before, by the Weight of a cubic Inch of Water, and therefore will ſink, according to the Laws of Hydroſtatics (Chap. II. §. 5.) till a cubic Inch of that Part of the Rod WS, which

the Sum of them all is $\frac{ds}{s-d}$. Hence we have $n = \frac{ds}{s-d}$, and therefore $ns = ds + dn$; that is, $1 : d :: n + s : ns :: \frac{n+s}{ns} : 1$; and therefore, by extracting the ſquare Roots of each Term in the Proportion, $1 : \sqrt{d}$ (that is, the Diameter of ST to the Diameter of HI) as $\sqrt{\frac{n+s}{ns}}$ to 1. *Q. E. D.*

Example 1. Putting $s = 14$ and $n = 1$, the Variation in each Barometer will be equal, by taking the Diameter of ST to the Diameter of HI, as $\sqrt{\frac{15}{14}}$ to 1, that is, as 30 to 29 nearly.

Example 2. If n be put infinite, the Diameter of ST will be to the Diameter of HI, as $\sqrt{\frac{1}{s}}$ to 1, or 1 to $\sqrt{14}$; that is, as 1 to $3\frac{3}{4}$ nearly.

was

was above the Surface of the Water at W, comes under it. Then, if we ſuppoſe this Rod ſo ſmall, that a cubic Inch of it ſhall be fourteen Inches in Length, the whole Machine will ſink fourteen Inches lower in the Fluid than before, and conſequently the Surface of the Mercury in the Baſon will be preſſed more than it was before, by a Column of Water fourteen Inches high. But the Preſſure of fourteen Inches of Water is equivalent to one of Mercury (becauſe Water is about fourteen times lighter than Mercury;) this additional Preſſure therefore will make the Mercury aſcend at X, as much as the ſuppoſed Variation in the Weight of the Air did at firſt. This Aſcent will give Room for a ſecond cubic Inch of Water to enter the Baſon; the Machine will therefore be again rendered heavier by the Weight of ſo much Water, and accordingly will ſubſide fourteen Inches farther. This will occaſion another additional Preſſure of Water, which will raiſe another Inch of Mercury, and make the Machine ſink fourteen Inches more, and ſo on, without ever approaching nearer to an *Æquilibrium* with the external Air: And therefore a Scale, anſwering to the Variation of this Barometer, ought ſtrictly and properly to be of an infinite Length; becauſe, after this Barometer has ſunk or riſen thouſands of Miles (if that were poſſible) it would ſtill have the ſame Tendency to ſink or riſe on, as when it firſt ſet out.

Now,

Now, was the Rod WS ſo ſmall, that a cubic Inch of it ſhould be more than fourteen Inches long (the other Parts remaining as was ſuppoſed above) the Variation in this Barometer would be more than Infinite, or negative with Reſpect to thoſe of the common Barometer. The Meaning of which is, that whereas, in the common Barometer, the ſuſpended Column of Mercury, by its riſing or falling, approaches nearer to an *Æquilibrium* with the external Air, this Barometer would continually recede from an *Æquilibrium* with it; ſo that the farther it ſhould move up or down, inſtead of acquiring by that Means a leſs Tendency to move on, as the Mercury in the common Barometer does, it would acquire a greater.

On the contrary, when a cubic Inch of the Rod is leſs than fourteen Inches in Length, the Variation will be finite; and may be made to bear any Proportion to thoſe of the common Barometer whatever, as demonſtrated in the foregoing Note.

While I am writing this, another Method occurs to me of making a Barometer, wherein the Scale of Variation ſhall bear any Proportion to that of the common one. It is this: Let there be a compound Tube, as ABC (*Fig.* 36.) hermetically ſealed at A, and open at C, empty from A to D, filled with Mercury from thence to B, and from thence to E with Water;

ter: Then, if the Tube F C be a little more than five Times leſs in Diameter than the Tube FA, the Variation in the lower Surface of the Water at E will be infinite; if it be above ſo many Times leſs, it will be more than infinite, otherwiſe it will be finite. See the Demonſtration in the Note *. That

* Let v denote a given Variation in the common Barometer, x the correſpondent Variation at E ſought. Let the Ratio of m to 1, expreſs that of the ſpecific Gravity of Mercury, to that of Water; and d to 1, that of the Diameter of the Tube FA to FC. Then the Variation at E, the lower Surface of the Water, being ſuppoſed x, the Variation of it at B, the upper Surface of it, will be $\frac{x}{dd}$, and conſequently GE, the Difference of the Legs EK and KB, will vary $x + \frac{x}{dd}$, becauſe as E riſes the Height x, G comes down $\frac{x}{dd}$. Again, the Variation of the Surface of the Mercury at B will be the ſame with that of the Water in the ſame Place, *viz.* $\frac{x}{dd}$; and, if the Tube is of the ſame Diameter at D, as at B, the Variation of the Surface at D will alſo be the ſame, that is, $\frac{x}{dd}$: The Sum of both Variations, or the Variation of HD the Difference of the Legs, will therefore be $\frac{2x}{dd}$. Now the Preſſure of the Mercury and Water together upon the Air at E, is owing to the Lengths of HD and GE; and ſince one of theſe will always ſhorten, when the other lengthens, the Variation in their Preſſure will depend on the Variation of the Difference of their Weights, that is, of the Difference between the Weight of $x + \frac{x}{dd}$ and of $\frac{2x}{dd}$. But the Weight of $x + \frac{x}{dd}$ (being the Weight of a Column of Water) compared to that of a Column of Mercury of the ſame Length, is only $\frac{x}{m} + \frac{x}{mdd}$. The Difference therefore between

That the Variation in this Barometer may be infinite, will appear from the following Computation.

Let the Proportion between the Bores of the Tube AF and FC be ſuch, that when HD, the Difference of the Legs wherein the Mercury is contained, is augmented one Inch, GE, the Difference of the Legs wherein the Water is contained, ſhall be diminiſhed fourteen; then, as much as the Preſſure of the Mercury is augmented, that of the Water will be diminiſhed, and ſo the Preſſure of both taken together will remain as it was. And conſequently, after it has begun to riſe, it will always have the ſame Tendency to riſe on, without ever coming to an *Æquilibrium* with the Air.

How far this Barometer will ſucceed in Practice muſt be left to experience to determine.

between $\frac{x}{m}+\frac{x}{mda}$ and $\frac{2x}{dd}$ will be always equal to the Variation in the common Barometer, and therefore $\frac{2x}{dd}-\frac{x}{m}-\frac{x}{mdd}=v$ and by the common Method of Reduction, $x=\frac{v\,m\,dd}{2m-dd-1}$: That is, $x : v :: mdd : 2m-dd-1$. Now, if we put $m=14$, and $d=\sqrt{27}$, $2m-dd-1$ will be found equal to nothing; and ſo x being by the Proportion as many Times more than v, as mdd is than nothing, 'tis infinite. And if m be put $=14$, and $d=5$, mdd will be equal to 350, and $2m-dd-1=2$; and therefore the Variations in this Caſe will be to thoſe in the common Barometer, as 175 to one.

Pro-

Probably, if the Bore of the Tube F C be made very ſmall, *viz.* about the twentieth Part of an Inch Diameter, the Air will not aſcend through the Water, as it is apt to do through the Mercury, in the pendant Barometer; and the Smallneſs of the Bore will not prevent the Water from moving, near ſo much as it does the Mercury in that Barometer.

There is an Improvement of another Kind in the common Barometer, whereby it is rendered *portable*. The Tube containing the Mercury, inſtead of having its lower End immerged in a Veſſel of that Fluid, has it tied up in a Leathern Bag, not quite full of Mercury. And though the external Air cannot get into the Bag to ſuſpend the Mercury in the Tube, by preſſing on its Surface, as in the common one; yet it has the ſame Effect, by preſſing on the Outſide of the Bag, which, being pliant, yields to the Preſſure, and keeps the Mercury ſuſpended in the Tube at its proper Height. This Bag is generally incloſed in a little Box, through the Bottom of which paſſes a Screw; with this Screw the Bag may be compreſſed, ſo as to force the Mercury up to the Top of the Tube; which keeps it ſteady, and hinders it from breaking the Tube by daſhing againſt the Top when it is carried about, as it otherwiſe would be apt to do.

See more on the Subject of this Diſſertation, *Weidleri* Inſtitutiones Mathemat. p. 608. *Melchior Verdries* Phyſ. Pars ſpecialis, Cap. IV. §. 15. Mr. *Paſchal*'s Traité de l'Equilibre des Liqueurs. *Sinclair*'s Ars magna Gravitatis & Levitatis. *Mariotte*'s Second Eſſay de la Nature de l'Air. Philoſoph. Burgund. Tom. II. p. 850. *Saul*'s Treatiſe on the Barometer. *Regnault*'s Philoſoph. Converſat. 22. *Clare*'s Motion of Fluids, p. 140. Mem. de l'Acad. 1705, 1711. Philoſophical Tranſactions, N° 9, 10, 11, 55, 86, 91, 165, 181, 185, 208, 229, 236, 237, 240, 243, 269, 351, 366, 385, 388, 405, 406, 427. *Cotes*'s Hydroſtatical and Pneumatical Lectures, Lect. 7. With ſeveral other Authors referred to in Mr. *Johnſon*'s Quæſtiones Philoſophicæ, Cap. IV. Quæſt. 36, 37.

DIS-

DISSERTATION V.

Of the Origin of Winds.

THE Wind is no other than the Motion of the Air upon the Surface of the Globe. Some of the Ancients took it to be Air, ruſhing out of the Bowels and Cavities of the Earth: And others thought it an Exhalation from its Surface. But theſe are *Hypotheſes* too chimerical to ſtand in Need of a particular Confutation. Some of the Moderns, who held a *Plenum*, have accounted for it thus: They imagined, that the Air being confined above, as it muſt be, if we ſuppoſe a *Plenum*, would, when more than ordinarily rarefied, or ſtocked with Vapours, drive away the neighbouring Air, in order to make Room for itſelf; and by this Means occaſion a Wind. Others, obſerving a conſtant and perpetual eaſterly Wind to blow at the *Equator*, aſcribed its Origin to the diurnal Rotation of the Earth about its Axis from Weſt to Eaſt; which they thought would occaſion the Air upon its Surface to ſeem to move the contrary Way, being in ſome Meaſure left behind. But, whereas there are Winds, in ſome Places near the *Equator*, that blow on other Points of the Compaſs

(as we ſhall ſee hereafter) this *Hypotheſis* is inſufficient. Beſides, the Air preſſing upon the Surface of the Earth by its Gravity, like other Bodies, and having nothing to hinder it from moving freely along with it, muſt neceſſarily, in Time, acquire an equal Degree of Velocity, and ſo keep Pace with it, all the Way round.

The principal Cauſe of the Wind, or, in other Words, of the Air's moving from Place to Place, upon the Surface of the Earth, is the Atmoſphere's being heated over one Part more than over another. For, in this Caſe, the warmer Air being rarefied, becomes ſpecifically lighter than the reſt; it is therefore overpoiſed by it, and raiſed upwards, the upper Parts of it diffuſing themſelves every Way over the Top of the Atmoſphere; while the neighbouring inferior Air ruſhes in from all Parts at the Bottom; which it continues to do, till the *Æquilibrium* is reſtored.

Upon this Principle it is, that moſt of the Winds may be accounted for.

To begin with thoſe which blow under the *Equator*.

1. Under the *Equator*, the Wind is always obſerved to blow from the Eaſt Point *.

For,

* For the Reader's Eaſe, (who perhaps is not furniſhed with the Philoſophical Tranſactions) I ſhall here inſert, by Way of Note, from Dr. *Halley*'s Account, ſo much of the Hiſtory of the Winds, as may be neceſſary for the underſtanding this Theory.

" The

For, ſuppoſing the Sun to continue vertical over ſome one Place, the Air will be moſt rarefied there; and conſequently, the neighbouring

" The univerſal Ocean, ſays he, may moſt properly be di-
" vided into three Parts, *viz.* 1. The *Atlantic* and *Æthiopic*
" Seas. 2. The *Indian* Ocean. 3. The great *South* Sea, or
" the *Pacific* Ocean.

" I. In the *Atlantic* and *Æthiopic* Seas, between the *Tropics*,
" there is a general eaſterly Wind all the Year long, without any
" conſiderable Variation; excepting, that it is ſubject to be de-
" flected therefrom, ſome few Points of the Compaſs, towards
" the North, or South, according to the Poſition of the Place.

" 1. Near the Coaſt of *Africa*, as ſoon as you have paſſed the
" *Canary* Iſles, you are ſure to meet a freſh Gale of North-eaſt
" Wind, about the Latitude of twenty-eight Degrees North;
" which ſeldom comes to the Eaſtward of the Eaſt-north-eaſt,
" or paſſes the North-north-eaſt. This Wind accompanies thoſe
" bound to the Southward, to the Latitude of ten Degrees
" North, and about an hundred Leagues from the *Guinea*
" Coaſt; where, till the fourth Degree of North Latitude, they
" fall into *Calms* and *Tornadoes*, or ſudden Storms.

" 2. Thoſe bound to the *Caribbee* Iſles, find, as they ap-
" proach the *American* Side, that the aforeſaid North-eaſt
" Wind becomes ſtill more and more eaſterly, ſo as ſometimes
" to be Eaſt, ſometimes Eaſt by South, but yet moſt commonly
" to the Northward of the Eaſt, a Point or two, ſeldom more.
" 'Tis likewiſe obſerved, that the Strength of theſe does gra-
" dually decreaſe, as you ſail to the Weſtward.

" 3. That the Limits of the *Trade* and *variable* Winds in
" this Ocean, are farther extended on the *American* Side, than
" the *African*; for, whereas you meet not with this certain
" Wind, till after you have paſs'd the Latitude of twenty-eight
" Degrees on this Side; on the contrary Side it commonly
" holds to thirty, thirty-one, or thirty-two Degrees of Lati-
" tude; and this is verified likewiſe to the Southward of the
" *Æquinoctial*; for near the *Cape of Good Hope*, the Limits of
" the *Trade* Wind are three or four Degrees nearer the *Line*,
" than on the Coaſt of *Braſil*.

ing Air will ruſh in from every Quarter with equal Force. But, as the Sun is continually ſhifting to the Weſtward, the Part where the Air

" 4. That from the Latitude of four Degrees North, to the " aforeſaid Limits on the South Side of the *Equator*, the Winds " are generally and perpetually between the South and Eaſt, " and moſt commonly between the South-eaſt and Eaſt; ob- " ſerving always this Rule, that on the *African* Side they are " more ſoutherly, on the *Braſilian* more eaſterly, ſo as to be- " come almoſt due Eaſt, the little Deflection they have being " ſtill to the Southward. In this Part of the Ocean, it has " been my Fortune to paſs a full Year, in an Employment that " obliged me to regard more than ordinarily the Weather, and " I found the Winds conſtantly about the South-eaſt, the moſt " uſual Point South-eaſt by Eaſt: When it was eaſterly, it ge- " nerally blew hard, and was gloomy, dark, and ſometimes " rainy Weather: If it came to the Southward, it was gene- " rally ſerene, and a ſmall Gale next to a Calm; but this not " very common. But I never ſaw it to the Weſtward of the " South, or Northward of the Eaſt.

" 5. That the Seaſon of the Year has ſome ſmall Effect on " theſe *Trade* Winds; for that, when the Sun is conſiderably to " the Northward of the *Equator*, the South-eaſt Winds, eſpe- " cially in the Streight of this Ocean (if I may ſo call it) be- " tween *Braſil*, and the Coaſt of *Guinea*, do vary a Point or " two to the Southward, and the North-eaſt become more " eaſterly; and, on the contrary, when the Sun is towards the " *Tropic* of *Capricorn*, the South-eaſterly Winds become more " eaſterly; and the North-eaſterly Winds, on this Side the " *Line*, veer more to the Northward.

" 6. That as there is no general Rule, that admits not of ſome " Exception, ſo there is in this Ocean a Tract of Sea, wherein " the ſoutherly and South-weſt Winds are perpetual, *viz.* all " along the Coaſt of *Guinea*, for above five hundred Leagues " together, from *Sierra Leona*, to the Iſle of St. *Thomas*: For " the South-eaſt *Trade* Wind having paſs'd the *Line*, and ap- " proaching the Coaſt of *Guinea* within eighty or an hundred " Leagues, inclines towards the Shore, and becomes South-ſouth- " eaſt; and by Degrees, as you come nearer, it veers about to " South, South-ſouth-weſt, and in with the Land South-weſt,

" and

Air is moſt rarefied, is carried the ſame Way; and therefore the Tendency of all the lower Air, taken together, is greater that Way, than any other.

" and ſometimes Weſt-ſouth-weſt. Theſe are the Winds, which " are obſerved on this Coaſt, when it blows true; but there are " frequent Calms, violent ſudden Guſts, called *Tornadoes*, from " all Points of the Compaſs, and ſometimes unwholeſome foggy " eaſterly Winds, called *Hermitæ*, by the Natives, which too " often infeſt the Navigation of theſe Parts.

" 7. That to the Northwards of the *Line*, between four and " ten Degrees of Latitude, and between the Meridians of *Cape* " *Verde*, and of the eaſtermoſt Iſlands that bear that Name, there " is a Tract of Sea, wherein it were improper to ſay, there is " any *Trade* Wind, or yet a *variable*; for it ſeems condemned to " perpetual Calms, attended with terrible Thunder and Light- " ning, and Rains ſo frequent, that our Navigators from thence " call this Part of the Sea, the *Rains*: The little Winds that " are, being only ſome certain Guſts, of very little Continu- " ance, and leſs Extent; ſo that ſometimes each Hour you " ſhall have a different Gale, which dies away into a Calm be- " fore another ſucceeds: And in a Fleet of Ships in Sight of " one another each ſhall have the Wind from a ſeveral Point of " the Compaſs: With theſe weak Breezes, Ships are obliged " to make the beſt of their Way to the Southward, through " the aforeſaid ſix Degrees; wherein it is reported ſome have " been detain'd whole Months for want of Wind.——

" II. In the *Indian* Ocean, the Winds are partly general, " as in the *Æthiopic* Ocean; partly periodical, that is, half " the Year they blow one Way, and the other half near upon " the oppoſite Points; and theſe Points, and Times of ſhifting, " are different in different Parts of this Ocean.

" 1. Between the Latitudes of ten Degrees and thirty De- " grees South, between *Madagaſcar* and *Hollandia Nova*, the " *General* Trade-Winds about the South-eaſt and by Eaſt, are " found to blow all the Year long, to all Intents and Purpoſes, " after the ſame Manner as in the ſame Latitudes in the " *Æthiopic* Ocean, as it is deſcribed in the fourth Remark " aforegoing.

other. Thus the Tendency of the Air towards the Weſt, becomes general, and its Parts impelling one another, and continuing to move

" 2. That the aforeſaid South-eaſt Winds extend to within " two Degrees of the *Equator*, during the Months of *June*, *July*, " and *Auguſt*, &c. to *November*; at which Time, between the " South Latitude of three and ten Degrees, being near the *Me-* " *ridian* of the North End of *Madagaſcar*, and between two " and twelve South Latitude, being near *Sumatra* and *Java*; " the contrary Winds from the North-weſt, or between the " North and Weſt, ſet in, and blow for half a Year, *viz.* from " the Beginning of *December* till *May*: And this *Monſoon* is " obſerved as far as the *Molucca* Iſles.

" 3. That to the Northward of three Degrees South Lati- " tude, over the whole *Arabian* and *Indian* Sea, and Gulf of " *Bengal*, from *Sumatra* to the Coaſt of *Africa*, there is ano- " ther *Monſoon*, blowing from *October* to *April*, upon the " North-eaſt Points: But in the other half Year, from *April* to " *October*, upon the oppoſite Points of South-weſt and Weſt- " ſouth-weſt, and that with rather more Force than the other, " accompanied with dark, rainy Weather; whereas the North- " eaſt blows clear. 'Tis likewiſe to be noted, that the Winds " are not ſo conſtant, either in Strength or Point, in the Gulf " of *Bengal*, as they are in the *Indian* Sea, where a certain " ſteady Gale ſcarce ever fails. 'Tis alſo remarkable, that " the South-weſt Winds, in theſe Seas, are generally more " ſoutherly on the *African* Side, and more weſterly on the " *Indian*.

" 4. There is a Tract of Sea to the Southward of the " *Equator*, ſubject to the ſame Changes of the Winds, *viz.* " near the *African* Coaſt, between it and the Iſland *Madagaſcar*, " or St. *Laurence*, and from thence Northwards, as far as the " *Line*; wherein, from *April* to *October*, there is found a con- " ſtant freſh South-ſouth-weſt Wind, which, as you go more " northerly, becomes ſtill more and more weſterly, ſo as to fall " in with the Weſt-ſouth-weſt Winds, mentioned before in thoſe " Months of the Year to be certain to the Northward of the " *Equator*. What Winds blow in thoſe Seas, for the other " half Year, I have not yet been able to obtain to my full Satiſ- " faction: The Account which has been given me, is only this,

" That

move till the next Return of the Sun, ſo much of its Motion, as was loſt by his Abſence, is again reſtored, and therefore the eaſterly Wind becomes *perpetual.*

Some

" That the Winds are much eaſterly hereabouts, and as often " to the North of the true Eaſt, as to the Southward thereof.

" 5. That to the Eaſtward of *Sumatra* and *Malacca*, to the " Northward of the *Line*, and along the Coaſt of *Camboia*, " and *China*, the *Monſoons* blow North and South; that is to " ſay, the North-eaſt Winds are much northerly, and the " South-weſt much ſoutherly. This Conſtitution reaches to the " eaſtward of the *Philippine* Iſles, and as far northerly as *Japan*; " the northern *Monſoon* ſetting in, in theſe Seas, in *October* or " *November*; and the ſouthern in *May*, blowing all the Summer " Months. Here it is to be noted, that the Points of the Com- " paſs from whence the Winds come, in theſe Parts of the " World, are not ſo fixed, as thoſe lately deſcribed; for the " ſoutherly will frequently paſs a Point or two to the Eaſtward " of the South, and the northerly as much to the Weſtward " of the North, which ſeems occaſioned by the great Quan- " tity of Land which is interſperſed in theſe Seas.

" 6. That in the ſame Meridians, but to the Southward of " the *Equator*, being that Tract lying between *Sumatra* and " *Java*, to the Weſt, and *New Guinea* to the Eaſt, the ſame " northerly and ſoutherly *Monſoons* are obſerved; but with this " Difference, that the Inclination of the northerly is towards " the North-weſt, and of the ſoutherly towards the South-eaſt: " But the *Plagæ Venti* are not more conſtant here than in the " former, *viz. variable* five or ſix Points. Beſides, the Times " of the Change of theſe Winds are not the ſame, as in the " *Chineſe* Seas, but about a Month, or ſix Weeks later.

" 7. That the contrary Winds do not ſhift all at once, but in " ſome Places the Time of the Change is attended with *Calms*, " in others with *variable* Winds; and it is particularly remark- " able, that the End of the weſterly *Monſoon*, in the Seas of " *China*, are very ſubject to be tempeſtuous. The Violence of " theſe Storms is ſuch, that they ſeem to be of the Nature of " the *Weſt-India* Hurricanes, and render the Navigation of " theſe Parts very unſafe about that Time of Year. Theſe

" Tempeſts

Some are inclined to think, that the continual ſhifting of the Sun to the Weſtward, ſhould produce a weſterly Wind under the *Equator*, by cauſing the Current of the Air from the Weſt to exceed and over-balance that,

" Tempeſts are by our Seamen uſually term'd the *Breaking up* " *of the Monſoons*.

" III. The Third Ocean, called *Mare Pacificum*, whoſe Ex- " tent is equal to that of the other two (it being from the Weſt " Coaſt of *America* to the *Philippine* Iſlands, not leſs than an " hundred and fifty Degrees of Longitude) is that which is leaſt " known to our own, or the neighbouring Nations: That Navi- " gation that there is on it, is by the *Spaniards*, who go yearly " from the Coaſt of *New Spain* to the *Manillas*: But that but " by one beaten Tract; ſo that I cannot be ſo particular here, " as in the other Two. What the *Spaniſh* Authors ſay of the " Winds, they find in their Courſes; and what is confirmed by " the old Accounts of *Drake* and *Candiſh*, and ſince by *Schooten*, " who ſailed the whole Breadth of this Sea, in the ſouthern " Latitude of fifteen or ſixteen Degrees, is, that there is a great " Conformity between the Winds of this Sea, and thoſe of the " *Atlantic* and *Æthiopic*; that is to ſay, that to the Northward " of the *Equator*, the predominant Wind is between the Eaſt and " North-eaſt; and to the Southward thereof, there is a con- " ſtant, ſteady Gale, between the Eaſt and South-eaſt, and that " on both Sides the *Line*, with ſo much Conſtancy, that they " ſcarce ever need to attend the Sails; and Strength, that it is " rare to fail of croſſing this vaſt Ocean in ten Weeks Time; " which is about an hundred and thirty Miles a Day: Beſides, " 'tis ſaid, that Storms and Tempeſts are never known in theſe " Parts; wherefore ſome have thought it might be as ſhort a " Voyage to *Japan* and *China*, to go by the Streights of *Ma-* " *gellan*, as by the Cape of *Good Hope*.

" The Limits of theſe *General* Winds are much the ſame as " in the *Atlantic* Sea, *viz.* about the thirtieth Degree of Lati- " tude on both Sides. Beſides, a farther Analogy between the " Winds of this Ocean, and the *Æthiopic*, appears in that, that " upon the Coaſts of *Peru* they are always much ſoutherly, like " as they are found near the Shores of *Angola*."

which

which oppoſes it from the Eaſt. For, whereas the Eaſtern Air retains its Heat ſome Time after the Sun is removed from it, it muſt always be rarefied to a greater Degree, and alſo to a greater Diſtance from the Place to which the Sun is vertical, than the weſtern Air is; and therefore the weſtern Air, being more ponderous, ſhould be an Over-balance for the eaſtern, and drive the Current before it.

But it is to be obſerved, that we are not to conſider the Point to which the Sun is vertical, but the Point of greateſt Rarefaction (which, upon Account of the Sun's Motion to the Weſtward, lies always to the Eaſtward;) and then ſee, which Side of the Column of Air, incumbent over that Point, ſuſtains the greater Preſſure from the neighbouring Columns. Now, although the Air is rarefied even farther to the Eaſt of this Point, than to the Weſt, yet ſtill, if we ſuppoſe the Point to keep its Place, the Air over it will ſuſtain an equal Degree of Preſſure on each Side. For, ſince no Column can be aſſigned on the weſtern Side, but one alſo on the eaſtern may be found under an equal Degree of Rarefaction, and therefore of the ſame ſpecific Gravity; and ſince Fluids of equal Heights, and the ſame ſpecific Gravities (whatever be the Breadth of their Columns) preſs equally againſt equal Objects, (Chap. I, §. 9.) 'tis very evident, that the Column of Air, over the Point

Point of greateſt Rarefaction, is preſſed equally both Ways; and ſo, upon this Suppoſition, each Wind will blow towards that Point with equal Force. But, if we ſuppoſe the Point of greateſt Rarefaction to ſhift towards the Weſt, we ſhall find, that this *Æquilibrium* will by that Means be deſtroyed, and the Motion of the Air (upon the whole) determined that Way. For let us conſider any Portion of the weſtern Air approaching towards the Point of greateſt Rarefaction, if that Point ſhifts, in the mean Time, towards the Weſt, then will that Portion reach the ſaid Point ſooner than it otherwiſe would have done; thereby loſing a Part of its Motion, by which Means the weſterly Current will be diminiſh'd. Again, if, while the Eaſt Wind blows towards the Point of greateſt Rarefaction, that Point moves on before it, then will the eaſtern Air have a greater Quantity of Motion, than it otherwiſe would have had; that, which ſhould have been an Impediment to it, being, upon this Suppoſition, in ſome Meaſure withdrawn; and ſo the Eaſt Wind will be augmented. Thus, the Weſt Wind having its Force leſſen'd by the Motion of the Sun, and the Eaſt one being increaſed, the latter at length as it were abſorbs the former, and carries it away in its own Direction.

2. On each Side of the *Equator*, to about the thirtieth Degree of Latitude, the Wind is found

found to vary from the Eaſt Point, ſo as to become North-eaſt on the northern Side, and South-eaſt on the ſouthern.

The Reaſon of which is, that, as the *equatorial* Parts are hotter than any other, both the northern and ſouthern Air ought to have a Tendency that Way; the northern Current therefore, meeting in this Paſſage with the eaſtern, produces a North-eaſt Wind on that Side; as the ſouthern Current, joining with the ſame, on the other Side the *Equator*, forms a South-eaſt Wind there.

Theſe two Propoſitions are to be underſtood of open Seas, and of ſuch Parts of them as are diſtant from the Land; for near the Shores, where the neighbouring Air is much rarefied, by the Reflection of the Sun's Heat from the Land, it frequently happens otherwiſe; particularly,

3. On the *Guinea* Coaſt, the Wind always ſets in upon the Land, blowing weſterly inſtead of eaſterly. This is becauſe the Deſerts of *Africa* lying near the *Equator*, and being a very ſandy Soil, reflect a greater Degree of Heat into the Air above them. This therefore being rendered lighter than that which is over the Sea, the Wind continually ruſhes in upon the Land to reſtore the *Æquilibrium*.

4. That Part of the Ocean, which is called the *Rains*, is attended with perpetual Calms, the Wind ſcarce blowing ſenſibly either one Way

Way or other. (See its Situation deſcribed in * 115 Note, Page 115, Remark 7th.) For this Tract being placed between the weſterly Wind blowing from thence towards the Coaſt of *Guinea*, and the eaſterly Wind blowing from the ſame Place to the Weſtward thereof, the Air ſtands *in Æquilibrio* between both, and its Gravity is ſo much diminiſhed thereby, that it is not able to ſupport the Vapour it contains, but lets it fall in continual Rain, from whence this Part of the Ocean has its Name.

5. There is a Species of Winds, obſervable in ſome Places within the *Tropics*, called by the Sailors *Monſoons*, which during ſix Months of the Year, blow one Way; and the remaining ſix the contrary.

The Occaſion of them in general is this: When the Sun approaches the northern *Tropic*, there are ſeveral Countries, as *Arabia*, *Perſia*, *India*, &c. which become hotter, and reflect more Heat than the Seas beyond the *Equator*, which the Sun has left; the Winds therefore, inſtead of blowing from thence to the Parts under the *Equator*, blow the contrary Way; and when the Sun leaves thoſe Countries, and draws near the other *Tropic*, the Winds turn about, and blow on the oppoſite Point of the Compaſs.

The Regularity of theſe Winds making them more than ordinarily uſeful in Navigation, they are from thence called *Trade* Winds.

Winds. As to other Circumſtances relating to them, and the particular Times and Places of the *Monſoons*, &c. ſee the Hiſtorical Account in the foregoing Note; all which might eaſily be ſolved upon the ſame Principle, had we *Data* to go upon, and were all the ſeveral Circumſtances of Situation, Heat, Cold, *&c.* ſufficiently known *.

From the Solution of the general *Trade* Winds, we may ſee the Reaſon, why in the *Atlantic* Ocean, a little on this Side the thirtieth Degree of North Latitude, or thereabouts, as was obſerved in the foregoing Diſſertation, there is generally a Weſt, or South-weſt Wind. For, as the inferior Air, within the Limits of thoſe Winds, is conſtantly ruſhing towards the *Equator*, from the North-eaſt Point, or thereabouts, the ſuperior Air moves the contrary Way; and therefore, after it has reached theſe Limits, and meets with Air, that has little or no Tendency to

* Some have thought, that the Regularity of the general *Trade* Winds is partly owing to the diurnal Motion of the Moon from Eaſt to Weſt. For, as the Sun renders the Air ſpecifically lighter by its Heat, ſo does the Moon by attracting it, in the ſame Manner, as it does the Sea, by raiſing the Tides: But it is to be obſerved, that as the Moon acts with the greateſt Force upon the ſuperior Parts of the Air, and as thoſe ſuperior Parts are unconfined, and therefore more at Liberty to ruſh in, in order to reſtore the *Æquilibrium*; it will from hence follow, that the ruſhing in of the ſuperior Parts of the Atmoſphere will principally contribute towards reſtoring the *Æquilibrium*; and ſo the Motion, produced below, will be very inconſiderable.

any

any one Point more than to another, it will determine it to move in the ſame Direction with itſelf.

In our own Climate we frequently experience, in calm Weather, gentle Breezes blowing from the Sea to the Land, in the Heat of the Day; which *Phænomenon* is very agreeable to the Principle laid down above: For the inferior Air over the Land being rarefied by the Beams of the Sun, reflected from its Surface, more than that which impends over the Water, which reflects fewer, the latter is conſtantly moving on to the Shore, in order to reſtore the *Æquilibrium*, when not diſturbed by ſtronger Winds from another Quarter †.

From what has been obſerved, nothing is more eaſy than to ſee, why the northern and ſouthern Parts of the World, beyond the Limits of the *Trade* Winds, are ſubject to ſuch

† In Confirmation of this, we have an eaſy, and very pertinent Experiment, related by Mr. *Clare*, in his *Motion of Fluids*. "Take, *ſays he*, a large Diſh, fill it with cold Water; "into the Middle of this put a Water-Plate, filled with warm "Water. The firſt will repreſent the Ocean; and the other "an Iſland, rarefying the Air above it. Blow out a Wax "Candle, and if the Place be ſtill, on applying it ſucceſſively "to every Side of the Diſh, the fuliginous Particles of the "Smoak, being viſible and very light, will be ſeen to move "towards the Plate, and riſing over it, point out the Courſe "of the Air from Sea to Land. Again, if the ambient Water "be warmed, and the Plate filled with cold Water, let the "ſmoaking Wick of the Candle be held over the Plate, and "the contrary will happen."

Variety

Variety of Winds. For the Air, upon Account of the leſſer Influence of the Sun in thoſe Parts, being undetermined to move towards any fixed Point, is continually ſhifting from Place to Place, in order to reſtore the *Æquilibrium*, wherever it is deſtroyed; whether by the Heat of the Sun, the riſing of Vapours, or Exhalations, the melting of Snow upon the Mountains, and a great Variety of other Circumſtances, more than can be eaſily enumerated.

We are told by Hiſtorians, of certain Caves that emit Wind; if ſo, it is when the included Air is rarefied by Heat, and therefore ruſhes out for want of Room; or, when the Preſſure of the external Air, incumbent upon the Mouth of the Cave, is diminiſhed, and ſo permits the internal Air to dilate itſelf, and iſſue out.

For more on this Subject, ſee *Nieuwentyt*'s Religious Philoſopher. *Regnault*'s Philoſophical Converſations. *Clare*'s Motion of Fluids. *Martin*'s Philoſophical Grammar. And the Authors referred to in Mr. *Johnſon*'s Quæſtiones Philoſoph. Cap. IV. Quæſt. 1, 2.

DISSERTATION VI.

Of the Formation and Aſcent of Vapours, and their Reſolution into Rain, Snow, *and* Hail.

THAT Vapours are raiſed from the Surface of Water by the Action of the Sun's Heat, is agreed on by all: But the Manner in which this is done, has ever been a Controverſy among Philoſophers; neither is it at this Time ſufficiently explained by any one.

If we conſult a *Carteſian* upon this Matter, he immediately tells us, that, by the Action of the Sun upon the Water, ſmall Particles of the Water are formed into hollow Spheres, filled with *Materia Subtilis*, and by that Means becoming lighter than an equal Bulk of Air, are eaſily buoyed up in it. But, as this *Materia Subtilis* is only a Fiction, this Solution is not to be regarded.

Dr. *Nieuwentyt*, and ſeveral other Philoſophers, who maintain, that Fire is a particular Subſtance, diſtinct from other Matter, account for the Formation and Aſcent of Vapours thus: They ſay, that the Rays of the Sun, or Particles of Fire ſeparated from them, adhering to Particles of the Water, make

up

up little Bodies, lighter than an equal Bulk of Air; which therefore, by the Laws of Hydroſtaticks, will aſcend in it, till they come to an Height where the Air is of the ſame ſpecific Gravity with themſelves. And, that Rain is produced by the Separation of the Particles of the Fire from thoſe of the Water; which laſt, being then left without Support, can no longer be ſuſtained by the Air, but falls down in Drops of Rain *.

This Opinion is liable to the following Difficulties; *Firſt*, Fire has never been yet proved to be a diſtinct Element, or a particular Subſtance†; and the Change of Weight in Bodies in chemical Preparations, heretofore thought to ariſe from the Adheſion of Particles of Fire, is found to proceed from the Adheſion of Particles of Air §.

Secondly, Should the above-mentioned Suppoſition be allowed, the fiery Particles, which are joined to the watery ones to buoy them up, muſt be conſiderably large, or elſe a very great Number muſt fix upon a ſingle Particle of Water; and then a Perſon, being on the Top of an Hill in a Cloud, would be ſenſible of the Heat, and find the Rain produced from that Vapour much colder than the Vapour it-

* See *Nieuwentyt*'s Religious Philoſopher, Contempl. 19.

† See the Authors referred to in Mr. *Johnſon*'s Quæſtiones Philoſoph. Cap. I. Quæſt. 30.

§ By Dr. *Hales*, in his *Vegetable Statics*.

ſelf: Whereas the contrary is evident to our Senſes; the Tops of Hills, though in the Clouds, being much colder than the Rain which falls below.

Beſides, the Manner in which the Particles of Water ſhould be ſeparated from thoſe of the Fire, ſo as to fall in Rain, is not eaſily to be conceived.

The moſt generally received Opinion is, That by the Action of the Sun, on the Surface of the Water, the aqueous Particles become formed into Bubbles, filled with a *Flatus*, or *warm Air*, which renders them ſpecifically lighter than common Air, and makes them riſe therein, till they meet with ſuch as is of the ſame ſpecific Gravity with themſelves *. But I aſk,

Firſt, How comes the Air in the Bubbles to be ſpecifically lighter than that without, ſince the Sun's Rays, which act upon the Water from whence they are raiſed, are equally denſe over all its Surface?

Secondly, If it could be poſſible for rarer Air to be ſeparated from the denſer ambient Air, to form the Bubbles (as Bubbles of ſoaped Water are blown up by warm Air from the Lungs, whilſt the ambient Air is colder and denſer) what would hinder the external Air from reducing that, which is incloſed in the Bubbles, immediately to the ſame Degree of

* Philoſophical Tranſactions, N° 192.

Cold-

Coldneſs, and ſpecific Gravity with itſelf; (Cold being readily communicated through ſuch thin Shells of Water.) By which Means, the Bubbles would become ſpecifically heavier than the circumambient Air, and would no longer be ſupported therein; but fall down, almoſt as ſoon as they were formed?

Thirdly, If we ſhould grant all the reſt of the Suppoſition, yet the following Difficulty will ſtill remain. If Clouds are made up of Bubbles of Water filled with Air, why do not thoſe Bubbles always expand when the ambient Air is rarefied, and preſſes leſs upon them than it did before; and why are they not condenſed, when the ambient Air is condenſed by the Accumulation of the ſuperior Air? But if this Condenſation and Rarefaction ſhould happen to them, the Clouds would always continue at the ſame Height, contrary to Obſervation; and we ſhould never have any Rain.

The two laſt Opinions are more largely examined by Dr. *Deſaguliers*, in the Philoſophical Tranſactions, N° 407. After which he endeavours to eſtabliſh one of his own.

He obſerves, with Sir *Iſaac Newton*, that, when by Heat or Fermentation the Particles of a Body are ſeparated from their Contact, their repulſive Force grows ſtronger, and the Particles exert that Force at greater Diſtances; ſo that the ſame Body ſhall be expanded into a very large Space, by becoming fluid; and

may ſometimes take up more than a Million of Times the Room it did in a ſolid and incompreſſible State. "Thus, ſays he, if the "Particles of Water are turned into Vapour, "by repelling each other ſtrongly, and repel "Air more than they repel each other; Ag- "gregates of ſuch Particles, made up of Va- "pour and Vacuity, may riſe in Air of different "Denſities, according to their own Denſity "depending on their Degree of Heat." He obſerves farther, that Heat acts more powerfully on Water, than on common Air; for that the ſame Degree of Heat, which rarefies Air two Thirds, will rarefy Water near fourteen thouſand Times, changing it into Steam or Vapour, as it boils it. And in Winter, that ſmall Degree of Heat, which in Reſpect of our Bodies appears cold, will raiſe a Steam or Vapour from Water, at the ſame Time that it condenſes Air. Laſtly, he obſerves, That the Denſity and Rarity of this Vapour depends chiefly on its Degree of Heat, and but little on the Preſſure of the circumambient Air. From all which he infers, that the Vapour being more rarefied near the Surface of the Earth, than the Air is there by the ſame Degree of Heat, muſt neceſſarily be buoyed up into the Atmoſphere; and ſince it does not expand itſelf much, though the Preſſure of the incumbent Air grows leſs, at length it finds a Place where the Atmoſphere is of the ſame ſpecific Gravity

Gravity with itſelf, and there floats, till, by ſome Accident or other, it is converted again into Drops of Water, and falls down in Rain.

And to ſhew that Air is not neceſſary for the Formation of Steam or Vapour, he gives us this Experiment.

" ABCD (*Fig.* 37.) is a pretty large Veſ-
" ſel of Water, which muſt be ſet upon the
" Fire to boil. In this Veſſel muſt be ſuſpend-
" ed the Glaſs Bell E, made heavy enough
" to ſink in Water; but put in in ſuch a
" Manner that it be filled with Water when
" upright, without any Bubbles of Air at its
" Crown within, the Crown being all under
" Water. As the Water boils, the Bell will
" by Degrees be emptied of its Water, the
" Water in the Bell being preſſed down by
" the Steam which riſes from it; but, as that
" Steam has the Appearance of Air, in order
" to know whether it be Air or not, take the
" Veſſel off the Fire, and draw up the Bell
" by a String faſtened to its Knob at Top, till
" only the Mouth remains under Water;
" then as the Steam condenſes by the cold Air
" on the Outſide of the Bell, the Water will
" riſe up into the Bell at F, quite to the Top,
" without any Bubble above it; which ſhews
" that the Steam, which kept out the Water,
" was not Air."

This Hypotheſis, however plauſible it may appear, is attended with Difficulties, as well as the other. For,

Firſt, If the repulſive Power of the Particles of Water is ſufficiently augmented by Heat as ſuch (and that by a very ſmall Degree of it) to make them recede from each other, and become ſpecifically lighter than common Air; how comes it to paſs, that all the Particles of Water, as ſoon as, or before it boils, have not their repulſive Forces thus augmented, ſince they are all under a much greater Degree of Heat, than is neceſſary to raiſe Vapour?

Secondly, Allowing that they may riſe off from the Surface of the Water, and float in the circumambient Air, as being lighter than it, why do not their repulſive Forces, as they riſe up into the Air, and the ſuperincumbent Preſſure becomes leſs, drive them to greater Diſtances from each other, and ſo cauſe them to continue lighter than the Air about them at all Heights?

Thirdly, If the Preſſure of the Air about them, when near the Surface of the Earth, is not ſufficient to bring them into ſo cloſe Contact, as to form Drops of Water there, what Force will they find ſufficient for it, when they are carried aloft into a Region of Air, where the Preſſure is not near ſo great? The Doctor hints, that they are formed into Rain, "when "by the great Diminution of the ſpecific Gra-

"vity

" vity of the Air about a Cloud, it has a great " Way to fall, in which Caſe, he ſays, the " Reſiſtance of the Air, which increaſes as " the Square of the Velocity of the deſcend- " ing Cloud, cauſes the floating Particles of " Water to come within the Power of each " other's Attraction, and form ſuch big Drops, " as, being ſpecifically heavier than any Air, " muſt fall in Rain." But as the inferior Air, from the Cloud to a conſiderable Depth below it, cannot be ſuppoſed to leave it all at once, the Vapours muſt neceſſarily fall all the Way through a reſiſting *Medium*; ſo that the little Velocity the Cloud can acquire, while it is in the Form of Vapour, will never produce a Reſiſtance from the Air greater than the Preſſure which the Vapours ſuſtained below.

Laſtly, The Experiment, brought to make Way for this Hypotheſis, ſhews clearly, that Vapour, formed without the Aſſiſtance of Air, is immediately condenſed into Water again, as ſoon as it is ſuffered to cool: Which is widely different from what happens to Vapours buoyed up into the colders Regions of the Air, and floating there, till they fall in Rain.

Theſe are all, or the principal *Hypotheſes*, that have been framed for the Solution of this common *Phænomenon*: Which, as they ſeem inadequate to the Effects produced, and therefore unſatisfactory, I thought it not amiſs to ſug-

ſuggeſt to the Reader the chief Difficulties with which I conceive them to be attended. But as it is eaſier to pull down, than to build up; to overturn a weak *Hypotheſis*, than to raiſe and ſupport a ſtrong and *ſufficient* one; ſo, I muſt own, in this Caſe, I can think of no Way of accounting for the *Riſe* of Vapours, according to the received Principles of Philoſophy, or ſay *wherein* their Nature conſiſts. Upon this Account it is impoſſible I ſhould give a Philoſophical Account of their Reſolution into Rain. It muſt ſuffice juſt to mention the Cauſes which are obſerved to produce that Change.

The firſt is, Part of the Air beneath them being carried away by contrary Winds blowing from the ſame Place *, the Remainder becomes too light to buoy them up, and ſo the upper ones, in all Probability, precipitate down upon the lower ones, unite with them, and form Drops of Rain †. Another is, great Quan-

* Perhaps it may be thought, that when the Air, which impends over any Place, is carried away from thence by contrary Winds, the Vapours which float in it ſhould be carried away too; and ſo the few that remain ſhould be eaſily ſupported; or, if they fall, ſhould not produce much Rain.

Now, in anſwer to this, it muſt be conſidered, that as the inferior Air is carried away from any Place by contrary Winds, the Height of the Atmoſphere is diminiſhed thereby; wherefore, in all Probability, the ſuperior Air ruſhes in by a contrary Current to fill up the Vacuity at the Top, which bringing in its Contents of Vapour, affords a continual Supply for the Rain that falls.

† A remarkable Inſtance we have of this, in that Part of the *Atlantic* Ocean, which the Sailors call the *Rains*, (See Diſſert. V.) from

Quantities of them, being driven by the Winds againſt the Sides of Mountains, are thereby made to coaleſce, and ſo conſtitute rain *.

It

from the frequent Rains that fall there; the Occaſion of which is, that the Atmoſphere is diminiſhed by continual contrary Winds blowing from thence, ſo that it is not able to ſuſtain the Vapour it receives.

* Upon this depends the Difference of the Seaſons of the Year at *Malabar* and *Coromandel* in the *Eaſt-Indies*, and at *Jamaica* in the Weſt. See Dr. *Gordon*'s Diſcourſe on the Cauſes of the Change of Weather, Philoſophical Tranſactions, N° 17.—— "The Rivers of *Indus* and *Ganges*, ſays he, where they enter the "Ocean, contain between them a large *Cherſoneſus*, which is di- "vided in the Middle by a Ridge of Hills, which they call the "*Gate*, which run along from Eaſt to Weſt, and quite through "to Cape *Comori*. On the one Side is *Malabar*, and on the "other *Coromandel*. On the *Malabar* Side, between that "Ridge of Mountains and the Sea, it is after their Appella- "tion Summer from *September* till *April*; in which Time it is "always a clear Sky, without once, or very little, Raining. "On the other Side the Hills, on the Coaſt of *Coromandel*, it "is at the ſame Time their Winter, every Day and Night "yielding Abundance of Rain; and from *April* to *September* it "is on the *Malabar* Side their Winter, and on the other Side "their Summer: So that in little more than twenty Leagues "Journey in ſome Places, as where they croſs the Hills to St. "*Thomas*, on the one Side of the Hill you aſcend with a fair "Summer; on the other you deſcend with a ſtormy Winter. "The like is ſaid to be at Cape *Razalgate* in *Arabia*. And Dr. "*Trapham* relates the ſame of *Jamaica*, intimating that there "is a Ridge of Hills which runs from Eaſt to Weſt, through "the Midſt of the Iſland, and that the Plantations on the South "Side of the Hills have, from *November* to *April*, a continual "Summer, whilſt thoſe on the North Side have as conſtant a "Winter, and the contrary from *April* to *November*.

"From theſe and ſuch like Accounts it ſeems evident, that a "bare leſſening of the Atmoſphere's Gravity will not occaſion "Rain, but that there is alſo needful either a ſudden Change "of Winds, or a Ridge of Hills to meet the Current of the Air "and Vapours, whereby the Particles of the Vapours are driven "together, and ſo fall down in Drops of Rain. And hence

"it

It is generally thought, That if the Vapours in their Deſcent paſs through a Region of Air ſufficiently cold, they are there frozen into Icicles,

" it is, that whilſt the Wind blows from the North-eaſt, *viz.* " from *November* to *April* (ſee the Account of the Monſoons in the foregoing Diſſertation) " there are continual Rains in the " northerly Plantations of *Jamaica*, and on the Side of *Coro-* " *mandel* in the *Eaſt-Indies*, becauſe the Winds beat againſt " that Side of the Hills; and ſo there is fair Weather on the " other Side of theſe Hills, in *Malabar* and in the ſouthern " Plantations of *Jamaica*, there being no Winds to drive the " Vapours together. But in the ſoutherly Monſoons, *viz.* " from *April* to *November*, *Malabar* and the ſoutherly Plantati- " ons of *Jamaica* have Floods of Rain, the Wind beating " againſt that Side of the Hills, whilſt in *Coromandel*, and the " other Side of *Jamaica*, there is fair and clear Weather.

" This ſerves alſo to clear the *Singularity* of *Seaſons* in *Peru*, " beyond any other Parts of the Earth, and ſeems to be aſſigned " by *Acoſta* as the Cauſe of it. *Peru* runs along from the *Line* " Southward about a thouſand Leagues. It is ſaid to be divided " into three Parts, long and narrow, which they call *Lanos*, " *Sierras*, and *Andes*; the *Lanos*, or Plains, run along the South " Sea Coaſt; the *Sierras* are moſtly Hills with ſome Vallies; " and the *Andes* ſteep and craggy Mountains. The *Lanos* have " ſome ten Leagues in Breadth, in ſome Parts leſs, and in ſome " more; the *Sierras* contain ſome twenty Leagues in Breadth, " the *Andes* as much, ſometimes more, ſometimes leſs; they " run in Length from North to South, and in Breadth from " Eaſt to Weſt. This Part of the World is ſaid to have theſe " remarkable Things; 1. All along the Coaſt, in the *Lanos*, it " blows continually with one only Wind, which is South and " South-weſt, contrary to that which uſually blows under the " torrid Zone. 2. It never rains, thunders, ſnows, or hails in " all this Coaſt, or *Lanos*, though there falls ſometimes a ſmall " Dew. 3. Upon the *Andes* it rains almoſt continually, though " it be ſometimes more clear than others. 4. In the *Sierras*, " which lie between both Extremes, it rains from *September* to " *April*, but in the other Seaſons it is more clear, which is " when the Sun is fartheſt off, and the contrary when it is " neareſt. Now the Reaſon of all ſeems to be this; The eaſt- " ern Breezes, which blow conſtantly under the *Line*, being " ſtopp'd

Icicles, and form Snow. But this Opinion ſeems to be falſe; becauſe it frequently ſnows when the Barometer is high, at which Time the Vapours cannot begin to fall. It is therefore more probable, that they are firſt frozen into Icicles, and by that Means ſhooting forth into ſeveral Points every Way from the Center (agreeable to the Nature of Freezing) loſe their Form; and ſo becoming ſpecifically heavier than Air, fall down, and in their Paſſage, ſeveral being congealed together, form Fleeces of Snow †.

Hail is evidently no other than Drops of Rain congealed into Ice. This happens, when in their Paſſage through the inferior Air they

" ſtopp'd in their Courſe by the *Sierras* and *Andes*, and yet the " ſame Breezes being to be found in the South-Sea beyond " *Peru*, as appears by the eaſy Voyages from *Peru* to the *Phi-* " *lippines*, a Current of Wind blows from the South on the " Plains of *Peru*, to ſupply the eaſtern Breeze on the South- " Seas, and there being but one conſtant Gale on theſe Plains, " and no contrary Winds or Hills for it to beat upon, this " ſeems to be the Reaſon why the Vapours are never or very " ſeldom driven into Rain. And the *Andes* being as high per- " haps in many Places as the Vapours aſcend in the higheſt " Degree of the Atmoſphere's Gravity, this may probably be " the Reaſon, why the eaſtern Breeze, beating conſtantly " againſt theſe Hills, occaſions Rain upon them at all Seaſons " of the Year. And the *Sierras* being it ſeems lower than the " *Andes*, therefore from *September* to *April*, when the Sun is " neareſt, and ſo the Atmoſphere's Gravity leſs, and the Va- " pours lower, they are driven againſt the *Sierras* into Rain."

The like is to be ſaid of other Countries. They are all, *cæteris paribus*, more or leſs rainy, as they are more or leſs mountainous. *Ægypt*, which is quite without Mountains, has ſeldom or never any Rain; but is plentifully watered by the *Nile*, which yearly riſes above its Banks, and overflows the Country.

† See a Diſcourſe on the Nature of Snow, Philoſophical Tranſactions, N° 92.

meet

meet with *nitrous* Particles, which are known to contribute greatly to Freezing. Their Magnitude is owing to a freſh Acceſſion of Matter, as they paſs along. Hence we ſee the Reaſon why Hail is ſo frequent in Summer, becauſe at that Time greater Quantities of Nitre are exhaled from the Earth, and float up and down in the Air.

If the Vapours, after they are exhaled from off the Waters, do not riſe very high in the Atmoſphere, but hover near the Surface of the Earth, they then conſtitute what we call a Fog. And, if they aſcend higher, they ſtill appear in the ſame Form to thoſe, who, being upon the Tops or Sides of Mountains, are at an equal Height with them; though to thoſe who are below, they appear as Clouds.

If they fall to the Earth, without uniting into Drops large enough to be called Rain, they are then ſaid to fall in Dew.

See farther on this Subject, *Derham*'s Phyſ. Theolog. Book I. Chap. 3. and Book II. Chap. 5. Spectacle de la Nature, Dialog. 21. and 23. *Nieuwentyt*, Contempl. 19. *Clare*'s Motion of Fluids. *Regnault*, Vol. III. Converſat. 7. *Muſſchenbroek* Epitom. Phyſ. Mathemat. Par. II. Cap. 24. *Melchior Verdries* Phyſic. Pars ſpecial. Cap. V. §. 8. And the Authors referred to in Mr. *Johnſon*'s Quæſtiones Philoſoph. Cap. IV. Quæſt. 7.

DIS-

DISSERTATION VII.

Of the Cauſes of Thunder and Lightning, and of the Aurora Borealis.

THOSE Philoſophers, who maintain that Vapours are buoyed up into the Air, by Particles of Fire adhering to them (as explained in the foregoing Diſſertation) account for the *Phænomena* of Thunder and Lightning, in the following Manner: They ſuppoſe, that from the, Particles of Sulphur, Nitre and other combuſtible Matter, which are exhaled from the Earth, and carried into the higher Regions of the Atmoſphere, together with the aſcending Vapours, is formed an inflammable Subſtance; which, when a ſufficient Quantity of fiery Particles is ſeparated from the Vapour by the Colliſion of two Clouds, or otherwiſe, takes Fire, and ſhoots out into a Train of Light, greater or leſs, according to the Strength and Quantity of the Materials.

The Opinion is falſe for the Reaſon mentioned in the foregoing Diſſertation, which plainly ſhow, that it is impoſſible the Vapours ſhould be attended with ſuch fiery Particles, as is here ſuppoſed.

Neither

Neither have we Occaſion to fly to ſuch an *Hypotheſis*; for, as Vapours exhaled from the Surface of the Water are carried up into the Atmoſphere; in like Manner, the *Effluvia* of ſolid Bodies are continually aſcending thither: Now we find by Experiment, that there are ſeveral inflammable Bodies, which, being mixed together in due Proportion, will kindle into Flame by Fermentation alone*, without the Help of any fiery Particles †. When therefore

* See the Theory of Fermentation explained in the following Diſſertation.

† Monſieur *Lemery* having covered up in the Earth about fifty Pounds of a Mixture compoſed of equal Parts of Sulphur, and Filings of Iron tempered with Water; after eight or nine Hours Time, the Earth, where it was laid, vomited up Flames. Hiſt. de l'Acad. 1700. p. 574.

From this Experiment we ſee the true Cauſe of the Fire of *Ætna* and *Veſuvius*, and other burning Mountains. They probably are Mountains of Sulphur, and ſome other Matter proper to ferment with it, and take Fire. From like Cauſes proceed the Heat of Bath-Waters, and other hot Springs.

Mix a ſmall Quantity of Gunpowder with Oil of Cloves, pour gently upon this Mixture two or three times as much Spirit of Nitre, and you will obſerve a bright Inflammation ſuddenly ariſing from it. A Mixture of the two Fluids alone will take Fire; the Powder is added only to augment the Inflammation.

Take two Pounds of Nitre, or refined Salt-Petre, well dried and reduced to the fineſt Powder, with a Pound of Oil of common Vitriol: From this Mixture may be drawn by Diſtillation a Spirit of Nitre capable of inflaming Oil of Turpentine. Mem. de l'Acad. 1726. p. 97, *&c.* Put into a Glaſs an Ounce of this Spirit of Nitre, with an Ounce of Oil of Vitriol; pour upon it an equal Quantity of Oil of Turpentine, and a very fine Flame will ariſe ſuddenly with a great Exploſion. When the Liquors are freſh the Effect is much greater. If we mix a Dram of the Spirit of Nitre, and three of Oil of Turpentine, with

fore there happens to be a proper Mixture of the *Effluvia* of ſuch Bodies floating in the Air, they ferment, kindle, and flaſhing like Gunpowder, occaſion thoſe Exploſions, and Streams of Fire, which we call Thunder and Lightning.

As to the particular Species of the *Effluvia*, which compoſe this Mixture, that cannot exactly be determined; they are thought to be chiefly ſulphureous and nitrous: *Sulphureous*, becauſe of the ſulphureous Smell which Lightning generally leaves behind it; and of that ſultry Heat in the Air which is commonly the Forerunner of it: *Nitrous*, becauſe we don't know of any Body ſo liable to a ſudden and violent Exploſion, as Nitre is *.

The

with only one of the Spirit of Vitriol, the Mixture will take Fire immediately. If the ſame Experiment be made with the Balm of *Mecca*, a ſudden Flame will ariſe, with a Noiſe like that of the Report of a Gun.

There are two celebrated Experiments of this Kind, though they do not come up exactly to the preſent Purpoſe, becauſe they will not ſucceed, unleſs the Ingredients be firſt heated, the one of *Aurum fulminans*, and the other of *Pulvis fulminans*. The firſt is a Mixture of Salt of Tartar, and Gold diſſolved by *Aqua Regia*. A ſmall Quantity of this, if put into a Braſs Spoon, and heated with the Flame of a Candle, cauſes a ſudden Noiſe reſembling that of Thunder; and goes off with great Violence. The other is a Mixture of three Parts of Nitre, two of Salt of Tartar, and one of Sulphur. This Mixture, when ſet upon the Fire, and warmed to a certain Degree, is diſſipated all on a ſudden with great Thundering, like the *Aurum fulminans*.

See an account of Exhalations taking Fire of their own Accord in Coal-Pits. *Power*'s Experimental Philoſophy, p. 62. and 181.

* Dr. *Liſter* is of Opinion, That the Matter both of Thunder and Lightning, and alſo of Earthquakes, is the *Effluvia* of the

The Effects of Thunder and Lightning are owing to the ſudden and violent Agitation the Air is put into thereby, together with the Force of the Exploſion*; and not to Thunderbolts falling from the Clouds, as ſuppoſed by the Vulgar †.

Pyrites; as he is, that the Matter of *Volcano*'s is the *Pyrites* itſelf. This is a Mineral that emits copious Exhalations, and is exceedingly apt to take Fire upon the Admiſſion of Moiſture. See the Doctor's Defence of his Notion in the Philoſophical Tranſactions, N° 157. He thinks this may be the Reaſon why *England* is ſo little troubled with Earthquakes, and *Italy*, and almoſt all Places round the *Mediterranean* Sea, ſo very much, *viz.* becauſe the *Pyrites* are rarely found in *England*; and where they are, they lie very thin, in Compariſon of what they do in thoſe Countries; as the vaſt Quantity of Sulphur, emitted from the burning Mountains there, ſeems to ſhew.

* Lightning is ſaid to have diſſolved Silver, without burning the Purſe it was in; and to have melted the Sword, without touching the Scabbard, and the like. The Occaſion of this may poſſibly be, that the Matter of the Exhalation may be ſo ſubtle and penetrating, that, as we ſee it happens with *Aqua Fortis*, or *volatile Salts*, it may paſs through ſoft Bodies without altering their Texture, while it ſpends its whole Force on hard ones, in which it finds the greater Reſiſtance.

† Some are inclined to think, that Thunderbolts are artificial, and that they were applied by the Ancients to ſome Uſe. What confirms them in their Opinion, is, that they are found more frequently where Sepulchres have been, than in other Places. And, as it was the Cuſtom of the Ancients to have their Arms buried with their Aſhes, they think they might be of ſome Uſe in War. Some are of Opinion, they were uſed in Sacrifices. See *Melchior Verdries* Phyſic. Pars ſpecial. Cap. V. §. 9. *Wedelius* Exercit. Med. Philol. Cont. II. Dec. I. p. 103. *Schminckius* Prof. Marpurg. Diſſertat. de Urnis Sepulchralibus, & Armis Lapideis, A. 1714. *Herman Nunningius* Sepulchret. Weſtphal. Mimigard. Gentil. p. 44. *Jo. Henr. Cohauſen* Oſſileg. Hiſtor. Phyſic. p. 44.

The

The Diſtance that Thunder is from us, may nearly be eſtimated by the Interval of Time between our ſeeing the Lightning, and hearing the Thunder. For, as the Motion of Light is ſo very quick, that the Time it takes up, in coming to us from the Cloud, is not perceptible; and as that of Sound is about a thouſand Feet in a Second; allowing a thouſand Feet for every Second, that paſſes between our ſeeing the one, and hearing the other; we have the Diſtance of the Cloud, pretty nearly, from whence the Thunder comes.

We ſometimes ſee Flaſhes of Lightning, though the Sky be clear and free from Clouds; in this Caſe they proceed from Clouds that lie immediately below our Horizon.

Of Affinity with the *Phænomena* of Lightning are thoſe of the *Aurora Borealis*, or *Northern Lights*, which, of late Years, have very frequently appeared in our Climate *. Theſe Lights differ ſo much from each other, that to give a Deſcription of one alone, would not be

* *Phænomena* of this Kind are reported to have been very frequent in *Greenland*, *Iceland*, and *Norway*, Countries near the Pole. The only ones, that are upon Record, as having appeared in *England*, before that of *March* the 6th, $17\frac{1}{18}$, are thoſe of *January* the 30th, 1560, *October* the 7th, 1564, *November* 14th and 15th, 1574, and a ſmall one ſeen near *London* on the 9th of *Auguſt*, 1708. On *November* the 16th, 1707, a ſmall one appeared in *Ireland*. Since that of *March* the 6th, $17\frac{1}{18}$, they have been, and ſtill continue very common.

ſufficient to acquaint the Reader with all the Circumſtances wherewith they are attended. I ſhall therefore collect together ſuch *Phænomena* as have been moſt generally obſerved, and reduce them to the ten following Propoſitions, adding in the Notes, by Way of *Specimen*, a full Account of that moſt remarkable *Aurora*, which was ſeen *March* the 6th, $17\frac{15}{16}$, as it was laid before the *Royal Society* by Dr. *Halley*, at their Requeſt†.

The

† "On *Tueſday* the 6th of *March*, in the year 1716 (the Afternoon having been very ſerene and calm, and ſomewhat warmer than ordinary) about the Time it began to grow dark (much about ſeven of the Clock) not only in *London*, but in all Parts of *England*, where the Beginning of this wonderful Sight was ſeen; out of what ſeemed a duſky Cloud, in the North-eaſt Parts of the Horizon, and ſcarce ten Degrees high, the Edges whereof were tinged with a reddiſh Yellow, like as if the Moon had been hid behind it, there aroſe very long luminous Rays, or Streaks perpendicular to the Horizon, ſome of which ſeemed nearly to aſcend to the *Zenith*. Preſently after, that reddiſh Cloud was ſwiftly propagated along the northern Horizon into the North-weſt, and ſtill farther weſterly; and immediately ſent forth its Rays from all Parts, now here, now there, they obſerving no Rule or Order in their riſing. Many of thoſe Rays, ſeeming to concur near the *Zenith*, formed there a *Corona*, or Image, which drew the Attention of all Spectators. Some likened it to that Repreſentation of Glory, wherewith our Painters in Churches ſurround the Holy Name of God. Others to thoſe radiating *Stars*, wherewith the Breaſts of *Knights* of the Order of the *Garter* are adorned. Many compared it to the *Concave* of the great *Cupola* of St. *Paul*'s Church, diſtinguiſhed with Streaks alternately light and obſcure, and having in the Middle a Space leſs bright than the reſt reſembling the Lanthorn. Whilſt others, to expreſs as well the Motion as Figure thereof, would have it to be like the Flame in an Oven, reverberated and rolling againſt the arched

" Roof

The moſt general *Phænomena* of an *Aurora Borealis* are theſe that follow.

1. In the northern Parts of the *Horizon*, there is commonly the Appearance of a very black

" Roof thereof: Some thought it liker to that tremulous Light, " which is caſt againſt the Cieling by the Beams of the Sun, re- " flected from the Surface of the Water in a Baſon, that's a little " ſhaken. But all agree, that this *Spectrum* laſted only a few " Minutes, and exhibited itſelf variouſly tinged with Colours, " Yellow, Red, and a duſky Green: Nor did it keep in the " ſame Place; for when firſt it began, it appeared a little to the " Northward of the *Zenith*, but by Degrees declining towards " the South, the long *Striæ* of Light, which aroſe from all " Parts of the Northern Semicircle of the *Horizon*, ſeemed to " meet together, not much above the Head of *Caſtor*, or the " northern *Twin*, and there ſoon diſappeared.

" After the firſt *Impetus* of the aſcending Vapour was over, " the *Corona* appeared no more; but ſtill, without any Order, " as to Time or Place, or Size, luminous *Radii*, like the former, " continued to ariſe perpendicularly, now oftner, and again " ſeldomer; now here, now there; now larger, now ſhorter. " Nor did they proceed, as at firſt, out of a Cloud, but oftner " would emerge at once out of the pure Sky, which was more " than ordinary ſerene and ſtill. Nor were they all of the ſame " Form. Moſt of them ſeemed to end in a Point upwards, " like erect Cones; others like truncate Cones, or Cylinders, " ſo much reſembling the long Tails of Comets, that at firſt " Sight they might well be taken for ſuch. Some of thoſe " Rays would continue viſible for ſeveral Minutes; when others, " and thoſe the much greater Part, juſt ſhewed themſelves, and " died away. Some ſeemed to have little Motion, and to ſtand, " as it were, fixed among the Stars, whilſt others, with a very " perceptible Tranſlation, moved from Eaſt to Weſt under the " Pole, contrary to the Motion of the Heavens; by which " Means they would ſometimes ſeem to run together, and at " other Times to fly one another.

" After this Sight had continued about an Hour and a half, " thoſe Beams began to riſe much fewer in Number, and not near " ſo high; and by Degrees, that diffuſed Light, which had il-

black Cloud; but it is evident that it is no real Cloud, becauſe the ſmalleſt Stars are viſible through it. This apparent Cloud is extended ſometimes farther towards the Weſt, than

" luſtrated the northern Parts of the Hemiſphere, ſeemed to " ſubſide, and ſettling on the Horizon, formed the Reſemblance " of a very bright *Crepuſculum*. That this was the State of this " *Phænomenon*, in the firſt Hours, is abundantly confirmed by " the unanimous Conſent of ſeveral. For, by the Letters we " have received from almoſt all the extreme Parts of the King- " dom, there is found very little Difference from what appeared " at *London* and *Oxford*; unleſs that in the North of *England* " and in *Scotland*, the Light ſeemed ſomewhat ſtronger and " brighter.

" Hitherto I have related the Obſervations of others; as to " myſelf, I had no Notice of this Matter, till about nine of the " Clock: I immediately perceived towards the South and South " weſt Quarter, that though the Sky was clear, yet it was tinged " with a ſtrange Sort of Light; ſo that the ſmaller Stars were " ſcarce to be ſeen, and much as it is when the Moon of four " Days old appears after Twilight. I perceived at the ſame " Time a very thin Vapour to paſs before us, which aroſe from " the preciſe Eaſt Part of the Horizon, aſcending obliquely, ſo " as to leave the *Zenith* about fifteen or twenty Degrees to the " Northward. But the Swiftneſs wherewith it proceeded was " ſcarce to be believed, ſeeming not inferior to that of Light- " ning; and exhibiting as it paſſed on a Sort of momentaneous " *Nubecula*, which diſcover'd itſelf by a very diluted and feint " Whiteneſs; and was no ſooner formed, but, before the Eye " could well take it, it was gone, and left no Signs behind it. " Nor was this a ſingle Appearance; but for ſeveral Minutes, " about ſix or ſeven Times in a Minute, the ſame was again and " again repeated; theſe Waves of Vapour regularly ſucceeding " one another, and at Intervals very nearly equal; all of " them in their Aſcent producing a like tranſient *Nubecula*.

" By this Particular we were firſt aſſured, that the Vapour we " ſaw, became conſpicuous by its own proper Light, without " the Help of the Sun's Beams; for theſe *Nubeculæ* did not " diſcover themſelves in any other Part of their Paſſage, but " only between the South-eaſt and South, where being oppoſite

" to

than the Eaſt; ſometimes farther towards the Eaſt, than to the Weſt; and generally takes up a Quarter of the Horizon, more or leſs.

2. The

" to the Sun, they were deepeſt immerſed in the Cone of the " Earth's Shadow; nor were they viſible before or after. " Whereas the contrary muſt have happened, had they bor- " rowed their Light from the Sun.

" On the weſtern Side of the northern Horizon, *viz.* between " Weſt and North-weſt, not much paſt ten of the Clock, I ob- " ſerved the Repreſentation of a very bright Twilight, con- " tiguous to the Horizon, out of which aroſe very long Beams " of Light, not exactly erect towards the *Vertex*, but ſomething " declining towards the South; which, aſcending by a quick " and undulating Motion to a conſiderable Height, vaniſhed in " a little Time; whilſt others, though at uncertain Intervals, " ſupplied their Place. But at the ſame Time, through all the " reſt of the northern Horizon, *viz.* from the North-weſt to " the true Eaſt, there did not appear any Sign of Light to ariſe " from, or join to, the *Horizon*; but what appeared to be an " exceeding black and diſmal Cloud, ſeemed to hang over all " that Part of it. Yet was it no Cloud, but only the ſerene Sky, " more than ordinary pure and limpid, ſo that the bright Stars " ſhone clearly in it, and particularly *Canuda Cygni*, then very " low in the North; the great Blackneſs manifeſtly proceeding " from the Neighbourhood of the Light, which was collected " above it. For the Light had now put on a Form quite different " from all that we have been deſcribing, and had faſhioned itſelf " into the Shape of two *Laminæ*, or Streaks, lying in a Poſition " parallel to the *Horizon*, whoſe Edges were but ill terminated. " They extended themſelves from the North by Eaſt to the " North-eaſt, and were each about a Degree broad; the under- " moſt about eight or nine Degrees high, and the other about " four or five Degrees over it; theſe kept their Places for a long " Time, and made the Sky ſo light, that I believe a Man might " eaſily have read an ordinary Print by the Help thereof.

" Whilſt I was viewing this ſurpriſing Light, and expecting " what was farther to come, the northern End of the upper *La-* " *mina* by Degrees went downwards, and at length cloſed with " the End of the other that was under it, ſo as to ſhut up on the " North Side an intermediate Space, which ſtill continued open

2. The upper Edge of this Cloud (which is ſometimes within leſs than ſix Degrees of the Horizon, and ſometimes forty or fifty above it)

" to the Eaſt. Not long after this, in the ſaid included Space, I " ſaw a great Number of ſmall Columns, or whitiſh Streaks, to " appear ſuddenly erect to the Horizon, and reaching from the " one *Lamina* to the other; which inſtantly diſappearing, were " too quick for the Eye, ſo that I could not judge, whether they " aroſe from the under, or fell from the upper; by their ſudden " Alterations, they made ſuch an Appearance, as might well " enough be taken to reſemble the Conflict of Men in Battle.

" And much about the ſame Time, there began on a ſudden " to appear, low under the Pole, and very near due North, three " or four lucid *Areas*, like Clouds, diſcovering themſelves in " the pure but very black Sky, by their yellowiſh Light. Theſe, " as they broke out at once, ſo after they had continued a few " Minutes, diſappeared as quick as if a Curtain had been drawn " over them: Nor were they of any determined Figure, but " both in Shape and Size might properly be compared to ſmall " Clouds illuminated by the full Moon, but brighter.

" Not long after this, from above the foreſaid two *Laminæ*, " there aroſe a very great *Pyramidal* Figure, like a Spear, ſharp " at the Top, whoſe Sides were inclined to each other, with an " Angle of about four or five Degrees, and which ſeemed to " reach up to the *Zenith*, or beyond it. This was carried with " an equable, and not very ſlow Motion, from the North-eaſt, " where it aroſe into the North-weſt, where it diſappeared, ſtill " keeping in a perpendicular Situation, or very near it; and " paſſing ſucceſſively over all the Stars of the *little Bear*, did " not efface the ſmaller ones in the Tail, which are of the fifth " Magnitude; ſuch was the extreme Rarity, and Perſpicuity " of the Matter whereof it conſiſted.

" This ſingle Beam was very remarkable for its Height above " all thoſe that, for a great While before, had preceded it, or " that followed it.

" It being now paſt eleven of the Clock, and nothing new " offering itſelf to our View, but repeated *Phaſes* of the ſame " Spectacle; I obſerved, that the two *Laminæ*, or Streaks, pa- " rallel to the *Horizon*, had now wholly diſappeared; and the " whole Spectacle reduced itſelf to the Reſemblance of a very " bright

it) is generally terminated with a very lucid Arch, from one to four or five Degrees broad, whoſe Center is below the Horizon. Sometimes

" bright *Crepuſculum* ſettling on the Northern *Horizon*, ſo as to " be brighteſt and higheſt under the Pole itſelf; from whence it " ſpread both Ways into the North-eaſt and North-weſt. Un- " der this, in the Middle thereof, there appeared a very black " Space, as it were the Segment of a leſſer Circle of the Sphere " cut off by the *Horizon*. It ſeemed to the Eye like a dark " Cloud, but was not ſo; for by the Teleſcope the ſmall Stars " appeared through it more clearly than uſual, conſidering " how long they were: And upon this, as a *Baſis*, our *Lumen* " *Auroriforme* reſted, which was no other than a Segment of a " Ring, or *Zone* of the Sphere, intercepted between two pa- " rallel leſſer Circles, cut off likewiſe by the *Horizon*; or the " Segment of a very broad *Iris*, but of one uniform Colour, " *viz.* a Flame-Colour inclining to Yellow, the Center thereof " being about forty Degrees below the *Horizon*. And above " this there were ſeen ſome Rudiments of a much larger Seg- " ment, with an Interval of Sky between, but this was ſo ex- " ceeding feint and uncertain, that I could make no proper " Eſtimate thereof.

" I attended this *Phænomenon* till near three in the Morning, " and the Riſing of the Moon: But for above two Hours toge- " ther it had no Manner of Change in its Appearance, nor Di- " minution, nor Increaſe of Light; only ſometimes, for very " ſhort Intervals, as if new Fuel had been caſt on a Fire, the " Light ſeemed to undulate and ſparkle not unlike the riſing of " a vaporous Smoak out of a great Blaze when agitated. But " one Thing I aſſured myſelf of, that the *Iris*-like Figure did by " no Means owe its Origin to the Sun's Beams: For that about " three in the Morning, the Sun being in the Middle between " the North and Eaſt; our *Aurora* had not followed him, but " ended in that very Point where he then was: Whereas in the " true North, which the Sun had long paſſed, the Light re- " mained unchanged, and in its full Luſtre."

Appearances of this Kind have been taken Notice of both by *Pliny*, *Seneca*, and *Ariſtotle*; the laſt of which calls the vibrating Light near the *Zenith* Δαλοί; the more ſteady perpendicular Streams, Δοκοί; and the *Aurora* itſelf, from the apparent dark Cloud juſt below it Χάσμα. That *Aurora* which was obſerved by

times there are two or more of theſe Arches, one above another. In ſome, the Cloud is not terminated by an Arch, but by a long bright Streak

by Monſieur *Gaſſendi*, in *Provence*, on the 21ſt of *September*, in the Year 1621, was very remarkable at that Time.

He tells us, that about the End of Twilight in the Evening, when the Sky was very clear, and there was no Moon, there appeared in the North a Sort of a riſing Moon, which aſcending by Degrees, became intermingled with certain Streaks, as it were, or Rays perpendicular to the Horizon: And that at the ſame Time there appeared ſome ſmall *paſſing* whitiſh Clouds between the South and the Place of the Sun's ſetting in Winter: And that in the Place where the Sun ſets in Summer, a bright Redneſs ſeemed to ariſe in the Form of a *Pyramid*, which moved towards the ſetting of the Sun at the *Equinox*; and which was diſtinguiſhed into three ſeveral *Pyramids*, which in a little Time were confounded one with another, and at laſt diſappeared. When this Redneſs ceaſed, the northern Whiteneſs was riſen forty Degrees and more, that is about the Altitude of the Pole-Star, forming itſelf into an Arch, and taking up near ſixty Degrees of the *Horizon*. After this, certain *Chevrons*, or Columns of Rays, ſome whiter, and ſome a little darker, began more plainly to be diſtinguiſhed, of about two Degrees in Breadth, and perpendicularly poſited; ſo that all that Part appeared as it were fluted. The whole Circumference immediately appeared ſcalloped; and then ſome of thoſe Columns which were in the Middle, and that were the whiteſt, began as it were to leave their Places with great Impetuoſity, and in leſs than a Quarter of a Minute, raiſed themſelves almoſt to the Top, putting on the Form of *Pyramids*, which they would retain four or five Minutes. It was about nine of the Clock when the Arch of Whiteneſs began to decreaſe or ſink; at which Time certain very white Streams of Smoak began to iſſue out from the Columns which were remaining under the *Pyramids*, and darting upwards with very great Rapidity through the *Pyramids*, like Javelins, vaniſhed immediately when they came to the Tops of them. This Spectacle laſted about an Hour; after which the Whiteneſs ſunk down to about ſix Degrees of the *Horizon*. *Vide Abregé de Gaſſendi*, Tom. V. p. 245.

This

Streak of Light, lying parallel to the *Horizon*. The Limb of this luminous Arch, or parallel Streak, is not always even and regular, but ſinks lower in ſome Parts than in others.

3. Out of this Arch proceed Streams of Light, generally perpendicular to the *Horizon*, but ſometimes a little inclined to it. Moſt of them ſeem to end in a Point, like *Pyramids* or *Cones*; and often very much reſemble the Tails of *Comets*. Sometimes there is no luminous Arch, nor Streak of Light; and then the Streams ſeem to iſſue out from behind the dark Cloud, being diſtinct from each other at their *Baſes*.

4. The upper Ends of the Streams inceſſantly appear and vaniſh again, as quick as if a Curtain were drawn backwards and forwards before them; which ſometimes cauſes ſuch a ſeeming Trembling in the Air, that you would

This *Phænomenon* appeared not only to *Gaſſendi*, in *Provence*, but was ſeen at Places very diſtant from thence, as *Toloſe*, *Montaubon*, *Bourdeaux*, *Grenoble*, *Dijon*, *Paris*, and *Roan*, &c. and at moſt other Places in *France*, and the neighbouring Countries, that lie to the Northwards of *Provence*, unleſs where it was intercepted by Clouds; but no where in ſuch as lie at any great Diſtance to the Southward of it.

Monſieur *Gaſſendi* is thought to have given the Name of *Aurora Borealis* to this *Phænomenon*; but this is obſerved by Monſieur *Mairan*, to be a Miſtake. See Mr. *Mairan*'s Phyſical and Hiſtorical Treatiſe of the *Aurora Borealis*, in the *Memoires de l'Academie Royale des Sciences*, *Année* 1731, or an Abſtract of it in Philoſoph. Tranſact. N° 431.

imagine

imagine the upper Part of the Heavens to be, as it were, in Convulſions *.

5. They ſometimes ſeem to meet in the *Zenith*, or more commonly to the Southward of it, about ten Degrees, more or leſs (ſometimes they deviate a little to the South-eaſt of the *Meridian*, and ſometimes to the South-weſt;) and there curling round, in ſome Meaſure imitate Flame confined under an Arch; and being frequently tinged with various Orders of Colours, exhibit a moſt beautiful Appearance, reſembling a Canopy finely painted †, of about ten or twenty Degrees in Breadth.

In many *Aurora*'s, there are Streams aſcending from thoſe Parts of the Heavens, which lie ſeveral Degrees to the Southward of the Canopy; and in ſome they appear to ariſe, though very rarely, almoſt as large and numerous from the ſouthern, as from the northern Parts of the Horizon.

6. The Height of the *Aurora Borealis* is very great; for that of *March* the 6th, $17\frac{15}{16}$, was viſible from the Weſt Side of *Ireland*, to the Confines of *Ruſſia* and *Poland* on the Eaſt, and perhaps farther; extending at leaſt over thirty Degrees of Longitude, and in La-

* See their Motions well deſcribed in the Account we have of an *Aurora* in the Philoſoph. Tranſact. N° 395. Art. 2.

† See the various Colours of the Canopy, as well as thoſe of an *Aurora* itſelf, accurately deſcribed by *Greenwood*, in Philoſoph. Tranſact. N° 418. Art. 1.

titude

titude from about the fiftieth Degree over almoſt all the North of *Europe*, and at all Places exhibiting, nearly at the ſame Time, the ſame Appearances.

7. Theſe Appearances have always been very frequent (as far as the Accounts we have of them inform us) in Countries that lie in, or near the *frigid Zone*, but very rare in thoſe of our Latitude; they are now become very frequent with us, but always ſeem to proceed from the North; and are as yet unknown to the Inhabitants of the more ſouthern Parts of our *Hemiſphere*. Whether they are ſeen to thoſe, who inhabit in, or near the other *frigid Zone*, is to us unknown *.

8. In ſome there are Trains of Light running horizontally, ſometimes from the Middle to the Extremes, and ſometimes from one Extreme to the other. And from theſe Trains often ariſe Streams perpendicular to the *Horizon*, and accompanying them as they paſs along.

9. When all the Streaming is over, the *Aurora Borealis* commonly degenerates into a bright Twilight in the North, and there gradually dies away.

10. It is probable, that theſe *Phænomena* often happen in cloudy Nights, though we are not ſenſible of them; for 'tis obſervable, that

* They are now known to be as frequent in the ſouthern Regions as they are in the northern. See *Wales*'s Obſervations, in Capt. *Cook*'s Second Voyage, p. 209, 218, 343, 344.

in

in ſuch Nights there is frequently more Light than what uſually proceeds from the Stars alone.

The moſt obvious Solution of the *Aurora Borealis*, or at leaſt what would appear ſo, to ſuch as have only attended to the Circumſtances of ſome particular ones, and which has accordingly been affixed by ſeveral * to their Accounts of the *Aurora*'s they have ſeen, is that it is a thin Nitro-ſulphureous Vapour, raiſed in our Atmoſphere conſiderably higher than the Clouds; that this Vapour by Fermentation takes Fire, and the Exploſion of one Portion of it kindling the next, the Flaſhes ſucceed one another, till the whole Quantity of Vapour within their Reach is ſet on Fire.

* Profeſſor *Cotes*, at the End of his Deſcription of a *Phænomenon* of this Kind, inſerted in the Philoſophical Tranſactions, N° 395, obſerves, that, ſuppoſing a Fund of Vapours or Exhalations at a conſiderable Height above us to be diffuſed every Way into a large and ſpacious Plane, parallel to the *Horizon*, that Fund of fixed Matter by Fermentation will emit Streams; and that, if the Wind be ſtill, they will aſcend perpendicularly upwards; otherwiſe they will be inclined towards the Point of the Horizon which is oppoſite to that from which the Wind blows; and that they will appear, by the Rules of Perſpective, in the firſt Caſe, to converge to the Spectator's *Zenith*, in the other, to ſome Point not far from it; and that if this Fund of Vapours be ſituated more to the North than the South, it will produce Streams of Light, attended with ſuch Circumſtances as then appeared: But he does not ſay, why the Vapours ſhould be ſituated rather to the North than to the South, or proceed to account for all the *Phænomena* of the *Aurora Borealis* in general from thence.

Some

Some have thought, that Vapours rarefied exceedingly by ſubterraneous Fire, and tinged with ſulphureous Steams, might from thence be diſpoſed to ſhine in the Night, and riſing up to the Top of the Atmoſphere, or even beyond its Limits (as we find the Vapours of Gunpowder, when heated *in Vacuo*, will ſhine in the Dark, and aſcend to the Top of the Receiver, though exhauſted) might produce thoſe Undulations in the Air, which conſtitute this *Phænomena*.

But theſe *Hypotheſes* have been rejected, as inſufficient; it having been thought impoſſible to account for all the Circumſtances of the *Aurora* by them *.

* In the Commentaries of the Academy of Sciences at *Peterſburgh*, I find a late Solution of the *Aurora Borealis* from Exhalations fermenting and taking Fire in the Atmoſphere, which the Author, *Chr. Maier*, ſays, occaſion the Appearance of the lucid Arch in the North, and thinks that the Streams, which ſeem to iſſue from thence, are no other than the Light of that Arch reflected to us from the under Side of ſome thin Clouds that lie above it. As to its appearing in the North rather than in the South, he ſuppoſes that may be owing to the Cold of thoſe Regions condenſing the Exhalations, and thereby rendering them more liable to ferment than they are in the ſouthern; but acknowledges ingenuouſly, that he has no Reaſon to ſuppoſe this, but its being neceſſary to his Solution. At the End he tells us, That it was known in ancient Times as well as lately: But omits taking Notice, that it appears much oftener of late Years in our Climate than it uſed to do; and ſo avoids accounting for that Particular. *Vide Commentar. Academ. Scientiar. Imperial. Petropolitan.* Tom. I. p. 351.

Dr.

Dr. *Halley* therefore has Recourſe to the *magnetic Effluvia* of the Earth, which he ſuppoſes to perform the ſame Kind of Circulation with regard to it, as the *Effluvia* of any particular *Terrella* * do with reſpect to that, *viz.* that they enter the Earth near the South Pole, and pervading its Pores, paſs out again at the ſame Diſtance from the northern: And thinks, they may ſometimes, by the Concourſe of ſeveral Cauſes very rarely coincident, and to us as yet unknown, be capable of producing a ſmall Degree of Light, either from the greater Denſity of the Matter, or perhaps from the greater Velocity of its Motion; after the ſame Manner, as we ſee the *Effluvia* of *Electric* Bodies emit Light in the Dark.

Monſieur *de Mairan* has given us a Phyſical and Hiſtorical Treatiſe of the *Aurora Borealis*, and endeavours to prove that it is owing to the *Zodiacal Light*, or the Atmoſphere of the Sun, ſpread on each Side of it along the *Zodiac* in the Form of a *Pyramid*. This, he ſays, is ſometimes extended to ſuch a Length as to reach beyond the Orbit of our Earth, and then mixing itſelf with our Atmoſphere, and being of an *Heterogeneous* Nature, produces

* A round Magnet, ſo called from the Reſemblance it bears to the Earth.

the

the ſeveral Appearances, which are obſerved in the *Aurora Borealis* *.

I have juſt mentioned theſe two Solutions, becauſe they came from two very ingenious Philoſophers; though I doubt not but the Reader will agree with me, that they are much too fine ſpun to hold, and that they are no other than the ingenious *Reveries* of Perſons determined to frame an Hypotheſis at any Rate. I queſtion not, but we may find Materials for the *Aurora Borealis*, without going ſo far for them, as theſe Gentlemen have done; and in particular that we have no Occaſion to have Recourſe either to the *magnetic Effluvia* of the Earth, or the *Zodiacal* Light, the Nature of both which we are wholly unacquainted with. The Materials employed in the firſt Solution (I mean ſuch *Effluvia* as are continually exhaled from the Surface and Bowels of the Earth) if rightly conſidered, will afford a very eaſy and natural one, as I ſhall endeavour to ſhew in the following Manner.

Firſt, we are aſſured by Experiment, that there are ſome Steams (as inflammable ſulphureous ones) which are capable of ſo great a Degree of Expanſion, that they will render themſelves lighter than the Air they float in, though

* See his Account at large, referred to at the end of Note, Page 251.

it be as rare, as it can be made by Art; for they will riſe to the Top of the Receiver, when exhauſted *, that is, when as much Air, as is poſſible, is drawn out †: Such Steams therefore, or Exhalations, riſing out of the Earth from Mines, Volcano's, *&c.* muſt neceſſarily be buoyed up towards the Top of the Atmoſphere, at leaſt, till they come to a Region, where the Air is as rare and expanded, as it can be made by the *Air Pump*, here below. This Height, according to Dr. *Halley*'s Computation §, (which he founded upon the Air's Elaſticity) is about forty or fifty Miles: But probably it is much greater; for the Air, which is higher than Vapours, and other *Heterogeneous* Matter, that is not elaſtic, riſe to, being much purer than any we can make Experiments upon, may be indued with a much greater Degree of Elaſticity, and ſo the Atmoſphere may be conſiderably higher, than what he, upon that Principle, computes it to be.

Secondly, Theſe *Effluvia* being raiſed to the Top of the Atmoſphere, or near it, and floating there, will neceſſarily be carried towards the polar Parts thereof, for the following Reaſons. 1. Becauſe the ſeperior Current

* See Philoſoph. Tranſact. N° 347. and 360.

† It is impoſſible to extract all the Air out of a Veſſel, becauſe it is by the Spring of the Air remaining in the Veſſel, that the Valves of the Pump are opened at each Stroke.

§ Philoſoph. Tranſact. N° 181.

of

of the Air, to a great Diſtance from the *Equator*, is that Way*. 2. We know from Experiment, that whatever ſwims upon a Fluid which revolves upon an *Axis*, is thereby carried towards that *Axis*. This is exactly the Caſe of theſe *Effluvia*, for they ſwim near the Top of the Atmoſphere which continually revolves about the *Axis* of the Earth; they muſt therefore neceſſarily be carried towards the polar Parts thereof.

Thirdly, Theſe *Effluvia* being collected together at, or near the Poles, and of an inflammable Nature, may eaſily be ſuppoſed to ferment, when they meet with other *heterogeneous* ones proper to produce ſuch an Effect; and emit Streams of Fire; which Streams will naturally riſe up into ſuch Parts of the Atmoſphere as are ſtill lighter than that wherein the *Effluvia* reſt, that is, directly upwards from the Center of the Earth. But, according to the Rules of *Perſpective*, thoſe Streams, though they really diverge, as *Radii* from a Center, will appear to a Spectator on the Surface of the Earth to converge towards a Point: Which Point will ſeem to be directly over his Head, if the Streams aſcend in right Lines from the Center of the Earth: But if they deviate all one Way from that Direction, the Point will be on

* As explained in Diſſertation V.

that Side the *Zenith* towards which they incline *.

To illuſtrate this; ſuppoſe ſeveral Strings hung down from the Cieling of a Room, and a Candle placed upon a Table below them, the Shadows of them all will converge towards the Point that is over the Candle. And, if they are made to incline, ſuppoſe all one Way, the Point of Convergency will remove from over the Candle, towards that Side of the Room to which the upper Ends of the Strings incline: Now if a perſon had viewed them from the Place where the Candle was, and referred their Places to the Cieling, they would have ſeemed to him to have converged towards the Point where the Shadows did.

And if the Streams ſpread themſelves as they ariſe, but not too much, they will nevertheleſs appear tapering towards the upper Ends, like *Cones* or *Pyramids*; juſt as the Sides of a long Walk ſeem to a Perſon that views them

* This may be made to appear in the following Manner: Let ADB (*Fig.* 38.) repreſent the *Concave* of the Heavens, AB the *Horizon*, C the Place of the Spectator, TV a luminous Subſtance ſending forth the parallel Streams EG, LM, NO, *&c.* Theſe Streams will all ſeem to converge towards the Point D, if that Point be taken ſuch, that the Line DC drawn from thence to the Spectator's Eye, be parallel to the Streams. For Inſtance, the Stream EG will ſeem to riſe from *e* to *g*, LM from *l* to *m*, and FH from *f* to *h*, and ſo of the reſt. And NO will appear wholly in D, the Place to which the reſt ſeem to converge. And if the Streams are as large, or ſomewhat larger at the upper Ends, than at the lower, they will ſtill appear leſs there, thoſe Ends being fartheſt from the Spectator's Eye.

from

from one End of it, or from a diſtant Place, to approach each other at that which is fartheſt from him.

This being premiſed, we may now account for the ſeveral *Phænomena* of the *Aurora Borealis* before laid down. As,

1. The Blackneſs of the Sky, which generally appears in the northern Parts of the *Horizon*, like a dark Cloud, is occaſioned by the Brightneſs of the luminous Matter of the *Aurora* juſt above it. That the Sky is clear here, is evident (as was obſerved before) becauſe the ſmalleſt Stars are ſeen through it.

2. The lucid Arch immediately above, is the luminous Matter of the *Aurora* itſelf, which ſometimes exhibits the Appearance of a Curve, ſometimes of a ſtrait Line, according to its Form and Situation in the Atmoſphere, though generally that of a Curve: For, by the Rules of *Perſpective*, when a ſtrait Line is diſtended horizontally, and above the Spectator's Eye, it ought to appear bent into a Curve, whoſe Center is below the *Horizon* *. Sometimes it appears on one Side the north Point, more

* Thus, when a Perſon ſtands fronting a Row of Houſes, and looks over the Tops of them, if they are all of an equal Height, that Houſe which is neareſt him, will ſeem to cut the Heavens in a Point that will be higher than where it is cut by any of the reſt; and the Points where the Heavens will ſeem to be cut by the Tops of thoſe, which are on the right and left Hand of the Spectator, will deſcend lower and lower, as the Houſes are farther off; ſo that the Points, taken all together, will repreſent a Curve.

than on the other; ſometimes regular, ſometimes irregular, as the various Circumſtances of the Air's Motion at the Top of the Atmoſphere, and of the Situation of the flaming Matter, may be.

3. The Streams of Light, iſſuing out of the lucid Arch, are Streams of Fire emitted upwards from the Matter of the *Aurora*, and ſeem, for the Reaſons already laid down, to converge towards the *Zenith* of the Spectator. Why they incline a little ſometimes from the Perpendicular, will be explained in the fifth Remark, where we account for the Situation of the Canopy. When no luminous Arch appears, it is probable that it is intercepted by the *Horizon*, or by the Vapours which float in great Quantities therein.

4. The trembling obſerved in the upper Part of the Heavens, is owing to the Quickneſs wherewith the Flaſhes ſucceed one another, and alſo to the irregular Motions and Agitations of the ſuperior Parts of the Atmoſphere.

5. So long as the luminous Matter of the *Aurora* is all of it towards the North of us, the Streams cannot ſeem to meet in a Point at the Top, as will appear to any one that conſiders the Figure referred to in the Note (p. 260;) but after it has advanced forwards, or become kindled over our Heads, then they appear to meet, and form the Canopy already deſcribed; and when it has paſſed further ſtill, they ſeem

to ariſe from all Parts; though they are much fainter on the ſouthern than on the northern Side, ſo long as the main Body of the *Aurora* remains on the northern Side of the Canopy, which it rarely, if ever, paſſes. The Reaſon why the Center of the Canopy is generally a few Degrees to the South of the Spectator's *Zenith* * is, becauſe the luminous Streams, which iſſue forth from the extreme Parts of the Subſtance of the *Aurora*, will naturally diverge a little from the middle ones; and, as thoſe which appear to us, proceed chiefly from the ſouthern Side (that being neareſt to us) the Point of Convergency will neceſſarily be placed to the South of our *Zenith*, according to what was ſaid above about the Inclination of the Strings hanging from the Cieling of a Room. If the Center of the Canopy is ſometimes to the Eaſtward, and ſometimes to the Weſtward of the Meridian, that depends upon the Motion of that Part of the Air, which is above the Subſtance of the *Aurora*, and through which the Streams paſs, as they riſe. This alſo it is that makes the Streams ſeem to ariſe ſometimes a little obliquely.

* According to this Theory, the Center of the Canopy will always be near the Spectator's *Zenith*, wherever he is; which I believe is the Caſe, for I have met with no Account where it is otherwiſe; and ſo every Spectator ſees a different Canopy, juſt as, when ſeveral Perſons are viewing a Rainbow, no two Perſons ſee the ſame Rainbow at the ſame Time. See Part III. p. 108.

6. The great Height of the *Aurora* is owing to the exceeding Lightneſs of the *Effluvia*, which compoſe the Subſtance of it (as explained above) and the darting of the Streams upwards, into Regions perhaps quite above the Atmoſphere, occaſions it to exhibit at very diſtant Places the ſame Appearances at the ſame Time.

7. That the *Aurora* appears near the *Pole*, and never at or near the *Equator*, is becauſe of the Tendency the Matter of it has towards the Poles, as explained above. And that it appears in Places more diſtant from the *Pole*, than it formerly did, is becauſe the *Effluvia*, which are now raiſed from the Earth, are prevented from approaching ſo near the polar Parts of the Atmoſphere, as they uſed to do; thoſe Parts being already ſtocked with others, which were formerly raiſed, and are now grown *effete* by frequent Fermentations and Exploſions.

8. The *horizontal* Trains of Light are the Subſtance of the *Aurora* juſt taking Fire, which runs from one Part to another, as in a Train of Gunpowder kindled in any one Part; and ſends up Streams perpendicularly from Places, where it meets with a greater Quantity of Matter than ordinary.

9. When the Matter of the *Aurora* is ſo far ſpent, as to emit no more Streams, it appears only as a bright ſteady Light in the North, which

which gradually dies away, for Want of freſh Fewel to ſupport it.

10. As the Vapours, of which Clouds are formed, never raiſe ſo high, as where the Matter of the *Aurora Borealis* floats; it is not at all inconſiſtent with the foregoing Theory, if it is ſometimes intercepted from our Sight, by the Interpoſition of Clouds below.

See farther on this Subject, *Ariſtot.* Meteor. Lib. I. Cap. 4, 5. *Plinii* Hiſtor. Natural. Cap. 26, 27. *Senec.* Queſt. Natural. Lib. I. *Lycoſt.* Prodigiorum ac Oſtentorum Chronicon, paſſim. *Julius Obſequens* de Prodigiis, Cap. 13, 43, 88. *Gaſſendi* Animadverſ. in *Diog. Laert.* Lib. X. p. 1157. *Cornelius Gemma* de divinis Nature Characteriſmis. *Nicephari* Hiſtor. Eccleſiaſt. Lib. XII. Cap. 37. *Iſid. Hiſpal.* Hiſtor. Goth. Tom. I. p. 65. Bibliothec Orientalis Clementino-Vaticana, Tom. I. p. 407. *Gregor. Tur.* paſſim. Mem. de Lit. de l'Acad. des Inſcriptions & Belles Lettres, Tom. IV. p. 431, Miſcellan. *Berolin.* Tom. I. p. 137. Theatr. Comet. *Staniſ. Lubienietz*, p. 264, 348. Mem. pour ſervir à l'Hiſtor. de *France*, Tom. I. p. 168. Mem. de l'Acad. Royal de Sciences, for almoſt each Year ſince 1716. Philoſoph. Tranſ. N° 305, 310, 320, 347, 348, 349, 351, 352, 363, 365, 368, 376, 385, 395, 398, 399, 402, 410, 418, 431; and the Authors referred to by Mr. *Johnſon*, in his Quæſt. Philoſoph. Cap. IV. §. 3.

DIS-

DISSERTATION VIII.

Of Fermentation.

HAving had Occaſion to mention ſome of the Effects of Fermentation, it may not be amiſs, before I put an End to theſe Diſſertations, to add a ſhort Account of the Nature of it, and to ſhew how thoſe Effects are produced by it.

Fermentation is a mutual Commotion of the conſtituent Particles of Bodies, one among another; and ariſes from an Inequality in their Attractions of Coheſion. Authors diſtinguiſh it into two Kinds; the one is that which happens when a Solid is diſſolved by a Fluid; the other is, when two Fluids, being mixed together, ferment with each other.

Thoſe Authors, who have treated of the firſt of theſe, tell us, That to cauſe a Fermentation between a Solid and a Fluid, ſeveral Circumſtances are neceſſary. Dr. *Freind* *, and *Keil*, † particularly, are of Opinion,

1. That the Particles of the Solid muſt attract thoſe of the Fluid with a greater Force, than the Particles of the Fluid attract one another.

* See his Chemical Lectures.

† See his Letter to Dr. *Cockburn*, *De Legibus Attractionis*.

2. That

2. That the Pores of the Solid muſt not be too ſmall to admit the Particles of the Fluid into them.

3. That the Body be of ſo looſe a Contexture, that the Force of Impact, with which the Particles of the Fluid ruſh into its Pores, may be ſufficient to diſunite its Parts.

4. That the Elaſticity of the Particles tends very much to promote, and augment the Fermentation.

Dr. *Boerhaave* makes alſo four Conditions requiſite *.

1. That there be a due Proportion between the Size of the Particles of the Fluid, and the Pores of the Body to be diſſolved.

2. That the Figure of the Particles of the Fluid have a determinate Relation to that of the Pores of the Solid.

3. That the Particles of the Fluid be ſufficiently ſolid, that their Moment, or Force of Action, may not be too weak.

4. The laſt Qualification he mentions, is a fit Diſpoſition of the Particles of the Fluid, when received into the Pores of the Solid, to make ſome Stay there, and not immediately to paſs through; but to act every Way upon the Solid, as they move towards the external Surface thereof.

* Dr. *Boerhaave*'s Chemiſtry, by *Shaw*, p. 344.

But

But we have no Occaſion to have Recourſe to ſo many Suppoſitions: If the Particles of the Solid attract thoſe of the Fluid with a greater Degree of Force than either thoſe of the Fluid, or thoſe of the Solid attract one another *, it is ſufficient; and there will follow a Diſſolution of the Body, as may clearly be demonſtrated from the Laws of *Mechanics*, whatever the other Circumſtances, relating to the Figure or Magnitude of Pores, *&c.* may be †.

When

* This may be thought an impoſſible Suppoſition, for the Force of Attraction of Cohefion being as the Surfaces of the attracting Particles, whatever Size or Form the Particles of the Solid and Fluid are of, there cannot be a greater Quantity of Surface between every two Particles, one of which is a Particle of the Solid, and the other a Particle of the Fluid, than there is between every two Particles, which are either both of the Solid, or both of the Fluid; and therefore the Particles of the Solid cannot attract thoſe of the Fluid with greater Force than either thoſe of the Solid or thoſe of the Fluid attract one another. But it is to be conſidered, that we are not ſo well acquainted with the Nature of the Attraction of Cohefion, as to determine *exactly* in what Manner, and by what Laws it acts. The Experiments made Uſe of for this Purpoſe, only ſhew that ſo long as we try them with the ſame Kind of Bodies, the Attraction is larger where the Contact is ſo. See Part I. Chap. III. But we have no Method of determining, whether the Difference of Attraction, which *various* Bodies exert upon one another, ariſes ſolely from a Difference in their Surfaces, or not.

† *Dem.* Thus, let *f*, *f*, *f*, *&c.* (*Fig.* 39.) repreſent a Series of the Particles of a Fluid, and *s*, *s*, *s*, *&c.* a Series of thoſe of a ſolid Body, contiguous to one another: And let the pricked Lines *ff*, *ff*, *&c.* repreſent the Forces of Attraction between the Fluid Particles one among another, and *ss*, *ss*, *&c.* thoſe of the ſolid ones among themſelves; and let the black Lines *fs*, *sf*, *&c.* expreſs thoſe which are between the fluid and ſolid Particles. Now, the latter Forces being by the Suppoſition ſtronger than the former, the fluid Particles will recede from

When a Solid is put into a Fluid, if their Particles have the above-mentioned Relation to each other, thoſe of the Solid being attracted with greater Force towards the Fluid, than they are the contrary Way, they will fall off from the Solid, and enter in between the Particles of the Fluid; and for the like Reaſon, thoſe of the Fluid will open to themſelves a Way in between thoſe of the Solid, and will ſeparate them from each other. Neither will their reſpective Motions ceaſe, unleſs their

from each other, and ſuffer thoſe of the Solid to enter in between them; and for the ſame Reaſon the ſolid Particles will give Way to thoſe of the Fluid. By which Means, the Diſtances repreſented by the pricked Lines becoming greater, the Attractions, which they expreſs, will be diminiſhed; ſo that the fluid Particles will enter quite in between the ſolid ones, and the ſolid ones between the fluid ones; and both of them together will conſtitute ſuch a Series, as is repreſented in *Figure* 40, in the middle Row *s*, *f*, *s*, *f*, *&c.* where the ſolid and fluid Particles lie mixed interchangeably one with another in a right Line. Now let it be ſuppoſed, that this Series is contiguous to one which conſiſts wholly of Fluid above it, as is expreſſed in the Figure, and to another below, conſiſting of ſolid Particles only. Every ſolid Particle in this Series will be attracted upwards with greater Force, than it is downwards; and every Fluid one with greater Force downwards than it is upwards, as appears by bare Inſpection of the Figure, where the black Lines, as in the former, expreſs the ſtronger Attractions, and the pricked ones the weaker. And, if we ſuppoſe the Number of Particles in the Solid and in the Fluid to be nearly equal, thoſe of the Fluid will not ſtop, till they have quite paſſed through the Solid: for they will always find a Series wholly conſiſting of ſolid Particles before them, whilſt that which they leave behind will be a Mixture of both. In like Manner, the ſolid ones will paſs quite through the fluid ones; for they will always meet with more fluid ones before them, than they leave behind within the Sphere of their own Attraction.

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Quantities be very unequal, till they are diffuſed uniformly one among another, as we may very eaſily conceive; for till then there will always be ſome Particles attracted with a greater Degree of Force one Way than they are another.

And if more of the Solid be added to this Fluid, the Particles of the Fluid will alſo enter into that Solid, till each is ſurrounded on all Sides with ſolid Particles, as far as its attractive Force reaches. After which the Fluid will (as they ſay) be *ſaturated*, and will diſſolve no more.

Again, if more of the Fluid be poured upon that Solid, the ſolid Particles will diffuſe themſelves farther into the Fluid, till each of them is encompaſſed with Particles of the Fluid, as far as its attractive Force extends; and then they will ſpread themſelves no farther.

But in either Caſe, if another Solid, or Fluid, the attractive Force of whoſe Particles differ from thoſe of the former, be added, a freſh Fermentation will begin, provided the attractive Forces between the Particles of the former Mixture, and of thoſe which are now added, have ſuch a Relation to each other, as is neceſſary to produce it.

Upon this Principle it ſhould ſeem, that a Fluid ſhould always be capable of diſſolving more than an equal Quantity of a Solid; and that a Solid ſhould be capable of entering in

and

and diffuſing itſelf through more than an equal Quantity of Fluid. The Reaſon why it is frequently not ſo, is, becauſe it commonly happens, that the Fluid and the Solid are not of equal ſpecific Gravities. When the Solid is heavieſt, ſo many of its Particles will not aſcend and enter into the Fluid, as would otherwiſe have done; and on the contrary, when the Fluid is heavieſt, the Weight of its Particles will be an Impediment to their riſing into, and diſſolving ſo much of the Solid, as it otherwiſe might have done.

We have no Occaſion to diſtinguiſh Fermentation into two Kinds, with regard to its Cauſes; for according to the foregoing Theory, whenever two Fluids, or a Solid and a Fluid, are put together, if the Particles of the one attract thoſe of the other, with greater Force than either thoſe of the one, or thoſe of the other attract themſelves, a Fermentation will *equally* enſue, the Cauſe being the ſame in both Caſes.

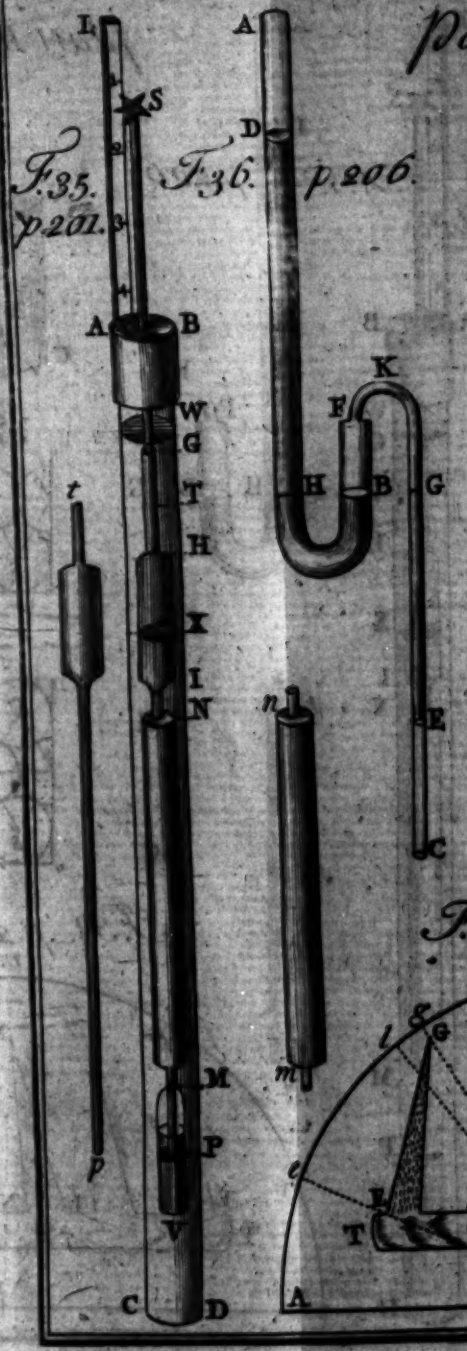
When two Fluids, or a Solid and a Fluid, ferment with each other, if the Agitation and *inteſtine* Motion of their Particles be very great, or continues a long Time, and if the Subſtance of them be of the inflammable Kind, they will, by continually rubbing one againſt another, be ſufficiently heated to take Fire, and burſt out into Flame; as was ſaid of the ſeveral Compoſitions mentioned in the foregoing Diſſertations.

See the Authors, who have explained and defended the old Solution, referred to by Mr. *Johnſon* in his Quæſtiones Philoſoph. Cap. III. Quæſt. 5, 6, 7.

The END *of the* SECOND PART.

A COM-

F.35. p.201.



F.36. p.206.

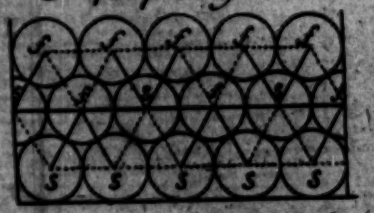
F.37. p.231.



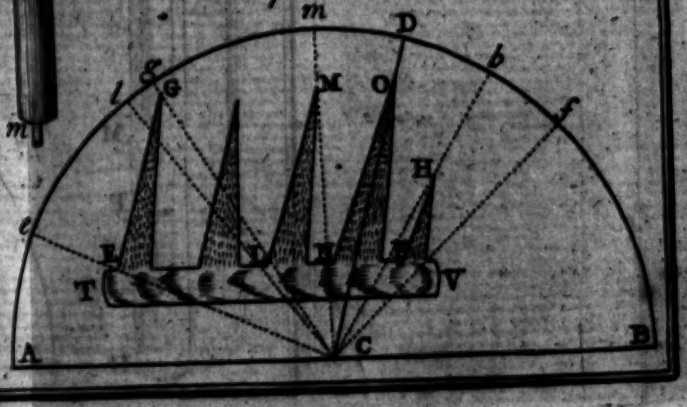
F.39. p.268

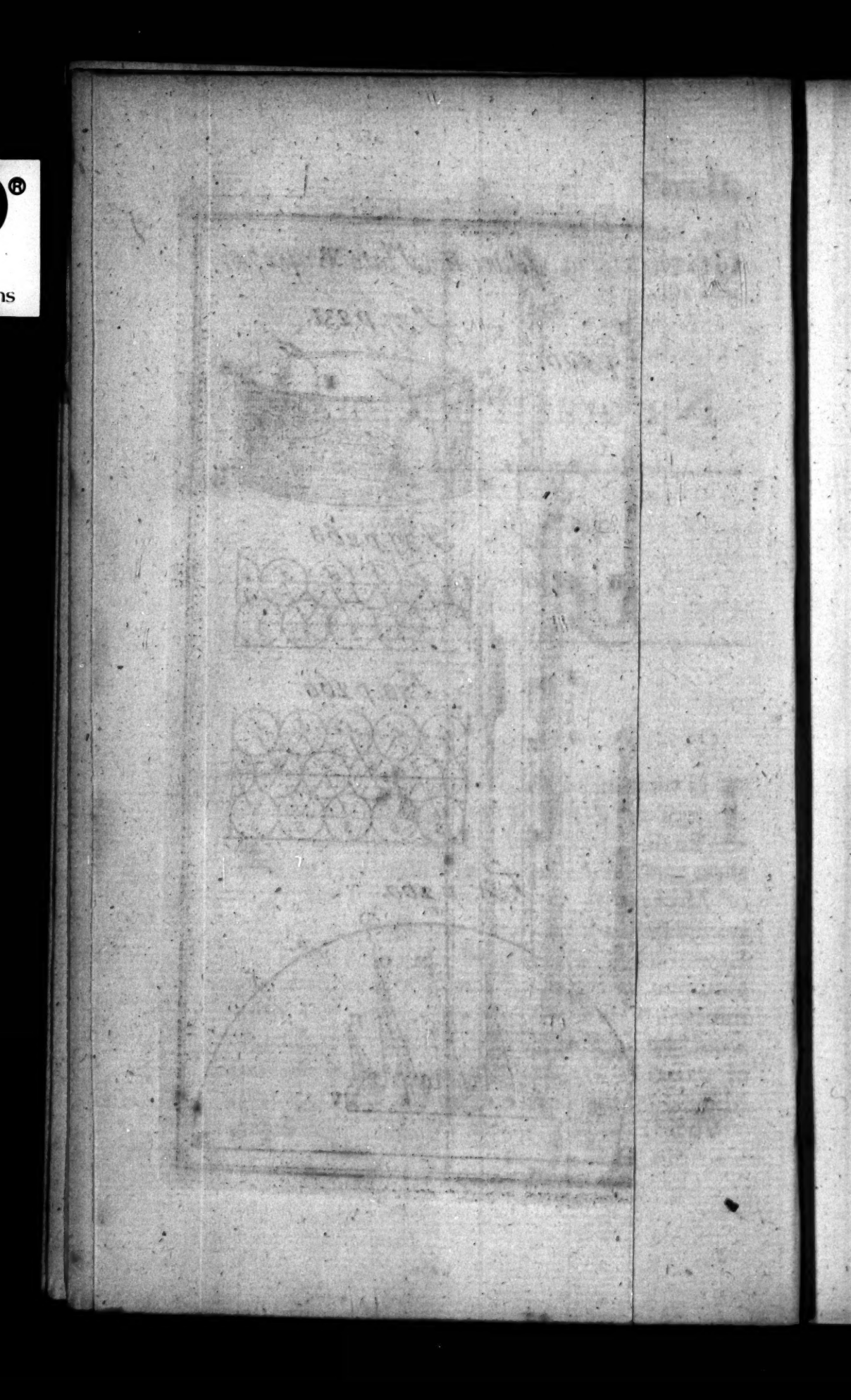


F.40. p.269.



F.38. p.260.





A COMPENDIOUS SYSTEM OF Natural Philoſophy.

PART III.

OPTICS.

CHAP. I.

Of the Nature and Propagation of Light.

IN treating of the Nature of Fluids, I have explained ſuch *Phænomena* as reſult from ſmall Particles of Matter collected together, and acted upon according to the Laws of *Mechaniſm*; the Order of my Deſign now brings me to ſhew how, according to the ſame Laws, ſuch *Phænomena* as reſult from the Emiſſion of infinitely ſmall Particles from luminous Bodies are produced; which *Phænomena*, being the Means whereby the Images of external Objects are repreſented to our Minds, by the Intervention of our Organs of

Sight, are for that Reaſon called *Optical*, and the Doctrine by which they are explained, the Science of *Optics* *.

Every viſible Body emits or reflects inconceivably ſmall Particles of Matter from each Point of its Surface, which iſſue from it continually, (not unlike Sparks from a Coal) in ſtrait Lines and in all Directions. Theſe Particles entering the Eye, and ſtriking upon the *Retina* (a Nerve expanded on the back Part of the Eye to receive their Impulſes) excite in our Minds the Idea of Light. And as they differ in Subſtance, Denſity, Velocity, or Magnitude †, they produce in us the Ideas of different Colours; as will be explained in its proper Place.

That

* The Science of Optics is generally divided into two Parts, *viz. Dioptrics*, under which is comprehended every thing that relates to the Appearances of Bodies ſeen through tranſparent Subſtances, and *Catoptrics*, or what relates to the ſeeing of Bodies by reflected Light. To theſe we may add a third, which properly comes under neither of the former Diſtinctions, and that is, *the Doctrine of Colours*, which explains every thing that relates to the Cauſes of the Diverſity of Colours obſervable in natural Bodies.

† It is more probable, that they differ either in Magnitude, or Denſity, than in Velocity or Subſtance. For, if the Difference of Colours ariſe from the different Velocity of the Rays of Light, then the Colours of Objects would appear changed to an Eye placed under Water, or within any *Medium* differing from the Air in Denſity: For when a Ray of Light paſſes out of a *Medium* into another of different Denſity, it undergoes an Alteration in its Velocity, as will be explained hereafter. And to ſuppoſe them to differ in Subſtance, is contrary to that Uniformity of Things, which is obſervable in the Univerſe; as well as repug-

nant

That the Particles, which conſtitute Light, are exceedingly ſmall, appears from hence, *viz.* that if a Hole be made through a Piece of Paper with a Needle, Rays of Light from every Object on the farther Side of it are capable of paſſing through it at once without the leaſt Confuſion; for any one of thoſe Objects may as clearly be ſeen through it as if no Rays paſſed through it from any of the reſt. Further, if a Candle is lighted, and there be no Obſtacle in the Way to obſtruct the Progreſs of its Rays, it will fill all the Space within two Miles of it every Way with luminous Particles, before it has loſt the leaſt ſenſible Part of its Subſtance thereby.

That theſe Particles proceed from every Point of the Surface of a viſible Body, and in all Directions, is clear from hence, *viz.* be-

nant to that *Homogeneity* in the primogeneal Parts of Matter, which from the Experiments hitherto made, is thought to exiſt every where. Whereas, if we ſuppoſe them to differ either in Magnitude or Denſity, nothing is more eaſy than to ſee how thoſe of the ſame Kind ſhould, however refracted, produce the ſame Colours; and alſo how thoſe which produce different Colours, ſhould ſuffer different Degrees of Refraction in paſſing through the ſame *Medium*. As to the firſt, it is ſelf-evident, becauſe Refraction cannot alter their Magnitudes or Denſities; as to the ſecond, it is probable, that the more intenſe and ſtronger Colours, the Rays of which ſuffer the leaſt Refraction, are produced by the larger, or more denſe Particles of Light: For, that ſuch Particles ſhould be leſs refracted than others, is quite conſonant to the Laws of Attraction of Coheſion, which, as it acts in Proportion to the Surfaces of Bodies only, muſt neceſſarily affect the larger, or the more denſe Particles leſs than it does the reſt; becauſe ſuch have larger *Momenta* or Forces in Proportion to their Surfaces, than others have.

cauſe where-ever a Spectator is placed with regard to the Body, every Point of that Part of the Surface which is turned towards him, is viſible to him. That they proceed from the Body in right Lines, we are aſſured, becauſe juſt ſo many and no more will be intercepted in their Paſſage to any Place, by an interpoſed Object, as that Object ought to intercept, ſuppoſing them to come in ſuch Lines.

The Velocity, with which they proceed from the Surface of the viſible Body, is no leſs ſurpriſing than their Minuteneſs: The Method whereby Philoſophers eſtimate their Swiftneſs, is by Obſervations made on the Eclipſes of *Jupiter*'s Satellites, which Eclipſes appear to us about ſeven Minutes ſooner than they ought to do by Calculation, when the Earth is placed between the Sun and him; that is when we are neareſt him, and as much later, when the Sun is between him and us, at which Time we are fartheſt from him; from whence it is concluded, that they require about ſeven Minutes to paſs over a Space equal to the Diſtance between the Sun and us, which is about eighty-one Millions of Miles *.

A Stream

* This affords us another Proof of the ſurpriſing Fineneſs of the Particles of Light; for the above-mentioned Velocity of the Rays is conſiderably more than a Million of Times greater than that of a Cannon Ball. Were they not therefore inconceivably ſmall, the Eye would be rather wounded than delighted with them;

A Stream of theſe Particles iſſuing from the Surface of a viſible Body in one and the ſame Direction, is called a *Ray of Light*.

As Rays proceed from a viſible Body in all Directions, they neceſſarily become thinner and thinner, continually ſpreading themſelves, as they paſs along, into a larger Space, and that in Proportion to the Squares of their Diſtances from the Body*; that is, at the Diſtance of two Spaces, they are four Times thinner, than they are at one; at the Diſtance of three Spaces, nine Times thinner, and ſo on: The Reaſon of which is, becauſe they ſpread themſelves in a twofold Manner, *viz.* upwards and downwards, as well as ſide-ways.

them; and the tender Flowers of Plants would be ſo far from being cheriſhed by them, that they would be torn in Pieces, and not able to ſtand before them.

* This Propoſition is demonſtrated mathematically thus; let us conceive two concentric Surfaces ABD, and EFG (*Fig.* 1.) and in theſe, two ſimilar Portions ELFI, and AHBK; let the Rays CE and CF, with the reſt proceeding from the Center C, fall upon the Portion ELFI, and cover it; it is evident from Inſpection of the Figure, that the ſame Rays at the Diſtance CH will cover the Portion AHBK only; now theſe Rays being the ſame in Number at each Place, will be thinner in the former, than they are in the latter, in Proportion as that is larger than this; but theſe Spaces being ſimilar Portions of the Surfaces of Spheres, bear the ſame Proportion to each other, that the Surfaces themſelves do, that is, they are to each other as the Squares of their *Radii* CL, CH; the Rays therefore are more diffuſed, or thinner in Proportion to the Squares of the ſame *Radii*, or of their Diſtances from the luminous Point C. *Q. E. D.*

The Particles of Light are ſubject to the Laws of Attraction of Coheſion like other ſmall Bodies, for if a Ray of Light be made to paſs by the Edge of a Knife, it will be diverted from its natural Courſe, and be inflected towards the Edge of the Knife. The like Inflection happens to a Ray when it enters obliquely into a denſer or rarer Subſtance than that in which it was before, in which Caſe it is ſaid to be *refracted*; the Laws of which Refraction are the Subject of the following Chapter *.

* The *Carteſian* Notion of Light, was not, that it is propagated from luminous Bodies by the Emiſſion of ſmall Particles, but that it was communicated to the Organ of Sight by their Preſſure upon the *Materia Subtilis*, with which they ſuppoſed the Univerſe to be full. But according to this *Hypotheſis*, it could never be dark; becauſe when a Fluid ſuſtains any Preſſure, if that Fluid fills all the Space it takes up, abſolutely, without leaving any Pores, which is the Caſe of the ſuppoſed *Materia Subtilis*; then that Preſſure muſt neceſſarily be communicated *equally* and *inſtantaneouſly* to every Part: And therefore, whether the Sun were above or below the Horizon, the Preſſure communicated, and conſequently the Light, would be the ſame. And farther, as the Preſſure would be inſtantaneous, ſo would the Light, which is contrary to what is collected, as we obſerved above, from the Eclipſes of *Jupiter*'s Satellites.

CHAP.

CHAP. II.

Of the Cauſe of Refraction, *and the* Law *by which it is performed.*

WHatever Subſtance a Ray of Light paſſes through, or if it paſſes through a Space void of all Subſtance, it is ſaid by Philoſophers to paſs through a *Medium*; and therefore if it paſſes out of any Subſtance, as Air or Glaſs, into a *Vacuum*, or the contrary, it is ſaid to paſs out of one *Medium* into another.

All Bodies being endued with an attractive Force, which is extended to ſome Diſtance beyond their Surfaces; when a Ray of Light paſſes out of a rarer into a denſer *Medium* (if this latter has a greater attractive Force than the former, as is commonly the Caſe *, and what we ſhall hereafter always ſuppoſe, unleſs it be mentioned to the contrary) the Ray juſt before its Entrance will begin to be attracted towards the denſer *Medium*, and this Attraction will continue to act upon it, till ſome Time after it has entered the *Medium*, as we ſhall ſhew by and by; and therefore if a Ray approaches a denſer *Medium* in a Direction perpendicular to its Surface, its Velocity will

* In oily and inflammable Bodies it happens otherwiſe; for they are obſerved to attract more ſtrongly than others of greater Denſity.

be continually accelerated during its Paſſage through the Space in which that Attraction exerts itſelf; and therefore, after it has paſſed that Space, it will move on, till it arrives at the oppoſite Side of the *Medium*, with a greater Degree of Velocity than it had before it entered. So that in this Caſe its Velocity only will be altered. Whereas, if a Ray enters a denſer *Medium* obliquely, it will not only have its Velocity augmented thereby, but its Direction will become leſs oblique to the Surface. Juſt as when a Stone is thrown downwards obliquely from a Precipice, it falls to the Surface of the Ground in a Direction nearer to a perpendicular one, than that with which it was thrown from the Hand. From hence we ſee a Ray of Light, in paſſing out of a rarer into a denſer *Medium*, is refracted *towards the Perpendicular*; that is, ſuppoſing a Line drawn perpendicularly to the Surface of the *Medium*, through the Point where the Ray enters, and extended both Ways, the Rays in paſſing through the Surface is refracted or bent towards the perpendicular Line; or, which is the ſame Thing, the Line which it deſcribes by its Motion after it has paſſed through the Surface, makes a leſs Angle with the Perpendicular, than the Line it deſcribed before. All which may be illuſtrated in the following Manner;

Let

Let us ſuppoſe, firſt, that the Ray paſſes out of a *Vacuum* into the denſer *Medium* A B C D, (*Fig.* 2.) and that the attractive Force of each Particle in the *Medium* is extended from its reſpective Center to a Diſtance equal to that which is between the Lines AB and EF, or AB and GH; and let KL be the Path deſcribed by a Ray of Light in its Progreſs towards the denſer *Medium*. This Ray, when it arrives at L, will enter the attractive Forces of thoſe Particles which lie in AB, the Surface of the denſer *Medium*, and will therefore ceaſe to proceed any longer in the right Line KLM, but will be diverted from its Courſe by being attracted towards the Line AB, and will begin to deſcribe the Curve LN, paſſing through the Surface AB in ſome new Direction, as OQ, thereby making a leſs Angle with a Line, as PR drawn perpendicularly through the Point N, than it would have done, had it proceeded in its firſt Direction K L M.

Farther, whereas we have ſuppoſed the attractive Force of each Particle to be extended through a Space equal to the Diſtance between A B and E F, it is evident the Ray, after it has entered the Surface, will ſtill be attracted downwards, till it has arrived at the Line E F; for till that Time there will not be ſo many Particles above it, which will attract it upwards, as below, that will attract it downwards. So that after it has entered

tered the Surface at N, in the Direction OQ, it will not proceed in that Direction, but will continue to deſcribe a Curve, as NS, after which it will proceed ſtrait on towards the oppoſite Side of the *Medium*, being attracted equally every Way; and therefore will at laſt proceed in the Direction XST, ſtill nearer the Perpendicular PR than before.

Now if we ſuppoſe the Space ABYZ not to be a *Vacuum*, but a rarer *Medium* than the other, the Caſe will ſtill be the ſame; but the Ray will not be ſo much refracted from its rectilineal Courſe, becauſe the Attraction of the Particles of the upper *Medium* being in a contrary Direction to that of the Attraction of thoſe in the Lower one, the Attraction of the denſer *Medium*, will in ſome Meaſure be deſtroyed by that of the rarer.

On the contrary, when a Ray paſſes out of a denſer into a rarer *Medium*, if its Direction be perpendicular to the Surface of the *Medium*, it will only loſe ſomewhat of its Velocity, in paſſing through the Spaces of Attraction of that *Medium* (that is, the Space wherein it is attracted more one Way than it is another.) If its Direction be oblique, it will continually recede from the Perpendicular during its Paſſage, and by that Means have its Obliquity increaſed, juſt as a Stone thrown up obliquely from the Surface of the Earth increaſes its Obliquity all the Time it riſes. Thus, ſuppoſing the Ray TS paſſing out of the denſer

denſer *Medium* ABCD into the rarer ABYZ, when it arrives at S it will begin to be attracted downwards, and ſo will deſcribe the Curve SNL, and then proceed in the right Line LK, making a larger Angle with the Perpendicular PR, than the Line TSX, in which it proceeded during its Paſſage through the other *Medium*.

The Space through which the Attraction of Coheſion of the Particles of Matter is extended is ſo very ſmall, that in conſidering the Progreſs of a Ray of Light out of one *Medium* into another, the Curvature it deſcribes in paſſing through the Space of Attraction is generally neglected; and its Path is ſuppoſed to be bent, or, in the uſual Terms, the Ray is ſuppoſed to be refracted only in the Point where it enters the denſer *Medium*.

Now the Line, which a Ray deſcribes before it enters a denſer or a rarer *Medium*, is called the *Incident Ray*; that which it deſcribes after it has entered, is the *Refracted Ray*.

The Angle comprehended between the Incident Ray and the Perpendicular, is the *Angle of Incidence*; and that between the Refracted Ray and the Perpendicular, is the *Angle of Refraction*.

There is a certain and immutable Law or Rule, by which Refraction is always performed; and that is this: Whatever Inclination a Ray of Light has to the Surface of any *Medi-*

um

um before it enters it, the Degree of Refraction will always be ſuch, that the Proportion between the Sine of the Angle of its Incidence, and that of the Angle of its Refraction, will always be the ſame in that *Medium* *.

To

* *Lemma.* If from a Point as M (*Fig.* 4.) taken any where without the Circle PNQ, a Line as MP be drawn paſſing thro' L the Center of the Circle, and terminated in the Circumference at P, the Product of MQ multiplied by MP is equal to the Difference between the Squares of ML and PL.

Demonſtration of the Lemma. Draw MT to touch the Circle in T, and join LT. LT is equal to LP (Def. 15. El. 1.) therefore the Square of LT is equal to the Square of LP; but (18 El. 3.) the Angle at T is a Right Angle, wherefore (47 El. 1.) the Square of MT is equal to the Difference between the Squares of ML and LT; it is therefore equal to the Difference between the Squares of ML and PL: now (36 El. 3.) the Square of MT is alſo equal to the Product of MQ into MP; conſequently the Difference between the Squares of ML and PL is equal to the Product of MQ into MP. *Q. E. D.*

Demonſtration of the Propoſition. When a Ray of Light paſſes through the Space of Attraction of any *Medium*, it is evident that its Motion will be ſubject to the like Laws with that of *Projectiles*, provided we ſuppoſe it to be acted upon with an equal Degree of Force during its whole Paſſage through that Space, as is commonly ſuppoſed to be the Caſe in *Projectiles*, to whatever Height they are thrown from the Earth. We will therefore put a Caſe as nearly parallel as may be to that which was demonſtrated of *Projectiles* in the ninth Chapter of the firſt Part; and ſuppoſe firſt, that the Force of Attraction of the denſer *Medium* is at all Diſtances the ſame as far as it reaches, and that the Ray proceeds out of a denſer into a rarer *Medium*; in which Caſe it will be attracted back towards the denſer *Medium*, during its Paſſage through the Space of Attraction, in like Manner as a *Projectile* thrown upwards is while it riſes from the Earth. Let then ABCD (*Fig.* 4.) repreſent the denſer *Medium*, and ABEF the Space of Attraction; and let GH be a Ray about to enter the Force of Attraction at H, and let GH be produced to M. Now it is evident, that in this Suppoſition, the Ray when at H, is in the ſame Circumſtances with a *Projectile*

To illuſtrate this, Let us ſuppoſe ABCD (*Fig.* 3.) to repreſent a rarer, and ABEF a denſer *Medium*; let GH be a Ray of Light paſſing

jectile about to be thrown upwards from H towards M, it will therefore deſcribe a Portion of a *Parabola* as HI; to which the Line HM will be a Tangent at H; and the Line IK, in which it would proceed after it has paſſed the Space of Attraction, a Tangent to it at I; for after having left the attractive Force at I, it goes ſtrait on in its laſt Direction. Let the Perpendicular IR be drawn meeting GH produced in M, and let KI be produced to L. On the Center L with the Radius LI, deſcribe the Circle PNQ, let fall the Perpendicular LO upon MR, and join the Points L and N. Now it is demonſtrated in the Caſe of Projectiles, that the *Parameter* of the Point H is equal to $\frac{HMq}{MI}$, and therefore the *Parameter* multiplied by MI is equal to HMq. And it is there farther demonſtrated, that the ſaid *Parameter* is equal to four Times the Height which a Body muſt fall from, to acquire the Velocity which the Projectile has at H; this Parameter therefore is a Quantity not at all depending on the Direction of the Projectile, but on its Velocity only; and conſequently in the preſent Suppoſition it is a given Quantity, the Ray GH being ſuppoſed to have the ſame Velocity, whatever is its Inclination to the Surface AB. Now the Tangent KI being produced to L, will by the Property of the *Parabola*, biſect the other Tangent HM, wherefore the Line LO being parallel to HR, MR will alſo be biſected in O; and adding the equal Lines OI and ON to each Part, MN will be equal to IR; but the Line IR is alſo a Line independent of the Inclination of the Ray GH, its Length being determined by the Breadth of the Space of Attraction ABEF only, and therefore MN is a given Quantity. Now, whereas MI, when multiplied by the Parameter of the Point H, which before was ſhewn to be a given Line, is equal to the Square of HM, therefore the ſame Line MI, when multiplied by any other given Line (*viz.* MN) if it is not equal to, will nevertheleſs bear a given Proportion to the Square of HM: But ſince MI multiplied by MN bears a given Proportion (*viz.* a Proportion that does not depend on the Inclination of the Ray GH) to the Square of MH, its Equal, *viz.* the Product MQ multiplied by MP (37 *El.* 3.) or what is equal to this, the Difference between the Squares of ML

paſſing through the firſt and entering the ſecond at H, and let HI be the refracted Ray; then ſuppoſing the Perpendicular PR drawn thro' the Point H, on the Center H, and with any

ML and PL (by the foregoing *Lemma*), or, which is the ſame Thing, of ML and LI, (becauſe PL and LI are *Radii* of the ſame Circle) does ſo too. Now the Square of ML bears alſo a given Proportion to the Square of MH (ML being equal to half MH) conſequently there is a given Proportion between the Square of ML and the Difference of the Squares of ML and LI; and therefore there is a certain Proportion between the Lines themſelves, *viz.* between ML and LI. But in every Triangle the Sides are proportional to the Sines of their oppoſite Angles, therefore in the Triangle MLI, the Sine of the Angle LMI has a given Proportion to the Sine of the Angle LIM, or of its Complement to two right ones MIK (for they have the ſame Sine:) But LMI being an Angle made by the incident Ray GH, produced with the Perpendicular RM, is the Angle of Incidence, and MIK being made by the refracted Ray IK, and the ſame Perpendicular, is the Angle of Refraction, therefore in this Caſe there is a conſtant *Ratio* between the Sine of the Angle of Incidence, and that of the Angle of Refraction.

We have here ſuppoſed that the Force of Attraction is every where uniform, but if it be otherwiſe, provided it be the ſame every where at the ſame Diſtances from the Surface AB, the Proportion between the forementioned Sines will ſtill be a given one. For, let us imagine the Space of Attraction divided into parallel Planes, and the Attraction to be the ſame through the whole Breadth of each Plane, tho' different in different Planes, the Sine of the Angle of Incidence out of each will, by what has been demonſtrated above, be to the Sine of the Angle of Refraction into the next in a given *Ratio*; and therefore, ſince the Sine of the Angle of Refraction out of one, will be the Sine of the Angle of Incidence into the next, it is evident that the Sine of the Angle of Incidence into the firſt, will be in a given *Ratio* to the Sine of the Angle of Refraction out of the laſt. Now let us ſuppoſe the Thickneſs of theſe Planes diminiſhed *in infinitum*, and their Number proportionally increaſed, the Law of Refraction will ſtill continue the ſame; and therefore whether the Attraction be uniform or not, there will be a conſtant *Ratio* between the Sine of the Angle of Incidence and of Refraction. *Q. E. D.*

Radius

Radius deſcribe the Circle APBR, and from G and I, where the incident and refracted Rays cut the Circle, let fall the Lines GK and IL perpendicularly upon the Line PR, the former of theſe will be the Sine of the Angle of Incidence, the latter of Refraction. Now if in this Caſe, the Ray GH is ſo refracted at H, that GK is double or triple, *&c.* of IL, then whatever other Inclination the Ray GH might have had, the Sine of its Angle of Incidence would have been double, or triple, *&c.* to that of its Angle of Refraction. For Inſtance, had the Ray paſſed in the Line MH before Refraction, it would have paſſed in ſome Line, as HN afterwards, ſo ſituated that MO ſhould have been double or triple, *&c.* of NQ.

When a Ray paſſes out of a *Vacuum* into Air, the Sine of the Angle of Incidence is found to be to that of Refraction, as 100036 to 100000.

When it paſſes out of Air into Water, as about 4 to 3.

When out of Air into Glaſs, as about 17 to 11.

When out of Air into a Diamond, as about 5 to 2.

CHAP.

CHAP. III.

Of the Refraction of Light *in paſſing thro' plain and ſpherical* Surfaces.

AS Rays of Light are capable of having their Progreſs altered by Refraction or Reflection, it is poſſible they may acquire various Inclinations and Directions different from thoſe which they had at their Emiſſion from the Surfaces of viſible Bodies.

When they recede from each other as they paſs along, they are ſaid to *diverge*; and the Point they proceed from is called the *Radiant Point*.

When they proceed towards any Point approaching nearer together in their Progreſs, they are then ſaid to *converge*; and the Point towards which they tend, is called the *Focus*.

This *Focus* may be either *real* or *imaginary*; it is ſaid to be real, when the Rays actually proceed to it; but if they are intercepted in their Progreſs, or turned another Way before they reach it, it is called their *imaginary Focus*.

Sometimes it happens, that Rays are ſo refracted or reflected, that they proceed afterwards, as from ſome Point, which is not their true Radiant, then alſo that Point is called their *imaginary Focus*.

When

When they proceed in parallel Lines, they are then called *parallel Rays*; and both their Focus and radiant Point is ſuppoſed to be at an infinite Diſtance.

When Rays paſs out of one *Medium* into another, they ſuffer various Alterations in their Motion. All which are expreſſed in the eighteen following Propoſitions.

I. When *parallel* Rays fall obliquely on a plain Surface of a *Medium* of different Denſity, they are *parallel* alſo after Refraction. For having all the ſame Inclination to the Surface, they ſuffer an equal Degree of Refraction.

II. When diverging Rays paſs out of a rarer into a denſer *Medium* through a plain Surface they are made thereby to diverge leſs.

For being all refracted towards their reſpective Perpendiculars, (but thoſe the moſt that are the moſt oblique to the Surface,) they are brought nearer to a Paralleliſm among themſelves; that is, they are made to diverge leſs than before.

See this and the following Caſes expreſſed more determinately, and demonſtrated in the Note below *.

III. When

* I. When Rays paſs out of one *Medium* into another of different Denſity through a plain Surface; if they diverge, the focal Diſtance will be to that of the radiant Point; if they converge, it will be to that of the imaginary Focus of the inci-

III. When they proceed out of a denſer into a rarer *Medium*, the contrary happens; for then, being refracted from their reſpective Perpendiculars, and thoſe the moſt that are the moſt oblique, they are made to diverge more.

IV. So

dent Rays, as the Sine of the Angle of Incidence is to that of the Angle of Refraction.

This Propoſition admits of four Caſes.

Caſe 1. Of diverging Rays paſſing out of a rarer into a denſer *Medium*.

Dem. Let X (*Fig.* 10.) repreſent a rarer, and Z a denſer *Medium*, ſeparated from each other by the plain Surface AB; ſuppoſe CE and CD to be two diverging Rays proceeding from the Point C, the one perpendicular to the Surface, the other oblique; through E draw the Perpendicular PK. The Ray CD being perpendicular to the Surface will proceed on in the right Line CQ, but the other falling on it obliquely at E, and there entering a denſer *Medium*, will ſuffer a Refraction towards the Perpendicular EK. Let then EG be the refracted Ray, and produce it back till it interſects DC produced alſo, in F; this will be the focal Point. On the Center E with the Radius EF, deſcribe the Circle AFBQ, and produced EC to H; draw HI the Sine of the Angle of Incidence, and GK that of Refraction, and which is equal to FP or CM. Now if we ſuppoſe the Points D and E contiguous, or nearly ſo, then will the Line HE be almoſt coincident with FD, and therefore FD will be to CD as HE to CE; but HE is to CE as HI to CM, becauſe the Triangles HIE and CME are ſimilar; that is, the focal Diſtance of the Ray CE is to the Diſtance of the Radiant Point, as the Sine of the Angle of Incidence is to that of the Angle of Refraction. *Q. E. D.*

Obſ. 1. *Whereas the* Ratio *of* FE *to* ME, *or which is the ſame Thing, that of* nD *to* CD *bears the exact Proportion of* HI *to* CM, *and becauſe this, (being the* Ratio *of the Sine of the Angle of Incidence to that of the Angle of Refraction) is always the ſame, the Line* In *is in all Inclinations of the Ray* CE, *at the ſame Diſtance from* CM; *conſequently had* CE *been coincident with* CD, *the Point* H *had fallen upon* n; *and becauſe the Circle paſſes through both* H *and* F, F *would alſo have fallen upon* n;

upon

IV. So when converging Rays paſs out of a rarer into a denſer *Medium*, through a plain Surface, they are made thereby to converge leſs.

For

upon which Account the Focus of the Ray CE *would have been there. But the Ray* CE *being oblique to the Surface* DB, *the Point* H *is at ſome Diſtance from* n; *and therefore the Point* F *is neceſſarily ſo too, and the more ſo by how much the greater that Diſtance is: from whence it is clear, that no two Rays flowing from the radiant Point* C, *and falling with different Obliquities on the Surface* BD, *will, after Refraction there, proceed as from the ſame Point; therefore ſtrictly ſpeaking, there is no one Point in the Line* D *produced, that can more properly be called the Focus of Rays flowing from* C, *than another: for thoſe which enter the refracting Surface near* D, *will after Refraction proceed, as has been obſerved, from the Parts about* n; *thoſe which enter near* E, *will flow as from the Parts about* F; *thoſe which enter about* T, *as from ſome Points in the Line* DF *produced, &c. And it is farther to be obſerved, that when the Angle* DCE *becomes large, the Line* nF *increaſes apace; wherefore thoſe Rays which fall near* T, *proceed after Refraction, as from a more* diffuſed *Space, than thoſe which fall at the ſame Diſtance from each other near the Point* D. *Upon which Account it is uſual with* optical *Writers to ſuppoſe the Diſtance between the Points where the Rays enter the plain Surface of a refracting* Medium, *to be inconſiderable with Regard to the Diſtance of the radiant Point, if they diverge; or to that of their imaginary Focus, if they converge: and unleſs there be ſome particular Reaſon to the contrary, they conſider them, as entering the refracting* Medium *in a Direction as nearly perpendicular to its Surfaces as may be.*

Caſe 2. Of diverging Rays proceeding out of a denſer into a rarer *Medium.*

Dem. Let X be the denſer, Z the rarer *Medium*, FD and FE two diverging Rays proceeding from the Point F; and ſuppoſing the Perpendicular PK drawn as before, FP will be the Sine of the Angle of Incidence of the oblique Ray FE, which in this Caſe being refracted from the Perpendicular, will paſs on in ſome Line as ER, which being produced back to the Circumference of the Circle will cut the Ray FD ſomewhere, ſup-

For being all refracted towards their reſpective Perpendiculars, and thoſe the moſt that are the moſt oblique, they themſelves are brought nearer to a Paralleliſm, and ſo converge leſs.

V. On

poſe in C, this therefore will be the imaginary Focus of the refracted Ray ER; draw RS the Sine of the Angle of Refraction to which HI will be equal: but here alſo FP or its equal CM, is to HI, as EC to EH, or (if the Point D and E be conſidered as contiguous) as DC to DF; that is, the Sine of the Angle of Incidence is to the Sine of the Angle of Refraction, as the focal Diſtance is to that of the radiant Point. *Q. E. D.*

Caſe 3. Of converging Rays paſſing out of a denſer *Medium* into a rarer.

Dem. Let Z be the denſer, X the rarer *Medium*, and GE the incident Ray; this will be refracted from the Perpendicular into a Line, as EH; then all Things remaining as before, GK, or its equal FP, or CM will be the Sine of the Angle of Incidence, and HI that of Refraction: but theſe Lines, as before, are to each other, as DC to DF; that is, the focal Diſtance is to the Diſtance of the imaginary Focus, as the Sine of the Angle of Incidence to that of the Angle of Refraction. *Q. E. D.*

Caſe 4. Of converging Rays paſſing out of a rarer into a denſer *Medium*.

Dem. Let Z be the rarer, X the denſer *Medium*, and RE the incident Ray; this will be refracted towards the Perpendicular into a Line, as EF; C will be the imaginary Focus, and F the real one, HI, which is equal to RS, the Sine of the Angle of Incidence, and FP that of the Angle of Refraction: but theſe are to each other, as DF to DC; and therefore the focal Diſtance is to that of the imaginary Focus, as the Sine of the Angle of Incidence is to that of the Angle of Refraction. *Q. E. D.*

II. When parallel Rays fall upon a ſpherical Surface of different Denſity, the focal Diſtance will be to the Diſtance of the Center of Convexity, as the Sine of the Angle of Incidence is to the Difference between that Sine and the Sine of the Angle of Refraction.

This

V. On the contrary, when they proceed out of a denſer into a rarer *Medium*, they are refracted the contrary Way, and ſo made to converge more.

All

This Propoſition admits of four Caſes.

Caſe 1. Of parallel Rays paſſing out of a rarer into a denſer *Medium* through a convex Surface of the denſer.

Dem. Let AB (*Fig.* 11.) repreſent a convex Surface, C its Center of Convexity; HA and DB two parallel Rays, paſſing out of the rarer *Medium* X into the denſer Z, the one perpendicular to the refracting Surface, the other oblique: draw CB, this being a Radius, will be perpendicular to the Surface at the Point B; and the oblique Ray DB being in this Caſe refracted towards the Perpendicular, will proceed in ſome Line, as BF, meeting the other Ray in F, which will therefore be the Focal Point: produce CB to N, then will DBN, or its equal BCA, be the Angle of Incidence, and FBC that of Refraction. Now, *whereas any Angle has the ſame Sine with its Complement to two right ones*, the Angle FCB *being the Complement of* ACB, which is equal to the Angle of Incidence, may here be taken for that Angle; and therefore, *as the Sides of a Triangle have the ſame Relation to each other, as the Sines of their oppoſite Angles have*, FB *being oppoſite to this Angle, and* FC *being oppoſite to the Angle of Refraction*, they may here be conſidered as the Sines of the Angles of Incidence and of Refraction; and for the ſame Reaſon CB may be conſidered as the Sine of the Angle CFB, which Angle *being together with the Angle* FBC, *equal to the external one* ACL (32 *El.* 1.) is itſelf equal to the Difference between thoſe two laſt Angles; and therefore the Line FB is to CB as the Sine of the Angle of Incidence is to the Sine of an Angle which is equal to the Difference between the Angle of Incidence and of Refraction. Now, becauſe in very ſmall Angles as theſe are, *for we ſuppoſe in this Caſe alſo the Diſtance* AB *to vaniſh, the Reaſon of which will be ſhewn by and by*, their Sines bear nearly the ſame Proportion to each other that they themſelves do, the Diſtance FB will be to CB as the Sine of the Angle of Incidence is to the Difference between that Sine and the Sine of the Angle of Refraction; but *becauſe* BA *vaniſhes*, FB and FA are equal, and therefore FA is to CA in that Proportion. *Q. E. D.*

All which may be illuſtrated, in the following Manner. 1. Let AB, CD, (*Fig.* 5.) be two parallel Rays falling on the plain Surface EF

Obſ. 2. *It appears from the foregoing Demonſtration, that the focal Diſtance of the Oblique Ray* DB, *is ſuch, that the Line* BF *ſhall be to the Line* CB *or* CA *as the Sine of the Angle of Incidence to the Sine of an Angle, which Angle is equal to the Difference between the Angle of Incidence and Refraction; therefore ſo long as the Angles* BCA, &c. *are ſmall, ſo long the Line* FB *is pretty much of the ſame Length, becauſe ſmall Angles have nearly the ſame Relation to each other that their Sines have. But when the Point* B *is removed far from* A, *ſo that the Ray* DB *enters the Surface, ſuppoſe about* O, *the Angles* BCA, &c. *becoming large, the Sine of the Angle of Incidence begins to bear a conſiderably leſs Proportion to the Sine of an Angle which is equal to the Difference between the Angle of Incidence and Refraction than before, and therefore the Line* BF *begins to bear a much leſs Proportion to* BC; *wherefore its Length decreaſes apace: Upon which Account thoſe Rays which enter the Surface about* O, *not only meet nearer the Center of Convexity than thoſe which enter at* A, *but are collected into a more* diffuſed *Space. From hence it is, that the Point where thoſe only which enter near* A, *are collected, is reckoned the true Focus; and the Diſtance* AB *in all Demonſtrations relating to the* Foci *of parallel Rays entering a ſpherical Surface, whether convex or concave, is ſuppoſed to* vaniſh.

Caſe 2. Of parallel Rays paſſing out of a denſer into a rarer *Medium* through a concave Surface of the Denſer.

Dem. Let X be the denſer, Z the rarer *Medium*. AB the Surface by which they are ſeparated, C the Center of Convexity, and HA and DB two parallel Rays, as before. Through B, the Point where the oblique Ray DB enters the rarer *Medium*, draw the Perpendicular CN; and let the Ray DB, which in this Caſe refracted from the Perpendicular, proceed in the Direction BM; produce BM back to H; this will be the imaginary Focus, and DBN, or its equal ACB, will be the Angle of Incidence, and CBM, or its equal HBN (for they are vertical) that of Refraction; produce DB to L, and draw BF ſuch, that the Angle LBF may be equal to DBH: then becauſe NBD and DBH together are equal to NBH the Angle of Refraction, therefore BCA, which is equal to the firſt, and LBF, which

is

EF of a *Medium* of a different Denſity: Now becauſe they both make equal Angles of Incidence with their reſpective Perpendiculars GH, IK,

is equal to the Second, are together equal to the Angle of Refraction; but LBF is equal to BFA (as being alternate to it) conſequently BFA and BCA together are equal to the Angle of Refraction; and therefore ſince one of them, *viz.* BCA, is equal to the Angle of Incidence, the other is the Difference between that Angle, and the Angle of Refraction. Now FB, the Sine of the Angle FCB, or which is the ſame Thing, of its Complement to two right ones BCA, the Angle of Incidence is to CB the Sine of the Angle BFC, as FB to CB, that is as HB to CB; for the Angles DBH and LBF being equal, the Lines BF and BH are ſo too; but the Diſtance BA vaniſhing, HB is to CB, as HA to CA; that is, the Sine of the Angle of Incidence is to the Sine of an Angle which is the Difference between the Angle of Incidence and Refraction, or becauſe the Angles are ſmall, to the Difference between the Sine of the Angle of Incidence and that of Refraction, as the Diſtance of the Focus from the Surface is to that of the Center from the ſame. *Q. E. D.*

Caſe 3. Of parallel Rays paſſing out of a rarer into a denſer *Medium*, through a concave Surface of the Denſer.

Dem. Let X be the denſer *Medium*, having the concave Surface AB, and let LB and FA be the incident Rays. Now whereas, when DB was the incident Ray, and paſſed out of a rarer into a denſer *Medium*, as in Caſe the firſt, it was refracted into a Line BF, this Ray LB having the ſame Inclination to the Perpendicular, will alſo ſuffer the ſame Degree of Refraction, and will therefore paſs on afterwards in the Line FB produced, *v. g.* towards P. So that, whereas in that Caſe the Point F was the real Focus of the incident Ray DB, the ſame Point will in this be the imaginary Focus of the incident Ray LB: But it was there demonſtrated, that the Diſtance FA is to CA, as the Sine of the Angle of Incidence is to the Difference between that and the Sine of the Angle of Refraction, therefore the focal Diſtance of the refracted Ray BP is to the Diſtance of the Center of Convexity in that Proportion. *Q. E. D.*

Caſe 4. Of parallel Rays paſſing out of a denſer into a rarer *Medium*, through a convex Surface of the denſer.

Dem. Let Z be the denſer *Medium*, having the convex Surface AB, and let LB and FA be the incident Rays, as before.

IK, before Refraction, they will make equal Angles of Refraction with them afterwards, and ſo proceed on in the parallel Lines BL, DM.

Now whereas when DB was the incident Ray paſſing out of a denſer into a rarer *Medium*, it was refracted into BM, as in Caſe the ſecond, having a Point, as H, in the Line MB produced, for its imaginary Focus; therefore LB, for the like Reaſon as was given in the laſt Caſe, will in this be refracted into BH, having the ſame Point H for its real Focus. So that here alſo the focal Diſtance will be to that of the Center of Convexity, as the Sine of the Angle of Incidence is to the Difference between that and the Sine of the Angle of Refraction. *Q. E. D.*

III. When diverging or converging Rays enter into a *Medium* of different Denſity through a ſpherical Surface, the *Ratio* compounded of that which the focal Diſtance bears to the Diſtance of the Radiant Point (or of the imaginary Focus of the incident Rays, if they converge;) and of that, which the Diſtance between the ſame radiant Point (or imaginary Focus) and the Center, bears to the Diſtance between the Center and the Focus, is equal to the *Ratio*, which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction.

This Propoſition admits of ſixteen Caſes.

Caſe 1. Of diverging Rays paſſing out of a rarer into a denſer *Medium*, thro' a convex Surface of the denſer, with ſuch a Degree of Divergency, that they ſhall converge after Refraction.

Dem. Let BD (*Fig.* 12.) repreſent a ſpherical Surface, C its Center of Convexity, and let there be two diverging Rays, AB, and AD, proceeding from the radiant Point A, the one perpendicular to the Surface, the other oblique. Through the Center C produce the perpendicular Ray AD to F, and draw the Radius CB, and produce it to K, and let BF be the refracted Ray; then will F be the focal Point; produce AB to H, and through the Point F draw the Line F G parallel to CB. A B being the incident Ray, and CK perpendicular to the Surface at the Point B, the Angle A B K, or which is equal to it, *becauſe of the parallel Lines* C B *and* F G, F G H is the Angle of Incidence. *Now whereas the Complement of any Angle to two right ones, has the ſame Sine with the Angle itſelf,* the Sine of the Angle F G B, *that being the Complement of* F G H *to two right ones, may be conſidered as the Sine of*

the

DM. 2. Let the diverging Rays AB, AE, AF, (*Fig.* 6.) paſs out of a rarer into a denſer *Medium*, through the plain Surface GH, and

the Angle of Incidence; which Sine the Line FB, *as the Sides of a Triangle have the ſame Relation to each other, that the Sines of their oppoſite Angles have*, may be taken for. Again, the Angle FBC is the Angle of Refraction, or its equal, *becauſe alternate to it*, BFG, to which BG *being an oppoſite Side*, may be looked upon as the Sine. But FB is to BG in a *Ratio* compounded of FB to BA, and of BA to BG, *for the* Ratio *that any two Quantities bear to each other, is compounded of the* Ratio *which the firſt bears to any other, and of the* Ratio *which that other bears to the ſecond.* Now FB is to BA, *ſuppoſing* BD *to vaniſh*, *as* FD to DA; and BA is to BG, *becauſe of the parallel Lines* CB *and* FG, as AC to CF. That is, the *Ratio* compounded of FD, *the focal Diſtance*, to DA, *the Diſtance of the radiant Point*, and of AC, *the Diſtance between the radiant Point and the Center to* CF, *the Diſtance between the Center and the Focus*, is equal to that which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

Obſ. 3. *Whereas the focal Diſtance of the oblique Ray* AB *is ſuch that the compound* Ratio *of* FB *to* BA *and of* AC *to* CF *ſhall be the ſame, whatever be the Diſtance between* B *and* D; *it is evident, that ſince* AC *is always of the ſame Length, the more the Line* AB *lengthens, the more* FB *muſt lengthen too, or elſe* FC *muſt ſhorten; but it appears by Inſpection of the Figure, that if* BF *lengthens,* CF *will do ſo too, and in a greater Proportion with reſpect to its own Length than* BF *will, therefore the lengthening of* BF *will conduce nothing towards preſerving the Equality of the Proportion: but as* AB *lengthens,* BF *and* CF *muſt both ſhorten, which is the only poſſible Way wherein the Proportion may be continued the ſame. And it is alſo apparent, that the farther* B *moves from* D *towards* O, *the faſter* AB *lengthens, and therefore the farther the Rays enter from* D, *the nearer to the refracting Surface is the Place where they meet, but the Space they are collected in is the more* diffuſed: *And therefore in this Caſe, as well as thoſe taken Notice of in the two foregoing Obſervations, different Rays, though flowing from the ſame Point, ſhall conſtitute different* Foci; *and none are ſo* effectual *as thoſe which enter at or very near the*

Point

and let the Ray AB be perpendicular to that Surface; the reſt being refracted towards their reſpective Perpendiculars EK, FM, and thoſe the moſt that fall the fartheſt from B, they will pro-

Point D. *And ſince the ſame is obſervable of converging as well as of diverging Rays, none except thoſe which enter very near that Point, are uſually taken into Conſideration; upon which Account it is, that the Diſtance* DB, *in determining the focal Diſtances of diverging or converging Rays entering a convex or concave Surface, is ſuppoſed to* vaniſh.

Thoſe who would ſee a Method of determining the preciſe Point which the Ray AB, whether it be parallel, converging, or diverging to the Ray AF, converges to or diverges from, after Refraction at B, or any other given Point in the Surface DO, may find it in the Appendix to *Molineux*'s Optics, which, for the Sake of thoſe who have not that Book, I ſhall ſubjoin at the End of this Note.

Caſe 2. Of converging Rays paſſing out of a rarer into a denſer *Medium* through a concave Surface of the denſer with ſuch a Degree of Convergency, that they ſhall diverge after Refraction.

Dem. Let the incident Rays be HB and FD paſſing out of a rarer into a denſer *Medium* through the concave Surface BD, and tending towards the Point A, from whence the diverging Rays flowed in the other Caſe; then the oblique Ray HB having its Angle of Incidence HBC equal to ABK the Angle of Incidence in the former Caſe, will be refracted into the Line BL, ſuch, that its refracted Angle KBL will be equal to FBC, the Angle of Refraction in the former Caſe; that is, it will proceed after Refraction in the Line FB produced, having the ſame focal Diſtance FD with the diverging Rays AB, AD, in the other Caſe. But, by what has been already demonſtrated, the *Ratio* compounded of FD, *the focal Diſtance, to* DA, *in this Caſe, the Diſtance of the imaginary Focus of the incident Rays,* and of AC, *the Diſtance between the ſame imaginary Focus and the Center, to* CF, *the Diſtance between the Center and the Focus,* is equal to that which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

Caſe 3. Of diverging Rays paſſing out of a rarer into a denſer *Medium* through a convex Surface of the denſer, with ſuch a Degree of Divergency as to continue diverging.

Dem.

proceed in the Directions EN and FO, diverging in a leſs Degree from the Ray AP, than they did before Refraction. 3. Had they proceeded

Dem. Let AB, AD, (*Fig.* 13.) be the diverging Rays, and let their Divergency be ſo great, that the refracted Ray BL ſhall alſo diverge from the other; produce LB back to F, which will be the focal Point; draw the Radius CB and produce it to K, produce BA likewiſe towards G, and draw FG parallel to BC. Then will ABK be the Angle of Incidence, whoſe Sine BF may be taken for, as being oppoſite to the Angle BGF, which is the Complement of the other to two right ones. And LBC is the Angle of Refraction, or its equal KBF, or which is equal to this, BFG, as being alternate; therefore BG, the oppoſite Side to this, may be taken for the Sine of the Angle of Refraction. But BF is to BG, for the like Reaſon as was given in Caſe the firſt, in a *Ratio* compounded of BF to BA, and of BA to BG. Now BF is to BA, (DB vaniſhing) as DF to DA, and becauſe of the parallel Lines FG and BC, the Triangles CBA and AGF are ſimilar, therefore BA is to AG as CA to AF, conſequently BA is to BA together with AG, that is, to BG, as CA is to CA together with AF, that is, CF. Therefore the *Ratio* compounded of DF the focal Diſtance to DA the Diſtance of the radiant Point, and of CA the Diſtance between the radiant Point and the Center, to CF the Diſtance between the Center and the Focus, is equal to that which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

Caſe 4. Of converging Rays paſſing out of a rarer into a denſer *Medium* through a concave Surface of the denſer, in ſuch Manner that they ſhall continue converging.

Dem. Let HB and CD be the incident Rays paſſing out of the rarer into the denſer *Medium* through the concave Surface BD, and tending towards A, the ſame Point from whence the diverging Rays flowed in the laſt Caſe. Then becauſe the Ray HB has the ſame Inclination to the Perpendicular CK that AB had before, it will ſuffer the ſame Degree of Refraction, and paſs on in the Line LB produced, having its Focus F at the ſame Diſtance from the refracting Surface as that of the diverging Ray AB in the other Caſe. Therefore, &c. *Q. E. D.*

Caſe 5. Of diverging Rays paſſing out of a denſer into a rarer *Medium* through a concave Surface of the denſer.

Dem.

ceeded out of a denſer into a rarer *Medium*, they would have been refracted from their Perpendiculars EK, FM, and thoſe the moſt which were the moſt oblique, and therefore would have

Dem. Let AB, AD (*Fig.* 14.) be the incident Rays paſſing out of a denſer into a rarer *Medium* through the concave Surface BD, whoſe Center is C; and let BL be the refracted Ray, which produce back to F, and draw FG parallel to CB. Here ABK is the Angle of Incidence, to which its alternate one FGB being equal, FB, the oppoſite Side, may be conſidered as the Sine of it. The Angle of Refraction is LBC or FBK, of which BFG being the Complement to two right ones, BG the oppoſite Side may be looked upon as its Sine. But BF is to BG, in the compound *Ratio* of BF to BA, and of BA to BG, for the Reaſon given above. Now (BD vaniſhing) BF is to BA as DF to DA, and BA is to BG as CA to CF. That is, the *Ratio* compounded of the focal Diſtance to the Diſtance of the radiant Point, *&c.* *Q. E. D.*

Caſe 6. Of converging Rays paſſing out of a denſer into a rarer *Medium* through a convex Surface of the denſer.

Dem. Let HB, CD, be the incident Rays tending towards the Point A which was the Radiant in the laſt Caſe. Then, for the Reaſon already given, the oblique Ray will ſuffer ſuch a Degree of Refraction, as to have its Focus F at the ſame Diſtance from the Surface, as the diverging Rays AB, AD, had in that Caſe. Therefore, *&c.* *Q. E. D.*

When the *Mediums* thro' which Rays paſs, and the refracting Surfaces are ſuch, that Rays flowing from A (*Fig.* 12.) are collected in F, then Rays flowing from F through the ſame *Mediums* the contrary Way, will be collected in A. For when Rays paſs out of one *Medium* into another, the Sine of the Angle of Incidence bears the ſame Proportion to the Sine of the Angle of Refraction, as the Sine of the Angle of Refraction does to the Sine of the Angle of Incidence, when they paſs the contrary Way. This is applicable to each of the ſix following Caſes compared reſpectively with the ſix foregoing; therefore they may be conſidered as the Converſe of them; or they may be demonſtrated independently of them, as follows:

Caſe 7. Of diverging Rays paſſing out of a denſer into a rarer *Medium* through a convex Surface of the denſer, ſo as to converge afterwards.

Dem.

have diverged more than before. 4. Let the converging Rays AB, CD, EF (*Fig.* 7.) paſs out of a rarer into a denſer *Medium*, through the

Dem. Let AB, AD (*Fig.* 15.) be two diverging Rays paſſing through the convex Surface BD into a rarer *Medium*. Let C be the Center of Convexity, and BF the refracted Ray. Draw CB and produce it to K, and draw FG parallel to it, meeting AB produced in G. Then will ABC be the Angle of Incidence, of which FB being oppoſite to its alternate and equal Angle FGB, may be conſidered as the Sine. The Angle of Refraction is FBK, of which GB being oppoſite to its Complement to two right ones GFB, may be taken for the Sine. Now FB is to BG, in a *Ratio* compounded of FB to BA, and of BA to BG. But (BD vaniſhing) FB is to BA as FD to DA, and becauſe of the parallel Lines CB and FG, BA is to BG as CA to CF. Therefore the focal Diſtance, &c. *Q. E. D.*

Caſe 8. Of converging Rays paſſing out of a denſer into a rarer *Medium* through a concave Surface of the denſer, ſo as to diverge afterwards.

Dem. Let GB and FD be the incident Rays tending towards A, and produce FB to L. Then as AB in the laſt Caſe was refracted into BF, GB will in this be refracted into BL, for the Reaſons already given, having F for its focal Point. Therefore, &c. *Q. E. D.*

Caſe 9. Of diverging Rays paſſing out of a denſer into a rarer *Medium* through a convex Surface of the denſer, in ſuch Manner as to continue diverging.

Dem. Let AB, AD (*Fig* 16.) be two Rays paſſing out of a denſer into a rarer *Medium*, through the convex Surface DB, whoſe Center of Convexity is C. Draw CB, produce it to K, and let BL be the refracted Ray, produce BL back to F, and draw FG parallel to CB meeting BA produced in G. Then will ABC be the Angle of Incidence, of which FB being oppoſite to its alternate and equal Angle FGB, may be conſidered as the Sine. The refracted Angle is LBK, or its equal CBF, of which, BG being oppoſite to BFG, its Complement to two right ones, is the Sine. Now BF is to BG in the compound *Ratio* of BF to BA and of BA to BG: but BF is to BA as DF to DA; and becauſe of the parallel Lines CB and FG, the Triangles BCA, AGF are ſimilar, therefore BA is to AG as CA to AF, and conſequently BA is to BG as CA to CF. Therefore, &c. *Q. E. D*

Caſe

the plain Surface GH, and let the Ray AB be perpendicular to that Surface, then the other Rays being refracted towards their reſpective

Caſe 10. Of converging Rays paſſing out of a denſer into a rarer *Medium* through a concave Surface of the denſer, in ſuch Manner as to continue converging.

Dem. Let HB, MD be the incident Rays tending towards the Point A. Then will the oblique Ray HB, for the Reaſons already given, be refracted into BF. Therefore, *&c. Q. E. D.*

Caſe 11. Of diverging Rays paſſing out of a rarer into a denſer *Medium* through a concave Surface of the denſer.

Dem. Let AB, AD (*Fig.* 17.) be the incident Rays paſſing out of a rarer into a denſer *Medium*, thro' the concave Surface BD, whoſe Center of Convexity is C, and ſuppoſing the Line CB drawn and produced to K, the refracted Ray BL drawn and produced back to F, and alſo FG drawn parallel to CB, ABC will be the Angle of Incidence, of which FB being oppoſite to its Complement to two right ones BGF, is the Sine. The Angle of Refraction will be LBK, or its equal FBC, of which BG being oppoſite to its equal and alternate one BFG, is the Sine. Now FB is to BG in the compound *Ratio* of FB to BA and of BA to BG. But (BD vaniſhing) FB is to BA as FD to DA, and becauſe of the parallel Lines FG and CB, BA is to BG as CA to CF. Therefore, *&c. Q. E. D.*

Caſe 12. Of converging Rays paſſing out of a rarer into a denſer *Medium* through a convex Surface of the denſer.

Dem. Let HB, MD be the incident Rays tending towards A, the radiant Point in the laſt Caſe; then, as was explained above, BF will be the refracted Ray. Therefore, *&c. Q. E. D.*

Caſe 13. Of Rays paſſing out of a rarer into a denſer *Medium* from a Point between the Center of Convexity and the Surface.

Dem. Let AB, AD (*Fig.* 18.) be two Rays paſſing out of a rarer into a denſer *Medium* from the Point A, which let be poſited between C, the Center of Convexity, and the refracting Surface BD; through B draw CK, and let BL be the refracted Ray; produce BL back to F, and draw FG parallel to BC. Then will ABC be the Angle of Incidence, of which BF being oppoſite to its Complement to two right ones BGF, is the Sine. LBK will be the Angle of Refraction, or its equal FBC, of which BG being oppoſite to its alternate and equal one BFG,

is

tive Perpendiculars DK, FM, and EF for Inſtance more than CD, they will proceed in the Directions DN, FN converging in a leſs Degree towards the Ray AN, than they did

is the Sine. But, as before, BF is to BG in a compound *Ratio* of BF to BA and of BA to BG; and (BD vaniſhing) BF is to BA as DF to DA, and becauſe the Lines CB and FG are parallel, BA is to BG as CA to CF. Therefore, &c. *Q. E. D.*

Caſe 14. Of Rays paſſing out of a rarer into a denſer *Medium* towards a Point between the Center of Convexity and the Surface.

Dem. Let the incident Rays be MD, HB, tending towards A from whence the other proceeded in the laſt Caſe. Then, as in that Caſe, the refracted Ray BL being produced back paſſed through F, in this the refracted Ray itſelf, for the like Reaſons as were given in the foregoing Caſes, will paſs through that Point. Therefore, &c. *Q. E. D.*

Caſe 15. Of Rays paſſing out of a denſer into a rarer *Medium* from a Point between the Center of Convexity and the Surface.

Dem. Let AB, AD (*Fig.* 19.) be two diverging Rays paſſing out of a denſer into a rarer *Medium* through the refracting Surface BD, whoſe Center of Convexity is C, a Point beyond that from whence the Rays flow. Through B draw CK, and let BL be the refracted Ray, produce it back to F, and draw FG parallel to BC meeting BA produced in G. ABC will be the Angle of Incidence, of which BF, being oppoſite to its alternate and equal Angle BFG, is the Sine. The Angle of Refraction is LBK or its equal FBC, of which BG, being oppoſite to its Complement to two right ones BFG, is the Sine. But BF is to BG in the compound *Ratio* of BF to BA and of BA to BG; and (BD vaniſhing) BF is to BA as DF to DA, and becauſe of the parallel Lines CB and GF, the Triangles AFG and ABC are ſimilar. BA therefore is to AG as CA to AF, conſequently BA is to BA and AG together, that is, to BG, as CA is to CA and AF together, that is, to CF; and therefore the focal Diſtance, &c. *Q. E. D.*

Caſe 16. Of Rays paſſing out of a denſer into a rarer *Medium* towards a Point between the Center of Convexity and the Surface.

Dem.

did before. 5. Laſtly, Had the firſt *Medium* been the denſer, they would have been refracted the other Way, and therefore have converged more.

VI. When

Dem. Let HB, MD be the incident Rays, having for their imaginary Focus the Point A which was the Radiant in the laſt Caſe, and let C, the Center of Convexity of the refracting Surface, be poſited beyond this Point. Then will HB, for the Reaſons already given, be refracted into BF, having the Point F for its real Focus which was the imaginary one of the diverging Rays AB, AD, in the former Caſe. Therefore, as before, the *Ratio* compounded of that which the focal Diſtance bears to the Diſtance of the imaginary Focus of the incident Rays, and of that which the Diſtance between the ſame imaginary Focus and the Center, bears to the Diſtance between the Center and the Focus, is equal to the *Ratio* which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

The firſt Term in the foregoing Proportion (*viz.* that in Propoſition the 3d of this Note) being always an unknown Quantity, thoſe who are not well verſed in the Uſe of ſuch Propoſitions, may think it impoſſible to inveſtigate the focal Diſtance of any refracting Surface by it; I ſhall therefore exemplify it in the following Inſtance, by which the Manner of doing it in all others will clearly be underſtood. *V. g.* Let it be required to determine the focal Diſtance of diverging Rays paſſing out of Air into Glaſs through a convex Surface, and let the Diſtance of the radiant Point be 20, and the *Radius* of Convexity be 5: Now becauſe we muſt make Uſe of the focal Diſtance before we know it; let that be expreſſed by ſome Symbol or Character as x: Then, becauſe by the aforeſaid Propoſition the *Ratio* compounded of that which the focal Diſtance bears to the Diſtance of the radiant Point (that is in this Suppoſition, of x to 20); and of the *Ratio* which the Diſtance of the ſame radiant Point from the Center bears to the Diſtance between the Center and the Focus (in this Caſe, of 25 to $x-5$) is equal to the *Ratio* which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction (that is, of 17 to 11), we ſhall have in the Inſtance before us, the following Proportion, *viz.*

$$\left.\begin{matrix} x : 20 \\ 25 : x-5 \end{matrix}\right\} :: 17 : 11,$$ and compounding them into one,

which

VI. When Rays proceed out of a rarer into a denſer *Medium*, through a convex Surface of the denſer, if they are parallel before Refraction, they become converging afterwards.

For

which is done by multiplying the two firſt Parts together, we have $25x : 20x - 100 :: 17 : 11$, and multiplying the extreme Terms and middle Terms together, $340x - 1700 = 275x$, which Equation, after due Reduction, gives $x = \frac{1700}{65}$.

In ſome Caſes which might have been put, the Quantity 65 would have been negative, and then the Quotient ariſing from 1700, divided by that, would have been ſo too: that is x the focal Diſtance would have been negative, in which Caſe the *Focus* muſt have been taken on the contrary Side the Surface to that on which it was ſuppoſed to fall in ſtating the Problem; that is, it muſt have been taken on the ſame Side with the radiant Point, for in calling the Diſtance between the Center and the *Focus* $x-5$ it was ſuppoſed the *Focus* would fall on the ſame Side with the Center, or on that which is oppoſite to the radiant Point, becauſe otherwiſe that Diſtance muſt have been expreſſed by $x+5$, as any one may ſee by Inſpection of the 13th or 14th Figure, in which the *Focus* of diverging Rays entering a convex Surface, is ſuppoſed to fall on the ſame Side with the radiant Point.

In likeManner as thisProblem was performed, a general Theorem may be raiſed to ſolve it in all Caſes whatſoever, by uſing Characters inſtead of Figures; as every one who is not unacquainted with algebraic Operations very well knows.

See this done, and applied to the Paſſage of Rays thro' the Surface of Lenſes, in the ſecond Note to the following Chapter.

A Method of determining the Point which a Ray, entering a ſpherical Surface at any given Diſtance from the Vertex of it, converges to, or diverges from, after Refraction at the ſame. *From the Appendix to* Molineux's *Dioptrics*.

" *Prop.* To find the *Focus* of any Parcel of Rays *diverging* " *from*, or *converging to* a given Point in the *Axis* of a *ſpherical* " *Lens* [Surface] and inclined thereto under the ſame Angle; " the *Ratio* of the Sines in Refraction being known.

" Let GL (*Fig.* 20.) be the *Lens*, P any Point in its Surface, " V the Pole [Vertex] thereof, C the Center of the Sphere " whereof it is a Segment, O the Object or Point in the Axis to

For in this Caſe, the Perpendiculars at the Points where the Rays enter the Surface, are all drawn from the Center of Convexity on the

" or from which the Rays do proceed, OP a given Ray; and " let the *Ratio* of Refraction be as *r* to *s*; make CR to CO as " *s* to *r* for the Immerſion of a Ray, or as *r* to *s* for the Emer- " ſion, (that is, as the Sines of the Angles in the *Medium* which " the Ray enters, to their correſponding Sines in the *Medium* " out of which it comes) and laying CR from C towards O, " the Point R ſhall be the ſame for all the Rays of the Point O. " Then draw the Radius PC (if Need be) continued, and with " the Center R and Diſtance OP deſcribe an Arch, in- " terſecting PC in Q; the Line QR being drawn ſhall be pa- " rallel to the refracted Ray, and PF being made parallel " thereto, ſhall interſect the Axis in the Point F, which is " the *Focus* ſought. Or make it as CQ : CP : : CR : CF, and " CF ſhall be the Diſtance of the *Focus* from the Center of " the Sphere.

" *Dem.* Let fall the Perpendiculars PX on the Axis, CY on the " given Ray, and CZ on the refracted Ray. By the Conſtruction " PF and QR are parallel, whence the Triangles QRC and PFC " are ſimilar, and CR to QR, as CF to PF, that is CR to OP as " CF to PF. Now CF : PF : : CZ : PX *ob ſimilia Triang.* " whence CR : OP : : CZ : PX, and CR : CZ : : OP : PX. Again " CR is to CO as the Sines of Refraction by Conſtruction, that " is, as *s* to *r*, or *r* to *s*; and as CR to CZ, ſo (CO =) $\frac{r}{s}$ or " $\frac{s}{r}$ CR to $\frac{r}{s}$, or $\frac{s}{r}$ CZ, and ſo is PO to PX: But as PO to PX, " ſo CO to CY. *Ergo* CY = $\frac{r}{s}$ or $\frac{s}{r}$ CZ, that is CY to CZ is " as the Sines of Refraction; but CY is the Sine of the Angle of " Incidence, and CZ of the refracted Angle. *Ergo conſtat Pro-* " *poſito.*

" Hitherto we have conſidered only *oblique Rays*; it now re- " mains to add ſomething concerning *Rays parallel* to the *Axis*: " In this Caſe the Point O muſt be conſidered as infinitely diſ- " tant, and conſequently OP, OC, and CR are all infinite; " and OP and OC are in this Caſe to be accounted as always " equal, (ſince they differ but by a Part of the *Radius* of the

" Sphere

the other Side; and therefore, as the Rays are refracted towards thoſe Perpendiculars, they are neceſſarily refracted towards each other, and thereby made to converge.

VII. If they enter diverging, then for the ſame Reaſon they are made to diverge leſs,

" Sphere GPVL, which is no Part of either of them) where-
" fore the *Ratio* of CR to OP will be always the ſame, *viz.*
" as *s* to *r* for immerging Rays, and as *r* to *s* for thoſe that
" emerge. And by this Propoſition CF is to PF in the ſame
" *Ratio*. It remains therefore to ſhew on the Baſe CP how to
" find all the Triangles CPF, wherein CF is to PF in the *Ratio*
" given by the Degree of Refraction. This Problem has been
" very fully conſidered by the celebrated Dr. *Wallis*, in his late
" Treatiſe of Algebra, p. 258, to which I refer; but I muſt
" here repeat the Conſtruction thereof. See *Fig.* 21 and 22.

" Let GPVL be a *Lens*, VC or PC the *Radius* of its Sphere,
" and let it be required to find all the Points *f*, *f*, ſuch as C*f*
" may be to P*f* in the given *Ratio* of *s* to *r* for immerging Rays,
" or as *r* to *s* for the emerging. Divide CV in K, and continue
" CV to F, that CK may be to VK, and CF to VF in the
" propoſed *Ratio*. Then divide KF equally in the Point *a*,
" and with that Center deſcribe the Circle FKF; this Circle be-
" ing drawn gives readily all the *Foci* of the parallel Rays OP,
" OP. For having continued CP till it interſect the Circle in
" F, PF ſhall be always equal to V*f* the Diſtance of the *Focus*
" of each reſpective Parcel of Rays OP from the Vertex or
" Pole of the *Lens*.

" To demonſtrate this, draw the pricked Line VF, and by
" what is delivered by Dr. *Wallis* in the above-cited Place, VF
" and CF will be always in the ſame propoſed *Ratio*. Again,
" V*f* being made equal to PF, CF and C*f* will be likewiſe
" equal, as are CP, VC; and the Angles PC*f*, VCF being *ad*
" *verticem*, are alſo equal: Wherefore P*f* will be equal to VF,
" and conſequently C*f* to P*f* in the ſame *Ratio* as CF to VF;
" whence, and by what foregoes, the Points *f*, *f*, are the ſe-
" veral reſpective *Foci* of the ſeveral Parcels of Rays OP, OP."
Q. E. D.

If any one would ſee how this is to be applied in all other Inſtances, he may conſult the Place.

to be parallel, or to converge, according to the Degree of Divergency they have before they enter.

For if they diverge very much, their being bent towards their reſpective Perpendiculars in paſſing through the Surface, may only diminiſh their Divergency; whereas, if they diverge in a ſmall Degree, it may make them parallel, or even to converge. What Degree of Divergency or Convergency before Refraction in this and the following Caſes, is neceſſary to make Rays become parallel, will be ſhewn at Section the 17th of this Chapter.

VIII. If they converge in ſuch Manner as to tend directly towards the Center of Convexity before they enter the Surface, they fall in with their reſpective Perpendiculars, and ſo paſs on to the Center without ſuffering any Refraction.

IX. If they converge leſs than their Perpendiculars, that is, if they tend to a Point beyond the Center of Convexity, they are made by Refraction to converge more; and if they converge more than their Perpendiculars, that is, if they tend towards a Point between the Center and the Surface, then by being refracted towards them, they are made to converge leſs.

This, and the three foregoing Propoſitions, may be illuſtrated in the following Manner. 1. Let AB, CD (*Fig.* 8.) be two parallel Rays entering a denſer *Medium* through the convex

convex Surface DB, whoſe Center of Convexity is E; and let one of theſe, *viz.* AB, be perpendicular to the Surface. This will paſs on through the Center without ſuffering any Refraction, but the other, being oblique to the Surface, will be refracted towards the Perpendicular ED, and will therefore be made to proceed in ſome Line, as DG, converging towards the other Ray, and meeting it in G, which Point, for that Reaſon, is called the *Focus*. 2. Had the Ray CD diverged from the other, ſuppoſe in the Line AD, it would, by being refracted towards its Perpendicular ED, have been made either to diverge leſs, be parallel, or to converge. 3. Let the Line ED be produced to F, and if the Ray had converged, ſo as to have deſcribed the Line FD, it would then have been coincident with its Perpendicular, and have ſuffered no Refraction at all. 4. If it had proceeded from any Point between C and F, as from H, or which is the ſame Thing, towards any Point beyond E in the Line BE produced, it would have been made to converge more, by being refracted towards the Perpendicular DE, which converges more than it; and had it proceeded from ſome Point, as I, on the other Side F, that is, towards any Point between B and E, it would then have converged more than its Perpendicular, and ſo, being refracted towards it, would have been made to have converged leſs.

X. When Rays proceed out of a denſer into a rarer *Medium*, through a concave Surface of the denſer, the contrary happens in each Caſe.

For being now refracted from their reſpective Perpendiculars, as they were before towards them, if they are parallel before Refraction, they diverge afterwards; if they diverge, their Divergency is increaſed; if they converge in the Direction of their Perpendiculars, they ſuffer no Refraction; if they converge leſs than their reſpective Perpendiculars, they are made to converge ſtill leſs, to be parallel, or to diverge; if they converge more, their Convergency is increaſed. All which may clearly be ſeen by the Figure, without any further Illuſtration, imagining the Rays AD, CD, &c. bent the contrary Way in their Refractions to what they were in the former Caſes.

XI. When Rays proceed out of a rarer into a denſer *Medium*, through a concave Surface of the denſer, if they are parallel before Refraction, they are made to diverge.

For in this Caſe, the Perpendiculars at the Points where the Rays enter the Surface, being drawn from a Point on that Side of the Surface from which the Rays tend, if we conceive them to paſs through the Surface, they will be ſo many diverging Lines on the other Side, and therefore the Rays, after they have paſſed thro' the ſame Points, muſt neceſſarily be rendered diverging in being refracted towards them.

XII. If

XII. If they diverge before Refraction, then for the ſame Reaſon, they are made to diverge more.

XIII. Unleſs they proceed directly from the Center, in which Caſe they fall in with their Perpendiculars, and ſuffer no Refraction: or from ſome Point between the Center of Convexity and the Surface, for then they diverge more than their reſpective Perpendiculars, and therefore, being by Refraction brought towards them, they become leſs diverging.

XIV. If they converge, then being refracted towards their Perpendiculars, they are either made leſs converging, parallel, or diverging, according to the Degree they converged in before Refraction.

To illuſtrate this, and the three foregoing Caſes. 1. Let AB, CD (*Fig.* 9.) be two parallel Rays entering the concave and denſer *Medium* X, the Center of whoſe Convexity is E, and the Perpendicular to the refracting Surface at the Point D, is EF; the Ray AB, if we ſuppoſe it perpendicular to the Surface at B, will proceed on directly to G; but the oblique one CD being refracted towards the Perpendicular DF, will recede from the other Ray AG in ſome Line as DH. 2. If the Ray CD had proceeded from A diverging in the Direction AD it would have been bent nearer to the Perpendicular, and therefore have diverged more. 3. But if it had diverged from the Cen-

ter E, it would have fallen in with the Perpendicular EF, and not have been refracted at all: and had it proceeded from I, a Point on the other Side the Center E, it would, by being refracted towards the Perpendicular DF, have proceeded in ſome Line nearer it than it otherwiſe would have done, and ſo would diverge leſs than before Refraction. 4. If it had converged in the Line LD, it would have been rendered leſs converging, parallel, or diverging, according to the Degree of Convergency which it had before it entered into the refracting Surface.

XV. If the ſame Rays proceed out of a denſer into a rarer *Medium* through a convex Surface of the denſer, the contrary happens in each Suppoſition: For the parallel Rays are made to converge; thoſe which diverge leſs than their reſpective Perpendiculars, that is, thoſe which proceed from a Point beyond the Center, are made leſs diverging, parallel, or converging, according to the Degree in which they diverged before Refraction; thoſe which diverge more than their reſpective Perpendiculars, that is, thoſe which proceed from a Point between the Center and the refracting Surface, are made to diverge ſtill more. And thoſe which converged, are made to converge more. All which may eaſily be ſeen by conſidering the Situation of the Rays AD, CD, *&c.* with Reſpect to the Perpendicular EF; and therefore requires no farther Illuſtration.

XVI. When

XVI. When diverging Rays are by Refraction made to converge, the nearer their radiant Point is to the refracting Surface, the farther is their *Focus* from it on the other Side, and *vice versâ*.

For the nearer the radiant Point is to the refracting Surface, the more the Rays which fall upon the ſame Points of it diverge before Refraction, upon which Account they converge the leſs afterwards.

XVII. When the radiant Point is at that Diſtance from the Surface, at which parallel Rays coming through it from the other Side would by Refraction be collected, then Rays flowing from that Point become parallel on the other Side, and are ſaid to have their *Focus* at an infinite Diſtance. For the Power of Refraction in the *Medium* is the ſame, whether the Ray paſſes one Way or the other. For Inſtance, if the parallel Rays AB, CD (*Fig.* 8.) in paſſing through the refracting Surface BD are brought to a *Focus* in G, then Rays flowing from G, as a radiant Point, will afterwards proceed in the parallel Lines BA and DC. And the Point G, where the parallel Rays AB and CD meet after Refraction, is called the *Focus of parallel Rays*.

XVIII. When Rays proceed from a Point nearer the refracting Surface than the *Focus* of parallel Rays, they continue to diverge after Refraction, and their *Focus* is then an imaginary

nary one, and ſituated on the ſame Side the Surface with the Radiant.

For in this Caſe, their Divergency being greater than that which they would have, if they had proceeded from the *Focus* of parallel Rays, they cannot be brought to a Paralleliſm with one another, much leſs be made to converge, and therefore they continue to diverge, though in a leſs Degree than before they paſſed through the refracting Surface; upon which Account they proceed after Refraction, as if they came from ſome Point farther diſtant from the refracting Surface than their Radiant.

CHAP. IV.

Of Lenſes, *and the Manner in which Rays are affected in paſſing through them.*

A *Lens* is a *Medium* terminated on one Side by a ſpherical Surface, on the other by a Surface either plain or ſpherical. And of theſe there are five Sorts. The firſt, as A, (*Fig.* 23.) is plain on one Side, and convex on the other; the ſecond, B, convex on both Sides; the third, plain on one Side and concave on the other, as C; the fourth, D, concave on both Sides; the fifth, convex on one Side and concave on the other, as E, which is by ſome called a *Meniſcus*.

The

Part III. Plate I. Pag. 314.

Fig. 1. P. 277

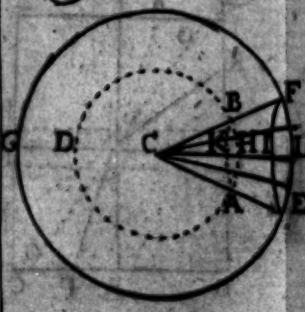


Fig. 2. P. 281

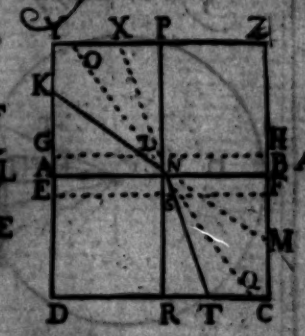


Fig. 3. P. 285

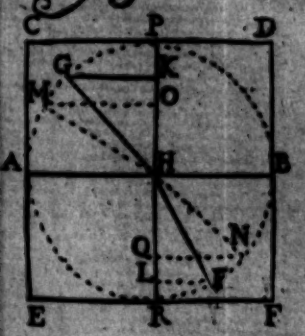


Fig. 4. P. 284

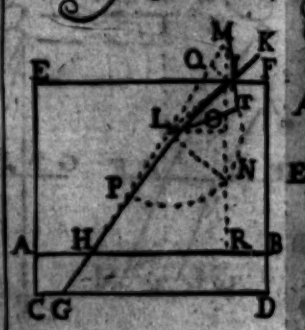


Fig. 5. P. 294

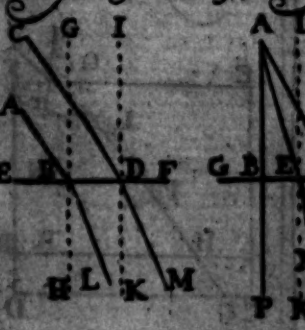


Fig. 6. P. 297



Fig. 7. P. 301

Fig. 8. P. 308.

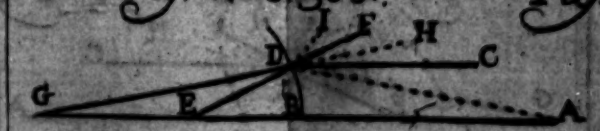
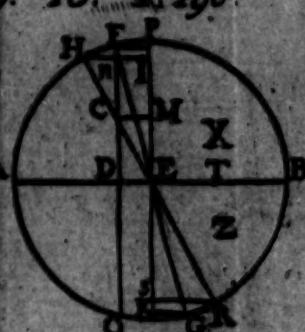


Fig. 9. P. 311.



Fig. 10. P. 290



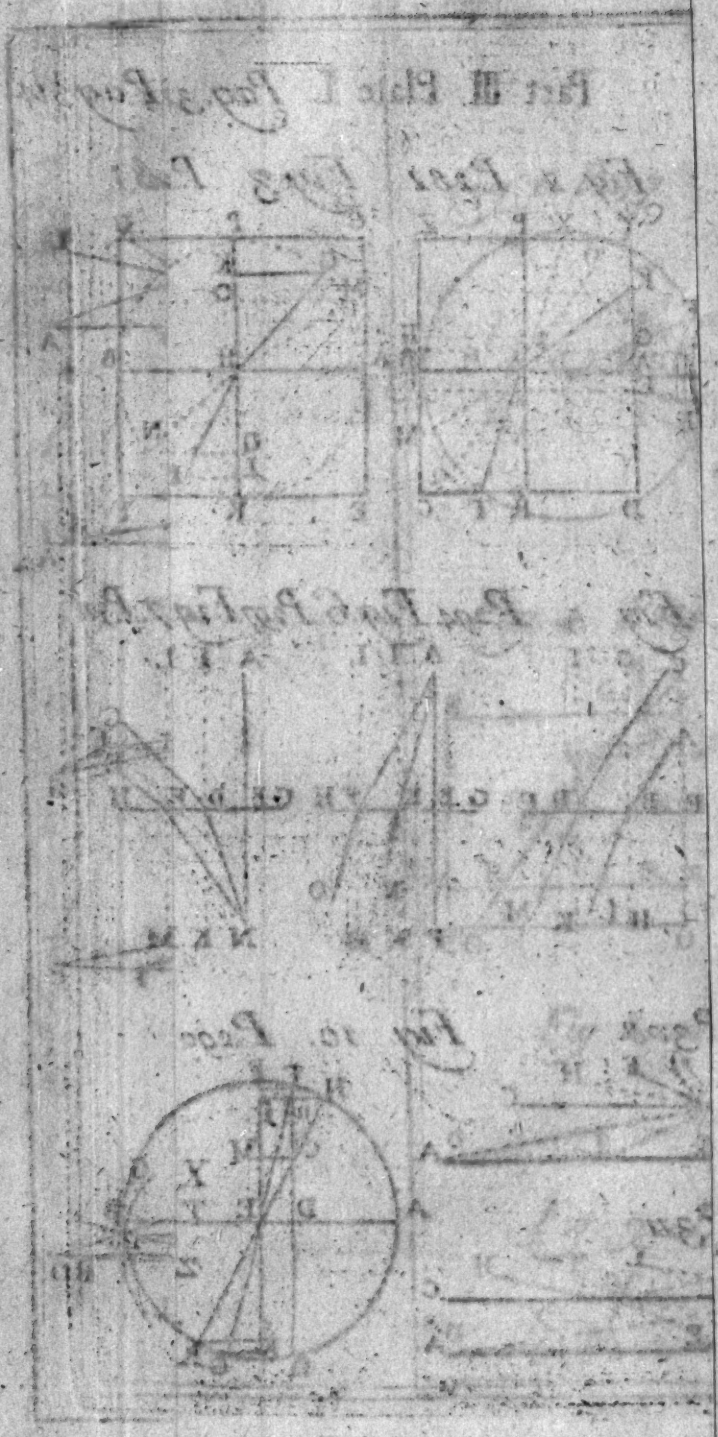


Fig. 11. P. 293

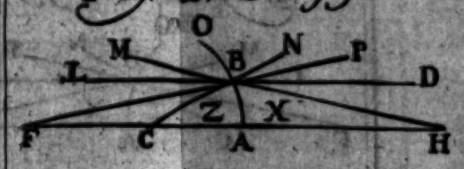


Fig. 12. P. 296

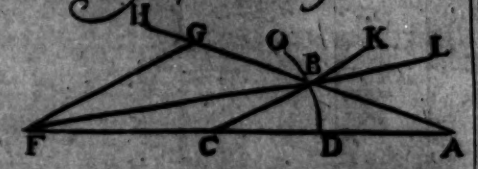


Fig. 13. P. 299

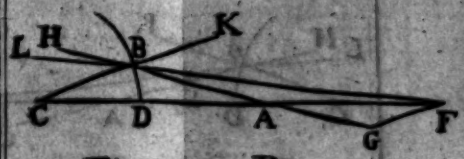


Fig. 14. P. 300.

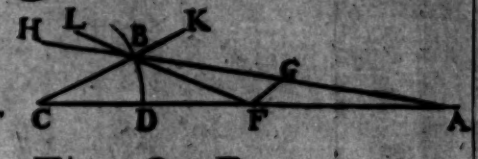


Fig. 15. P. 301

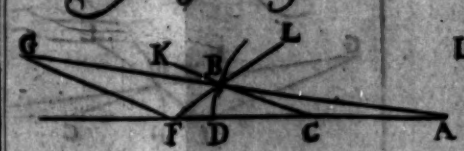


Fig. 16. P. 301.

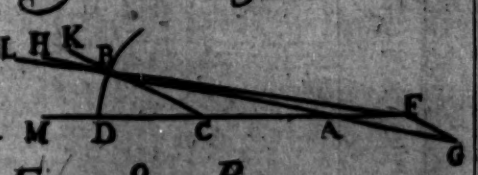


Fig. 17. P. 302

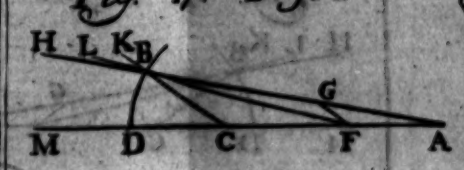


Fig. 18. P. 302.

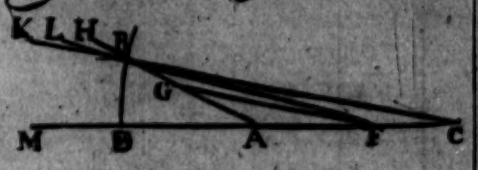


Fig. 19. P. 303.

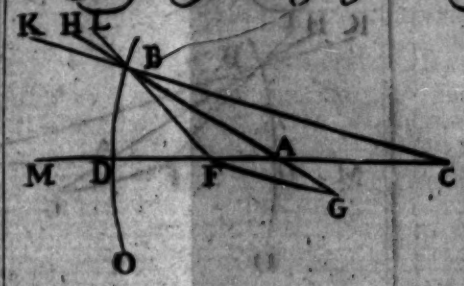
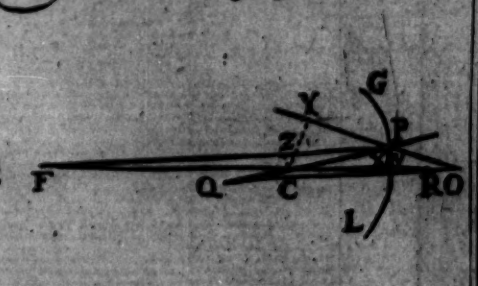
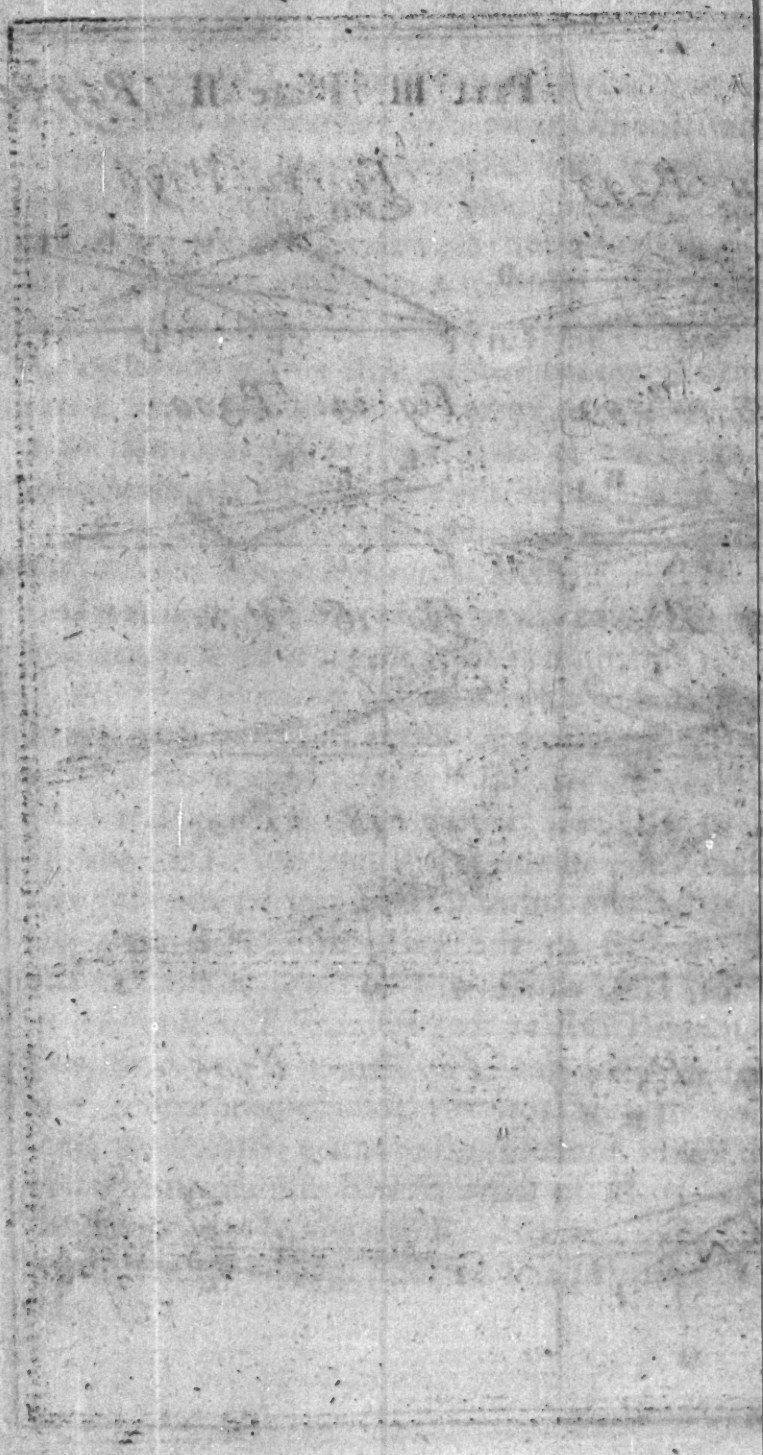


Fig. 20. P. 305.





The *Axis* of a *Lens* is a Line paſſing perpendicularly through both its Surfaces: Thus, the Line FG is an *Axis* common to all the five.

Lenſes are diſtinguiſhed into two general Kinds, convex and concave; the firſt and ſecond *Lenſes* are conſidered, as convex; the third and fourth, as concave: the laſt, if its Convexity is greater than its Concavity, is looked upon as convex; if, on the contrary, its Concavity be greater, it is conſidered as concave.

A *Lens* is always ſuppoſed to conſiſt of a *Medium* denſer than the circumambient one, unleſs where the contrary is expreſſed.

When parallel Rays fall upon the Surface of a convex *Lens*, they are refracted towards each other in paſſing through it, and thereby collected to a *Focus* on the other Side.

To explain this, let us trace the Progreſs of a Ray as AB (*Fig.* 24.) through the convex *Lens* CDEH, whoſe Axis is IK. Let L be the Center of the firſt Convexity CDE, and M that of the other CHE; and let the Ray AB be parallel to the Axis; through B draw the Line LN, which will be perpendicular to the Surface CDE at that Point. The Ray AB in entering the denſer Subſtance of the *Lens*, will be refracted towards the Perpendicular, and therefore proceed, after it has entered the Surface at B, in ſome Direction inclined towards the Axis, as BP. Through M, the Center of Convexity of this Surface, and the Point P, draw

the

the Line MR, which paſſing through the Center will be perpendicular to the Surface at P, and the Ray now entering a rarer *Medium* will be refracted from the Perpendicular into ſome Direction, as PF. In like Manner, and for the ſame Reaſons, the parallel Ray ST on the other Side of the Axis, and alſo all the intermediate ones as XZ, *&c.* will meet it in the ſame Point, unleſs the Rays AB and ST enter the Surface of the *Lens* at too great a Diſtance from the Axis IF, the Reaſon of which has already been fully explained *.

The Point F, where the parallel Rays AB, SF, *&c.* are ſuppoſed to be collected by paſſing through the *Lens* CE, is called the *Focus of parallel Rays of that Lens.*

If the Rays converge before they enter the *Lens*, they are then collected at a Point nearer to the *Lens* than the *Focus* of parallel Rays. If they diverge before they enter the *Lens*, they are then collected in a Point beyond F; unleſs they proceed from a Point on the other Side, at the ſame Diſtance with the *Focus* of parallel Rays, in which Caſe they are rendered parallel. If they proceed from a Point nearer than that, they diverge afterwards, but in a leſs Degree than before they enter the *Lens*.

If the *Lens* is plain on one Side, and convex on the other, the Rays are refracted the ſame Way, but in a leſs Degree.

* See *Obſervation* 3, in the foregoing Note.

If

If the Rays AB, ST, be ſuppoſed to proceed from a radiant Point on one Side of the *Lens*, and be refracted into a *Focus*, at F, on the other; then Rays proceeding from that focal Point, F, as from a Radiant, and ſuppoſed to paſs through the *Lens* the contrary Way, will be collected in that Point which was the Radiant in the other Caſe: and the nearer the radiant Point is to the *Lens*, the farther is the *Focus* from it on the other Side, and *vice verſâ*.

If the Rays AB, CD, EF, &c. (*Fig.* 25.) be parallel to each other, but oblique to GH the Axis of the *Lens* IK, or if the diverging Rays CB, CF, proceed as from ſome Point C, which is not ſituated in the Axis of the *Lens*, they will be collected into ſome Point as L, not directly oppoſite to the Radiant C, but nearly ſo: for the Ray CD which paſſes through M the Middle of the *Lens*, and falls upon the Surface of it with ſome Obliquity, will itſelf ſuffer a Refraction at D and N: but then it will be refracted the contrary Way in one Place to what it is in the other, and theſe Refractions will be equal in Degree if the *Lens* has an equal Convexity on each Side, as we may eaſily perceive if we imagine ND to be a Ray paſſing out of the *Lens* both at N and D, for it is evident the Line ND has an equal Inclination to each Surface at both its Extremities. Upon which Account the Difference between the Situation of the Point L, and one directly oppoſite

ſite to C, is ſo ſmall, that it is generally neglected; and the *Focus* is ſuppoſed to be in that Line, which a Ray, that would paſs through the middle Point of the *Lens*, were it to ſuffer no Refraction, would proceed in.

All which is ſufficiently clear, from what has been ſaid concerning the Laws of Refraction explained in the foregoing Chapter.

When parallel Rays fall upon a concave *Lens*, they are refracted from each other in paſſing through it, and thereby made to diverge, proceeding as from an imaginary *Focus* on the firſt Side of the *Lens*.

In order to comprehend this, let ABCD (*Fig.* 26.) repreſent a concave *Lens*, EO its Axis, GH the Radius of the firſt Concavity, IK that of the ſecond; produce HG to L, and let MG be a Ray of Light entering the *Lens* at the Point G. This Ray being refracted towards the Perpendicular GL, will paſs on to ſome Point, as K in the other Surface, more diſtant from the Axis than G, and being there refracted from the Perpendicular IK, will be diverted farther ſtill from the Axis, and proceed on in the Direction KN, as from ſome Point, O, on the firſt Side of the *Lens*. In like Manner other Rays, as PQ, parallel to the former, will proceed, after Refraction at both Surfaces, as from the ſame Point O; which, upon that Account, will be the imaginary *Focus* of parallel Rays of this *Lens*.

If

If the Rays diverge before they enter the *Lens*, their imaginary *Focus* is then nearer the *Lens* than that of the parallel Rays. If they converge before they enter the *Lens*, proceeding towards ſome diſtant Point in the Axis as E, they are then rendered leſs converging: if they converge to a Point at the ſame Diſtance from the *Lens* with the *Focus* of parallel Rays, they then go out parallel; if to a Point at a leſs Diſtance they remain converging, but in a leſs Degree than before they entered the *Lens*.

When the Rays enter the *Lens* diverging, the nearer their radiant Point is to it, the nearer alſo is their imaginary *Focus* after Refraction, and *vice versâ*.

If the *Lens* is plain on one Side, and concave on the other, the Rays ſuffer a like Refraction in each Caſe, but in a leſs Degree.

The Truth of what has been ſaid concerning the Paſſage of Rays through a concave *Lens*, is eaſily to be deduced from the Laws of Refraction delivered in the foregoing Chapter.

But the Method of determining the exact focal Diſtances of *Lenſes* is to be had from the Propoſitions laid down and demonſtrated in the Note in the foregoing Chapter. Thus, the Progreſs of the Rays, after their Refraction at the firſt Surface where they enter a *Lens*, is had by one of thoſe which determines the focal Diſtance of Rays entering a denſer *Medium* of ſuch Form: And their Progreſs, after their Re-

Refraction at the other Surface where they go out, is had by computing what Progreſs Rays, moving in the Direction they are found to have after their Entrance at the firſt Surface, will acquire by being refracted at the other; which is to be effected by one which determines the focal Diſtance of Rays paſſing out of a denſer *Medium* of like Form with that of the *Lens* *.

When

* Or a general Theorem may be made after the following Manner, to determine the Progreſs of Rays after Refraction at both Sides of the *Lens*, whatever be the Matter of it, or the Form wherein it is made.

Thus ſuppoſe GH (*Fig.* 28.) to be a given *Lens*, and E a Point in its Axis from whence the diverging Rays EL, *&c.* fall upon the *Lens*, AL, the *Radius* of the firſt Convexity, and CK that of the ſecond; let LK*f* be the Direction of the diverging Ray EL after its Refraction at the firſt Surface, and KF its Direction after Refraction at both. Then will *f* be the *Focus* of the Rays after their firſt Refraction, and F the Point they will meet in after both. Let BD be the Thickneſs of the *Lens*, and let the Proportion which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction be expreſſed by the *Ratio* of I to R. Call EB, d; BD, t; AB, r; CD, s; B*f*, x; DF, y: Now, to find *f* their *Focus* after Refraction at L where they enter the firſt Surface of the *Lens*, comes under the third Propoſition in the forementioned Note: According to which the *Ratio* compounded of x, *the focal Diſtance ſought*, to d, *the Diſtance of the radiant Point*; and of $d+r$, *the Diſtance between the ſame Point and the Center*, to $x-r$, *the Diſtance between the Center and the* Focus, is as I to R; compounding theſe two *Ratios* therefore (that is multiplying them together) we have $dx+rx : dx-dr :: I : R$; which Proportion being converted into an Equation, and duly reduced, gives $x = \frac{Idr}{Id-Rd-Rr}$.

Thus having found the Diſtance B*f*, and conſequently the Point *f*, to which the Rays converge from L, we muſt proceed to find F, that to which they will converge after having paſſed thro' K, where

When a Ray paſſes through a *Medium* terminated by two plain and parallel Surfaces, it is

where they ſuffer a ſecond Refraction: This comes under the ſame Propoſition; but, if we would uſe the ſame Letters as before, to expreſs the Proportion which the Sine of the Angle of Incidence bears to that of the Angle of Refraction, they muſt be put one for the other; becauſe, when Rays paſs out of a denſer into a rarer *Medium*, the Sine of the Angle of Incidence bears the ſame Proportion to the Sine of the Angle of Refraction, that the Sine of the Angle of Refraction does to the Sine of the Angle of Incidence, when they paſs out of a rarer into a denſer. This being obſerved, by the aforeſaid Propoſition, we ſhall have the *Ratio* compounded of y, *the focal Diſtance*, to $\frac{Idr}{Id-Rd-Rr}-t$, *the imaginary Focus of the incident Rays*, and of $\frac{Idr}{Id-Rd-Rr}-t+s$, *the Diſtance between the imaginary Focus and the Center*, to $y+s$, *the Diſtance between the Center and the Focus*, as R to I. Which Equation, if we reduce the mixed Quantities $\frac{Idr}{Id-Rd-Rr}-t$, and $\frac{Idr}{Id-Rd-Rr}-t+s$, into improper Fractions, will ſtand thus:

$$\left.\begin{array}{l} y:\frac{Idr-Idt+Rdt+Rrt}{Id-Rd-Rr} \\ \text{and} \\ \frac{Idr-Idt+Rdt+Rrt+Ids-Rds-Rrs}{Id-Rd-Rr}:y+s \end{array}\right\} :: R:I.$$

And, compounding theſe Ratios, we have

as $\frac{Idry-Idty+Rdty+Rrty+Idsy-Rdsy-Rrsy}{Id-Rd-Rr}$ is to $\frac{Idry-Idty+Rdty+Rrty+Idrs-Idts+Rdts+Rrts}{Id-Rd-Rr}$ as R to I.

And, throwing out the equal Denominators, multiplying Means together, and Extremes together, we ſhall have $IIdry-IIdyt+IRdty+IRrty+IIdsy-IRdsy-IRrsy=IRdry-IRdty+RRdty+RRrty+IRdrs-IRdts+RRdts+RRrts$; and this Equation being properly reduced, gives $y=\frac{IRdrs-IRdts+RRdts+RRrts}{IIdr-IIdt+2RIdt+IRrt+IIds-IRds-IRrs-IRdr-RRdt-RRrt}$.

is refracted one Way in going out of the ſecond, as much as it was the other in entering the firſt; and therefore proceeds afterwards not in the ſame Direction, but in one that is parallel to that which it had before. Thus if the Ray AB (*Fig.* 27.) enters the denſer *Medium* CDEF terminated by the parallel Surfaces CD and EF, it is refracted at B towards the Perpendicular BI, proceeding to a Point G, where it is as much refracted from the Perpendicular GK in going out, and proceeds in the Direction GH, not the ſame, but parallel to the former ABL.

This Theorem may be applied to all Caſes whatever, even to plain Surfaces *mutatis mutandis*; *v. g.* the *Radius* of a concave Surface being negative (as lying the contrary Way) with Reſpect to that of a convex, and the *Radius* of a plain Surface being an infinite Line; if we would apply this Theorem to a concave Surface, we muſt change all the Signs of thoſe Members wherein the Symbol expreſſing the *Radius* of that Surface occurs; and, if to a plain Surface, all the Members which involve the *Radius* muſt be conſidered as infinite Quantities; that is, all, except them, muſt be ſtruck out of the Equation as nothing. So, likewiſe, if we would have it extend to other Rays beſides diverging ones, the Point where converging Rays would meet, lying on the contrary Side to that from whence the diverging ones were ſuppoſed to flow, its Diſtance muſt be made negative; and, the Diſtance where parallel Rays meet being infinite, it is only changing the Signs of all thoſe Members in which *d* is found, if the Rays are ſuppoſed converging, or making thoſe Members infinite, in Caſe the Rays are ſuppoſed parallel; which is done by ſtriking out all the reſt, as bearing no Proportion to them.

See the Method of reducing this Equation to fewer Terms, where it is alſo illuſtrated with divers Inſtances, in Dr. *Brown*'s Appendix to *Gregory*'s Optics, or in Dr. *Halley*'s Method of finding the principal Focus of Optic Glaſſes univerſally, Philoſoph. Tranſact. N° 205.

CHAP.

Fig. 21. P. 307.

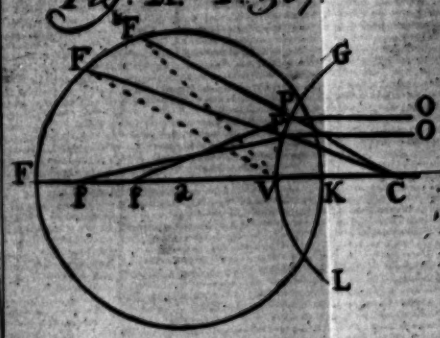


Fig. 22. P. 307.

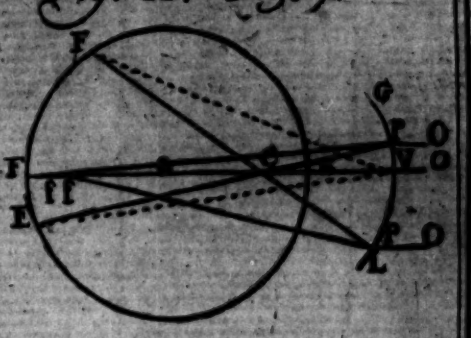


Fig. 23. P. 314.

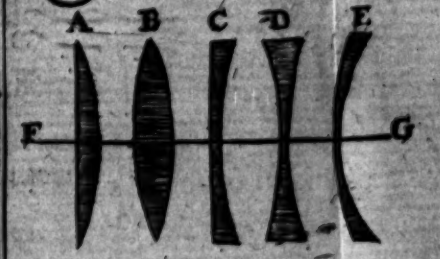


Fig. 24. P. 315.

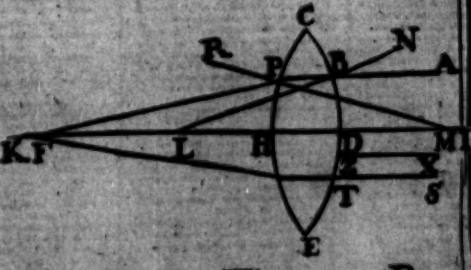


Fig. 25. P. 317

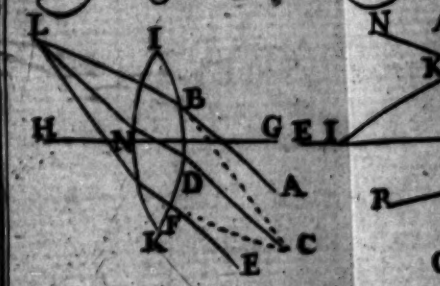


Fig. 26. P. 318

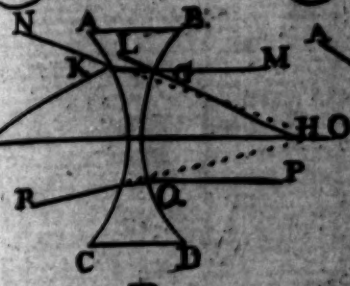


Fig. 27. P. 322

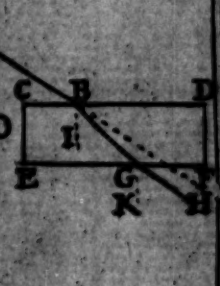
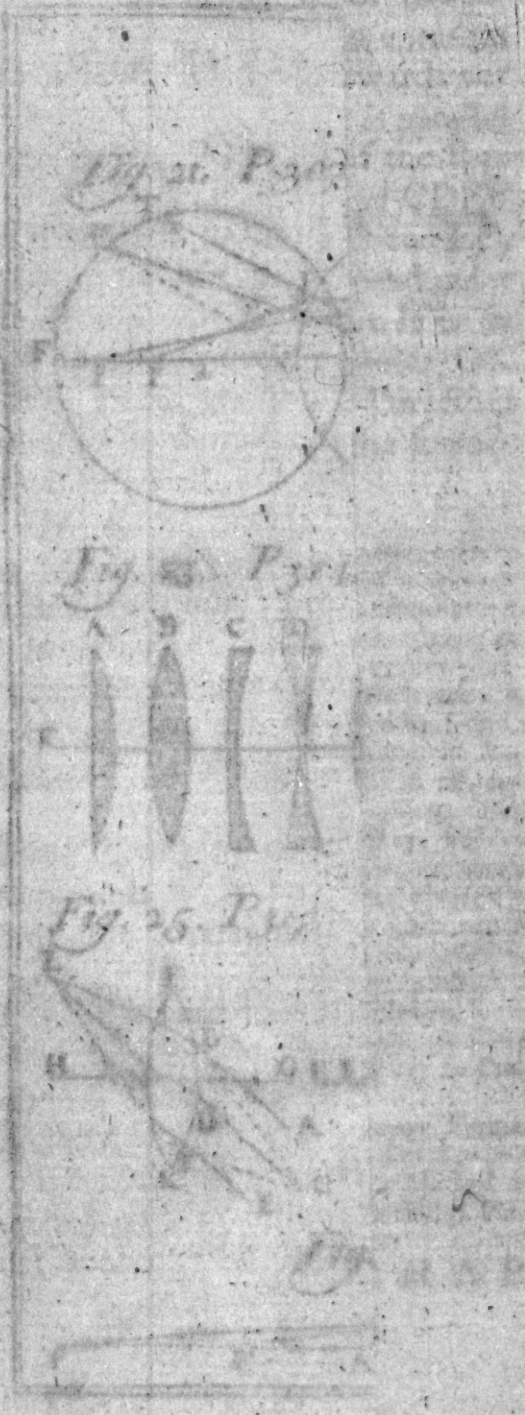


Fig. 28. P. 320




Fig. 24. Pag.
F
Fig. 25. Pag.
A
B
C
F
Fig. 26. Pag.
H

CHAP. V.

Of the Eye.

THE Form of the Eye is ſuch as is repreſented in Figure 29, and would be a perfect Sphere, were not the fore Part A A ſomewhat more protuberant than the reſt.

The Deſcription of it, ſo far as is neceſſary to explain the Nature of Viſion, is as follows:

It is incloſed in three diſtinct Coats or Teguments; the outermoſt of which, *viz. a a*, is called *Tunica Sclerotica*; the next, *c c*, *Choroides*, or *Uvea*; the third and innermoſt, *d d*, derives its Name from that of its Diſcoverer, and is called *Tunica Ruyſchiana*.

Theſe Coats are contiguous to each other every where, except on the fore Part of the Eye.

That Portion of the *Sclerotica* which lies between A and A is more protuberant than the reſt, is tranſparent, and has the Name of *Tunica Cornea*.

That Portion of the *Choroides* which is ſituated between *b* and *b* is called the *Iris*, and is that which by its Colour denominates an Eye black, grey, *&c.* In the Middle of this there is a round Hole as *p p*, called the *Pupil*.

The *Iris* conſiſts of two Kinds of muſcular Fibres; the firſt lie extended from its Extre-

mity like the *Radii* of a Circle, and point towards the Middle of the Pupil as towards a Center: the other are circular ones, and ſurround the Pupil, having the Middle of it for their common Center. Theſe are connected to the former where they croſs them: and therefore when theſe contract, the Pupil is diminiſhed; when the other contract, it is inlarged.

Within the Cavity of the Eye, and not far behind the Pupil, there is a ſoft tranſparent Subſtance CC, not unlike a double convex *Lens*, one of whoſe Surfaces as S, is more convex than the other. This is called the *Cryſtalline Humour*, and is ſuſpended within the Eye by certain Ligaments, as *Cl, Cl*, called *Ligamenta Ciliaria*, or *Proceſſus Ciliares:* theſe are convex towards the Pupil, as expreſſed in the Figure, and concave on the other Side, and are muſcular, and therefore capable of Contraction and Dilatation. The convex Sides of theſe Ligaments are lined with a very black Subſtance, as is alſo that Side of the *Iris* which is next them.

The *Tunica Ruyſchiana* leaves the *Choroides* at *l*, and, paſſing behind the *Ligamenta Ciliaria* and the Chryſtalline Humour, is contiguous to them, and joins the *Choroides* again at *l*, on the other Side the Pupil.

By Means of the forementioned Parts, the Cavity of the Eye is divided into two Portions; the one of which V V is filled with a Fluid nearly of the ſame Denſity with Water, and

is

is therefore called *Humor Aqueus*; the other TT contains a Fluid whoſe Conſiſtency is greater than that of the former, and is called *Humor Vitreus:* And both theſe Humors are rarer than the Subſtance of the Chryſtalline.

At the back Part of each Eye, but not directly oppoſite to the Pupil, there enters a Nerve as N N, which is called the *Optic Nerve.*

The Fibres of this Nerve, after their Entrance into the Eye at N, ſpread themſelves over the innermoſt Coat of it as far as the *Ligamenta Ciliaria*, and form a very thin Membrane, called *Tunica Retina.*

The innermoſt Coat of the Eye is every where covered (except that Part of it which is contiguous to the back Part of the Chryſtallin Humour) with a very black Subſtance, not unlike that with which the back Part of the Iris and fore Part of the *Ligamenta Ciliaria* were obſerved to be covered. This is to hinder any Light from being reflected from thoſe Parts to the *Retina*; for that would render the Images of Objects indiſtinct; as we ſhall ſee when we have explained the Nature of Viſion, which is the Subject of the next Chapter.

CHAP. VI.

Of the Nature of Viſion.

SUCH is the Subſtance and Form of the Humours of the Eye, when lodged in their proper Receptacles, that Rays of Light in paſſing through them are affected in the like Manner as in paſſing through a convex *Lens*; as we ſhall ſee immediately: and therefore, to underſtand the Nature of Viſion, which depends on the Paſſage of Rays flowing from the *ſeveral* Points of a diſtant Object through thoſe Humours, little more is required than to know how the ſame Rays would be affected, were they ſuppoſed to paſs through a convex *Lens*: Which may eaſily be done by applying to this Caſe what has been delivered in the fourth Chapter concerning the Manner in which Rays flowing from a *ſingle* Point are affected in paſſing through *Lenſes* of that Kind.

We have already ſeen, in the abovementioned Chapter, that Rays flowing from a *ſingle* radiant Point, and afterwards falling on a convex *Lens*, are collected to a *Focus* in ſome Point oppoſite, or nearly ſo, to the radiant. Let us now ſuppoſe an Object placed before a *Lens*, but farther from it than the focal Diſtance of its parallel Rays; and let it ſend forth

Rays

Rays from each Point of its Surface in every Direction, as from ſo many radiant Points. Some of the Rays which flow from each Point of that Surface of the Object which is turned towards the *Lens*, will neceſſarily fall upon it, and, paſſing through it, will be collected in ſo many diſtinct focal Points on the oppoſite Side, as there are diſtinct radiant Points in the Surface of the Object from whence they came. Now, as the radiant Points are contiguous to each other in the Surface of the Object on one Side of the *Lens*, the focal Points will alſo be contiguous on the other, and as each focal Point is oppoſite to its reſpective Radiant, their Places will have the ſame Relation to each other, that thoſe of the radiant have; and, conſequently theſe Points, taken together, will be a true Repreſentation and perfect Image of that Object; for each Point will exhibit the ſame Colour that its correſpondent Point in the Object is of: But, becauſe each Point in the Repreſentation is oppoſite to its reſpective one in the Object, the Image will be inverted. The Truth of this may eaſily be experienced, if we hold a clean white Paper facing the *Lens* in the Place where the focal Points are, and take Care to prevent all other Light from falling upon the Paper, except that which paſſes through the *Lens* *.

To

* On this depends the Structure of the *Camera Obſcura*, which is a Contrivance to exhibit the Repreſentation of ſuch

To illuſtrate this, let PQR (*Fig.* 30.) repreſent an Object placed before the *Lens* AB, and ſending forth Rays from each Point in its Surface; and let *q* be the focal Diſtance of Rays proceeding from Q, and paſſing through the ſaid *Lens*. Then will all the Rays that proceed from the Point Q, between the Lines QA and QB, be collected in *q*; in like Manner all that flow from P, between the Lines PA and PB, will meet in the oppoſite Point *p*; and ſo many as proceed from R, and paſs through the *Lens*, will be collected in *r*; and all the Rays that flow from the remaining Points between P and R and fall upon the *Lens*, will be collected in as many Points between *p* and *r*; and, if the Rays are received there upon a white Surface, there will be exhibited an Image of the Object PR, but inverted; becauſe the Rays PL*p* and RL*r*

Objects as may be ſeen from a Window upon ſome plain white Surface held before the Window within the Room. In order to do this, a common Spectacle Glaſs, or Burning Glaſs, (both which are convex *Lenſes*) muſt be fixed in an Hole in the Window-ſhutter; for then, if no Light be ſuffered to enter into the Room, but what paſſes thro' the Hole, and a Sheet of white Paper be held oppoſite to the Hole at that Diſtance where the Rays proceeding from the Objects abroad, and paſſing thro' the Glaſs, are collected into their reſpective *Foci*, we ſhall have the Images of all the Objects which lie before the Hole repreſented upon the Paper inverted; but in a much more lively and exact Manner than can be done by the Pencil; and not only the Objects and their reſpective Situations, but, what is peculiar to this Sort of Painting, their Motions will alſo be expreſſed.

croſs

croſs each other at L in paſſing through the *Lens* *.

Thoſe Rays which flow from the ſame Point of an Object, when conſidered together, are called a *Cone* or *Pencil* of Rays. Thus the Rays QA, QL, *&c.* conſtitute a Pencil flowing from the Point Q; ſo the Rays PA, PL, *&c.* a Pencil from the Point P; and the middle Ray of each Pencil, as PL, QL, *&c.* is called the *Axis* of that Pencil, to which it belongs.

Now in like Manner as the ſeveral Pencils of Rays flowing from the diſtinct Points in the Surface of an Object placed before a *Lens*, are collected in ſo many Points at a certain Diſtance on the other Side of the *Lens*, and form an Image there when received upon a white Paper; ſo Pencils proceeding from an Object placed before the Eye at a proper Diſtance from it, and being refracted in paſſing through the Humours of it, are collected into their reſpective *Foci* upon the *Retina*, where they form a Repreſentation of that Object; and by their Impulſes upon the tender Nerves of the *Retina*, an *Idea* of the Object is excited in the Mind.

* A burning Glaſs is no other than a Piece of Glaſs ground into the Form of a convex *Lens*; for if the Rays of the Sun are permitted to paſs through ſuch a one, they will burn very ſtrongly in the Place where they are collected into their reſpective *Foci*; upon which Account it is, that the Point where Rays in general are collected, is called their *Focus*, that is, their Place of Burning.

The

The Progreſs of the Rays through the Humours of the Eye, are expreſſed in the 31ſt Figure: where FG*t* is the Eye, FG the *Tunica Cornea*, *pp* the Pupil, AA the aqueous Humour, HH the chryſtalline, and VV the vitreous. And RS repreſents an Object placed before it, emitting Pencils of Rays from its ſeveral Points R, S, T, &c. The Rays which conſtitute the Pencil GTF, in entering the aqueous Humour, paſs out of a rarer into a denſer *Medium* through a convex Surface, in which Caſe diverging Rays are made to diverge leſs, to become parallel, or to converge (Chap. 3. Prop. 7.) in entering the Chryſtalline they do the like; and in paſſing out of that, they proceed out of a denſer into a rarer *Medium*, through a convex Surface of the denſer, which alſo has the ſame Effect, (Chap. 3. Prop. 15.) By which Means they are made to converge, as deſcribed in the Figure, and to meet together in a *Focus* at *t*, a Point in the *Retina*. In like Manner the Rays flowing from R, and conſtituting the Pencil GRF, will proceed as deſcribed in the Figure, and after Refraction meet in *r*; and the Rays proceeding from S will be collected in *s*, &c. by which Means an Image of the Object will be formed in *rts* upon the *Retina*, but becauſe the Pencils croſs each other in paſſing through the Pupil, it will be inverted *.

The

* Of this we have experimental Proof: For if we cut away the back Part of an Eye, and apply a Paper there, we ſhall ſee the Images

The leſs the Diſtance between the Object and the Eye is, the more the Rays which come from the Object are ſaid to diverge, and *è contra*: Not that the Situation of the Eye makes any Alteration in the Progreſs of thoſe Rays, but that, when the Eye is placed nearer the Object, it receives into its Pupil Rays which diverge in a greater Degree than thoſe which it can receive when placed farther off. The following Illuſtration will make this clear: Let AB (*Fig.* 32.) repreſent an Object emitting Rays from each Point of its Surface, and let C*d*, C*r*, &c. expreſs thoſe which flow from the Point C: let *m* be a Pupil of an Eye placed at the Diſtance C*m* from it; 'tis plain this Pupil will receive into it the diverging Rays C*r*, C*s*; whereas the Rays C*o*, C*t*, will diverge the moſt of any that can enter the ſame Pupil, when placed at the Diſtance C*n*; but theſe diverge leſs than the former, the Angle *o*C*t* being included in the Angle *r*C*s*.

Images of external Objects placed thereon, as accurately as in the *Camera Obſcura*, provided no Light is permitted to fall upon the Paper, except that which paſſes through the Humours of the Eye.

Optic Writers have made it Matter of great Difficulty to determine the Point where the *Axis* of the Pencils which enter the Eye, croſs each other; ſome placing it in the Center of the Eye, others in the Vertex of the Chryſtalline Humour, others in that of the *Tunica Cornea*: But as the Rays of each Pencil fill the Pupil, or as the Pupil itſelf is a common Baſe to each Pencil, it is inconceivable how the *Axis* of thoſe Pencils ſhould croſs each other in any other Place than the Center of the Pupil. See Fig. 31, or any other where ſeveral Pencils are repreſented, as paſſing through the Pupil of an Eye.

Viſion

Viſion is diſtinguiſhed into *bright* and *obſcure*; *diſtinct* and *confuſed*.

It is ſaid to be *bright*, when a ſufficient Number of Rays enter the Pupil at the ſame Time; *obſcure*, when too few. It is *diſtinct*, when each Pencil of Rays is collected into a *Focus* exactly upon the *Retina*; *confuſed*, when they meet before they come at it, or when they would paſs it before they meet; for in either of theſe laſt Caſes, the Rays flowing from different Points of the Object, will fall upon the ſame Part of the *Retina*, which muſt neceſſarily render the Image confuſed and indiſtinct.

Now that Objects may appear with a due *Brightneſs*, whether more or fewer Rays proceed from them, we have a Power of contracting, or dilating the Pupil by Means of the muſcular Fibres of the *Iris* (as explained in the foregoing Chapter,) in order to take in more or fewer Rays as Occaſion requires. But this Power has its Limits *.

And that the Rays may be collected into Points exactly upon the *Retina*, that is, that Objects may appear *diſtinct*, whether they be nearer or farther off, that is, whether the Rays proceeding from them diverge more or leſs, we have a Power of contracting or relaxing the *Ligamenta Ciliaria*, and thereby altering the

* In ſome Animals, this Power is much greater than in others; particularly in ſuch as are obliged to make Uſe of their Eyes by Night, as well as by Day, as in Cats, *&c.*

Form

Fig. 29. P. 323.

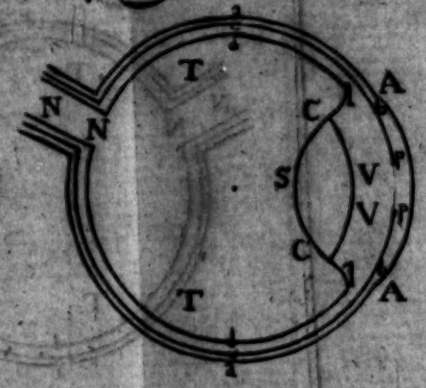


Fig. 31. P. 330.

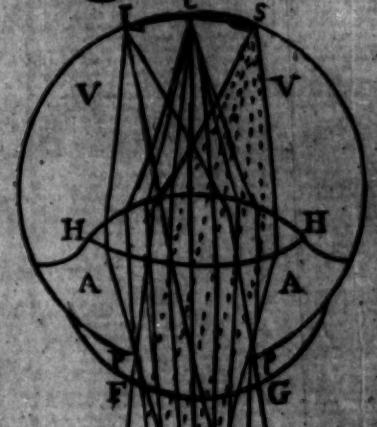
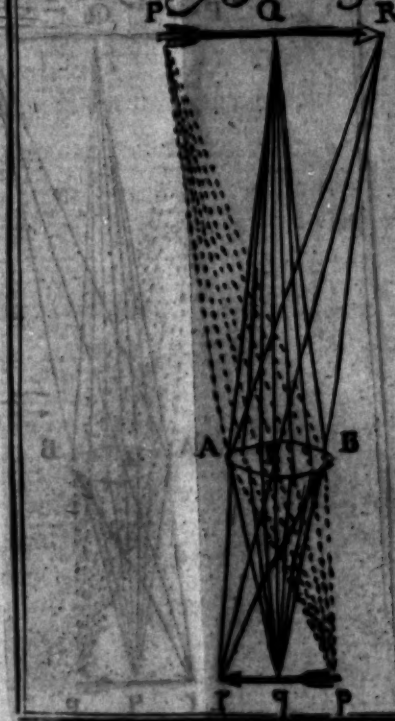
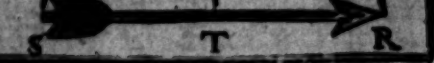
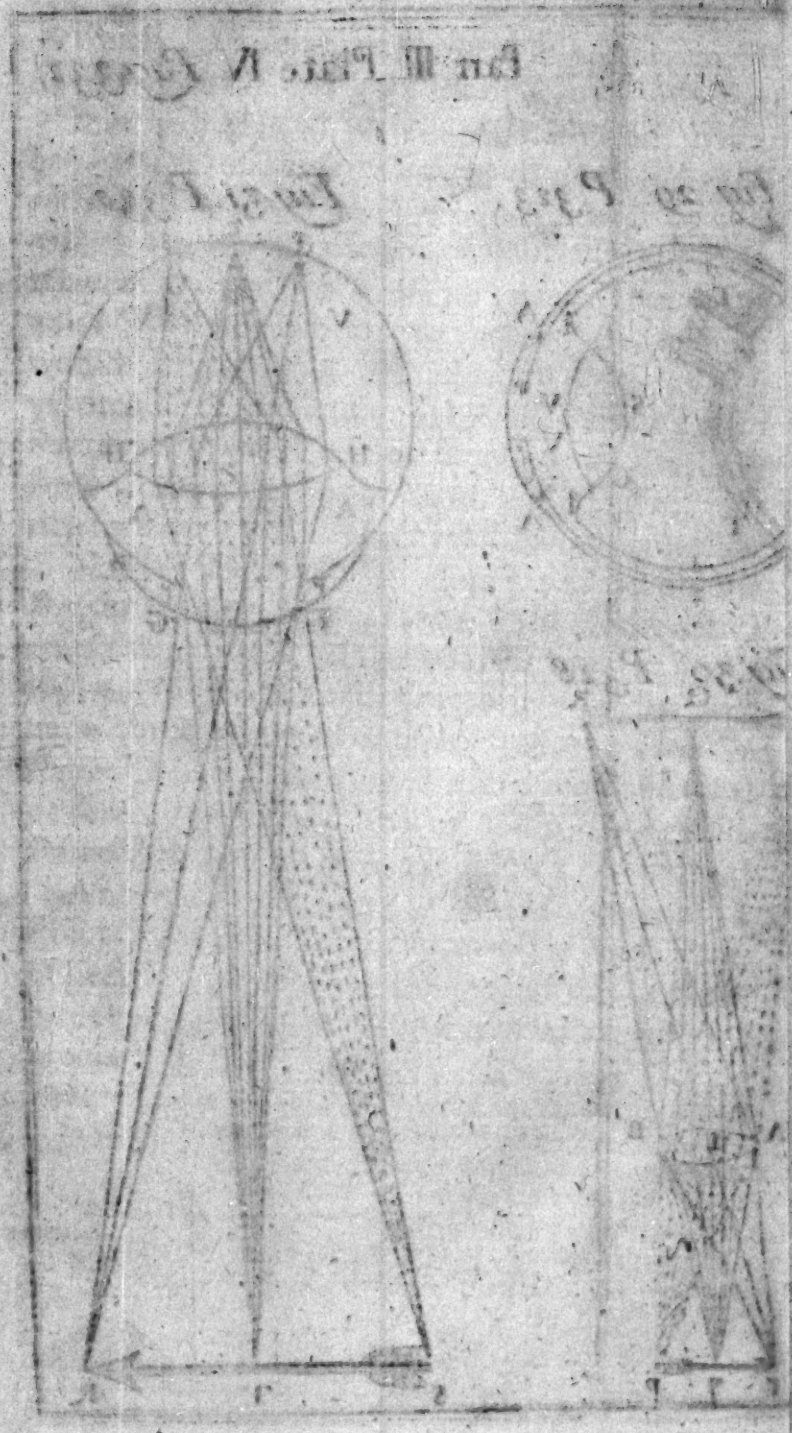


Fig. 30. P. 328.







Form of the Chryſtalline Humour, and with that the focal Diſtance of the Rays. Thus, when the Object we view is far off, and the Rays fall upon the Pupil with a very ſmall Degree of Divergency, we contract the *Ligamenta Ciliaria*, which being concave towards the vitreous Humour, do thereby compreſs it more than otherwiſe they would do; by this Means it is made to preſs harder upon the back Side of the Chryſtalline Humour, which is thereby rendered flatter; and ſo the Rays proceed farther before they meet in a *Focus* than otherwiſe they would have done. Add to this, that we dilate the Pupils of our Eyes (unleſs in Caſes where the Light is ſo ſtrong that it offends the Eye) and thereby admit Rays into them, that are more diverging than thoſe which would otherwiſe enter. And when the Rays come from an Object that is very near, and therefore diverge too much to be collected into their reſpective *Foci* upon the *Retina*, by relaxing the *Ligamenta Ciliaria* we give the Chryſtalline a more convex Form, by which Means the Rays are made to ſuffer a proportionably greater Degree of Refraction in paſſing through it *.

* Some Philoſophers are of Opinion, that we do this by a Power of altering the Form of the Eye; and others, by removing the Chryſtalline forwards or backwards as Occaſion requires; but neither of theſe Opinions is probable; for the Coats of the Eye are too hard, eſpecially in ſome Animals, for the firſt; and as to moving the Chryſtalline out of its Place, the Cavities of the Eye ſeem to be too well filled with the other Humours to admit of ſuch Removal.

And

And beſides this, by contracting the Pupils of our Eyes, we exclude the more diverging Rays, and admit only ſuch as are more eaſily refracted into their reſpective *Foci* *. But Viſion is not diſtinct at all Diſtances, for our Power of contracting and relaxing the *Ligamenta Ciliaria* is alſo circumſcribed within certain Limits.

The nearer an Object is placed to the Eye, the greater is the Image of it upon the *Retina*. Becauſe the Pencils flowing from the extreme Parts of the Object when near, make a larger Angle with each other in the Pupil where they croſs, than the ſame Pencils do when the Object is placed farther off. Thus AB (*Fig.* 33.) the Image of the Object CD, far exceeds EF that of the ſame Object GH, placed at a greater Diſtance from the Eye, as is evident from Inſpection of the Figure.

In thoſe Eyes where the *Tunica Cornea* is very protuberant and convex, the Rays of Light ſuffer a very conſiderable Refraction at their Entrance into the aqueous Humour, and are therefore collected to a *Focus* before they fall upon the *Retina*, unleſs the Object be placed very near, ſo that the Rays which enter the Eye, may have a conſiderable Degree of Divergency. People that have ſuch Eyes, are

* Accordingly it is obſervable, that if we make a ſmall Hole with the Point of a Needle thro' a Piece of Paper, and apply that Hole cloſe to the Eye, making Uſe of it, as it were, inſtead of a Pupil, we ſhall be able to ſee an Object diſtinctly through it, tho' the Object be placed within half an Inch of the Eye.

ſaid

ſaid to be *purblind*. Now the nearer an Object is placed to the Eye, the greater is the Image of it therein, as explained above; theſe People therefore can ſee much ſmaller Objects than others, as ſeeing much nearer ones with the ſame Diſtinctneſs. And their Sight continues good longer than that of other People, becauſe the *Tunica Cornea* of their Eyes, as they grow old, becomes plainer, for Want of that Redundancy of Humours with which they were filled before.

On the contrary, old Men having the *Cornea* of their Eyes too flat for want of a ſufficient Quantity of the Aqueous Humour to fill them out, if the Rays diverge too much before they enter the Eye, they cannot be brought to a *Focus* before they reach the *Retina*; on which Account thoſe People cannot ſee diſtinctly, unleſs the Object be ſituated at a greater Diſtance from the Eye, than is required for thoſe whoſe Eyes are of a due Form.

Since the Images of the Objects we look at are inverted in the Eye, it may be thought the Objects themſelves ought to appear ſo; but it muſt be conſidered, that there is no *natural* Connection between the *Idea* in our Mind, and the Image upon the *Retina*; we find by Experience, that when ſuch an *Idea* is excited in our Mind, ſuch an Object ſtands before us in ſuch a Poſition and of ſuch a Form; whenever therefore the like *Idea* is excited again, we con-

conclude there is a like Cauſe of it. For it is found by Obſervation, that People who have been born blind, and have afterwards received their Sight, have had no Information from their Eyes at firſt, concerning the particular Situation or Form of Bodies; but have been obliged to ſtay till Experience has taught them what Figures and Situation of Bodies correſpond to ſuch and ſuch Senſations in the Mind *.

In like Manner it is from Experience that an Object appears ſingle, though there be an Image of it in each Eye: for after we find, that its Place, according to the Repreſentation of it in each Eye, is the ſame, we neceſſarily conceive it to be but one. The Manner how we come to find this ſeems to be as follows: There is one Part of the *Retina* upon which when the Image falls, the Object appears brighter and more diſtinct, than when it falls upon any other, as is evident, becauſe we always ſee one Part of an Object with greater Diſtinctneſs than any of the reſt. This Point I ſhall hereafter call *the Point of Diſtinct Viſion*. This naturally leads us to turn our Eyes ſo, that the Object may be ſituated directly oppoſite to this Point. And this Action of ours is that which has given Riſe to thoſe imaginary Lines, which are ſuppoſed to paſs di-

* See Mr. *Cheſſelden*'s Obſervations on a young Gentleman couched by him at the Age of 13 Years. Philoſoph. Tranſact. N° 402.

rectly

rectly through the Eye and to terminate in the Object we view, and are commonly called the *optic Axes*. We therefore turn our Eyes ſo that the Object may appear in thoſe Lines. Therefore, ſince theſe Lines concur at the Object, when we endeavour to view it with Diſtinctneſs, each Eye affords us an *Idea* of the Object in the ſame Place, from whence it neceſſarily appears but one *.

When

* There are other Methods of accounting for theſe two laſt *Phænomena*, ſome of which, perhaps, the Reader may think more plauſible; for the Connection between the Image on the *Retina* and the Idea in the Mind being purely *metaphyſical*, we can never hope to arrive at Certainty in this Matter.

Some are of Opinion, that we judge thoſe Rays which paint the uppermoſt Part of the Image in the Eye to proceed from the lowermoſt Part of the Object, becauſe they ſtrike upon the *Retina*, as coming from that Part; and that we conclude from hence that the Object is erect, though the Image be inverted; as if the Direction, wherewith the Rays ſtrike the *Retina*, informed the Mind which Way they came. This Solution ſerves alſo to explicate the *Phænomenon* of ſeeing but one Object with both Eyes; for as the Mind is informed, by the Direction with which the Rays ſtrike the *Retina*, of the Place from whence they come; therefore when it appears that they enter each Eye as from the ſame Place, the Object neceſſarily ſeems to be but one; becauſe we can't ſuppoſe two to exiſt in the ſame Place at the ſame Time.

Some have been ſo abſurd as to embrace an Opinion, the firſt Author of which was *Gaſſendus*, that we ſee one and the ſame Point of an Object only with one Eye at a Time, (*otiante alio*, as they expreſs it) while the other does nothing. Vid. *Gaſſendi* Epiſtol. de Magnitud. Solis; or *Tacquet*. Optic. Lib. I. Prop. 2.

Some imagining that the *Optic* Nerves conſiſt of a Bundle of ſmall ones wrapped up in one common Tegument, are of Opinion, that ſuch as lie upon the *Retina* at equal Diſtances from the Point of diſtinct Viſion, and on the ſame Side of it in each Eye, are connected together in one, before they terminate in the Brain; and ſo, whether one or both are affected, only one *Idea*

Whenever the Eyes are ſo ſituated with reſpect to an Object, that the ſame Part of the *Retina* in each Eye is affected by the Rays that flow from it, which is wont to be affected when two Objects are placed before the Eyes, the Mind, receiving no Information from without, but by the Impulſes of the Ray upon the *Retina*, judges that there are two Objects. Thus, let A, B (*Fig.* 34.) repreſent two Eyes whoſe *optic Axes* are directed to the Point C, and let E be an Object on one Side the Point C, and F an Object on the other. Now, Objects thus ſituated muſt appear ſeparate: otherwiſe, every Time we viewed an Object we muſt imagine all the different Points in its Surface to be but one, which is contrary to Experience. In this Caſe, the Point *d* in each Eye will be affected by the Rays which flow from thoſe Objects; but ſo it will, if a ſingle Object be placed at D; and therefore, for the Reaſon given above, an Object in that Situation ſhall appear as the two ſeparate ones E and F, that is, double. A-

is excited in the Mind. *Graveſand* confirms this Opinion, by aſſerting that, in all Animals which look at the ſame Object with both Eyes, the optic Nerves concur, before they enter the Brain; and that, in ſuch as look at one Object with one Eye, and at a different one with the other, they are ſeparate all the Way.

Others, with *Briggius* (ſee his *Ophthalmog.* Chap. 11.) do not contend, that the forementioned correſponding Parts of the *optic* Nerves are connected before they terminate in the Brain; but that they are of an equal Tenſion, and therefore excite the ſame Senſation in the Mind.

gain,

gain, let there be an Object placed without the *optic Axes*, as at G, Rays flowing from this will affect the ſame Part in each Eye, as if there were two diſtinct Objects, *viz.* one at E, and the other at H; this therefore will alſo appear double. Farther, as the Objects D and G are ſituated in this Figure, if both are attended to at the ſame Time, (the *optic Axes* being ſtill directed to the ſame Point C) they will appear as three, being ſituated oppoſite to the three Points F, E, and H. And what has been ſaid of the Appearance of the Objects D and G, as they are ſituated in this Figure, may be applied to their Appearance as they are placed in the next, where they are repreſented as being beyond the Line HF. So that wherever an Object is placed, provided it be nearer to the Eyes than the Point where the *optic Axes* concur, or farther from them, it appears double.

There is one Part of the *Retina* of each Eye, upon which, when the Image falls, the Object cannot be ſeen at all with that Eye; the Proof of this we have from the following Experiment. Fix two Objects upon a Wall, of ſuch Bigneſs that each may hide a ſquare Inch of it, or thereabouts, and at the Diſtance of about a Foot or two from each other, and go back from the Wall about three Times that Diſtance; then ſhutting the left Eye, look at the left Object with the right one, and while the right Eye is in that Poſition, the right Object will not be

ſeen. So, if in that Station we look at the right Object with only the left Eye, the left Object will diſappear. The Reaſon of this is ſuppoſed to be, that the Image of that Object which diſappears falls upon the Blood-veſſels of the *optic* Nerve, from which no Senſation is conveyed to the Brain.

The Angle comprehended between the Rays which flow from the extreme Parts of the Object, and croſs in the Pupil, is called the *optic Angle*.

'Tis by Means of this Angle that we are able to form ſome Judgment of the Magnitude of an Object; becauſe, the larger this is, the larger is the Image upon the *Retina*, that is, a greater Portion of it is affected by the Rays which flow from that Object. But this is not ſufficient alone, becauſe different Objects, at different Diſtances from the Eye, may ſubtend equal Angles at the Pupil. We ought therefore to know alſo the Diſtance of the Object.

This, if the Object be very near, we are able to form a tolerable Judgment of, by the Degree of Divergency, wherein the Rays which flow from the ſame Point of the Object enter the Eye; becauſe we find it neceſſary to adapt the Eye accordingly, in order to bring them to a *Focus* upon the *Retina*.

But, when the Object is at greater Diſtance from us, a conſiderable Variation in the Diſtance of it makes but a very ſmall one in the Diver-

Divergency of thoſe Rays, and therefore this Rule of judging ceaſes to be of Uſe. The only Expedient then is the Angle comprehended between the *optic Axes* at the Object to which they are directed, or, which is the ſame Thing, the Poſition of the Eyes with reſpect to each other when they view the Object *. But in very large Diſtances this Poſition varies ſo little, that it is alſo of no Uſe; in which Caſe, we make the beſt Judgment we can from the Brightneſs, Diſtinctneſs, and apparent Magnitude of the Object, and likewiſe from its Situation with reſpect to others which are interpoſed †.

When we are unable to judge rightly concerning the Diſtance of an Object, we conceive it greater, the farther we imagine it to be from us, and *vice verſâ*; becauſe it requires a larger Object to exhibit the ſame Image upon the *Retina*, when it is ſituated at a great Diſtance, than when near. Thus we imagine the Sun and Moon to be farther off, when they are in the *Horizon*, than when they are near the *Meridian*, and accordingly think them

* That the Poſition of the *optic Axes* is a Means whereby we judge of Diſtances, is evident from hence, *viz.* that they who have loſt the Sight of one Eye, find it much more difficult to eſtimate the Diſtances of Objects, than they did when they had the Uſe of both.

† We have a remarkable Inſtance of the Error of our Judgment concerning the Diſtances of very remote Bodies, in that we look upon the Sun, Moon, and Stars to be all at the ſame Diſtance, whereas ſome of them are a thouſand Times farther from us than others.

proportionally larger in one Situation than in the other, though they are found to exhibit the ſame Image upon the *Retina* in both Caſes*.

We are never able to ſee very diſtant Objects with Diſtinctneſs; this is not ſolely owing to the Pupil's not receiving into it a ſufficient Number of Rays for that Purpoſe, or becauſe they are not collected into *Foci* upon the *Retina*, but becauſe the Object being very far off, the Rays which flow from Points of the Object that are contiguous, fall too near each other upon the *Retina* to excite diſtinct Senſations in the Mind, ſo that the *Idea* of the Whole is confuſed.

CHAP. VII.

Of the Appearance of Objects *ſeen through* Media *of different Forms.*

THAT what we ſhall ſay upon the Subject of this Chapter may more readily be underſtood, we ſhall premiſe the five following Particulars, which are all compriſed in the foregoing Chapter, or follow immediately from what has been there laid down, *viz.*

1. That, as each Point of an Object, when viewed by the naked Eye, appears in its pro-

* See the Diſſertation on the *Horizontal* Moon, annexed to this Part.

per

Fig. 32. P. 337



Fig. 33. P. 334.

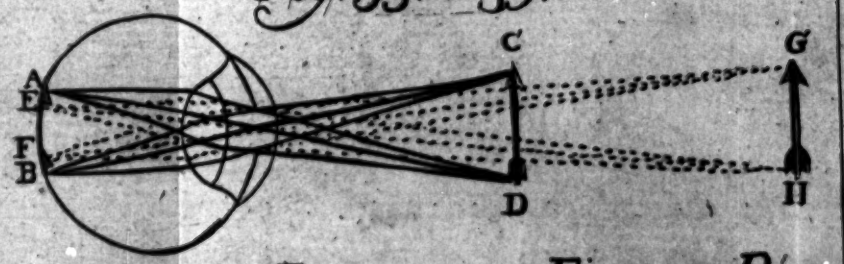


Fig. 34. P. 338

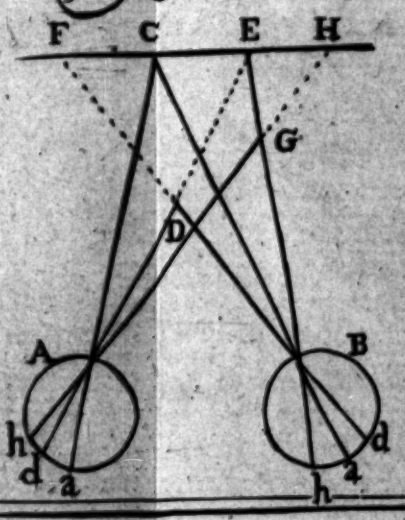
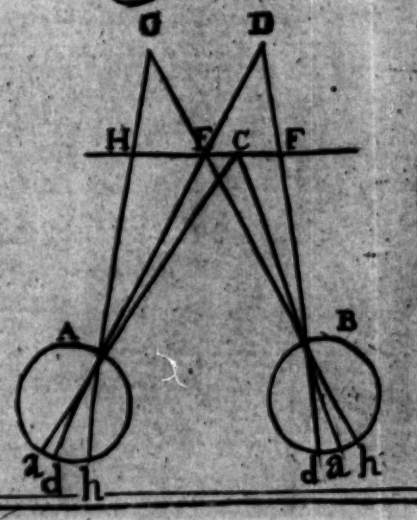


Fig. 35. P. 339



per Place, and as that Place is always to be found in the Line in which the *Axis* of a Pencil of Rays flowing from it enters the Eye, we from hence acquire an Habit of judging the Point to be ſituated in that Line; and, becauſe the Mind is unacquainted with what Refractions the Rays ſuffer before they enter the Eye, therefore, in Caſes where they are diverted from their natural Courſe by paſſing through any *Medium*, it judges the Point to be in that Line produced back in which the *Axis* of a Pencil of Rays flowing from it is ſituated the Inſtant they enter the Eye, and not in that it was in before Refraction. We ſhall therefore, in what follows, ſuppoſe the apparent Place of an Object, when ſeen through a refracting *Medium*, to be ſomewhere in that Line produced back in which the *Axis* of a Pencil of Rays flowing from it proceeds after they have paſſed through the *Medium*.

2, That we are able to judge, though imperfectly, of the Diſtance of an Object by the Degree of Divergency, wherein the Rays flowing from the ſame Point of the Object enter the Pupil of the Eye, in Caſes where that Divergency is conſiderable; but becauſe in what follows, it will be neceſſary to ſuppoſe an Object, when ſeen through a *Medium* whereby its apparent Diſtance is altered, to appear in ſome determinate Situation; in thoſe Caſes where the Divergency of the Rays at their Entrance

into the Eye is conſiderable, we will ſuppoſe the Object to appear where thoſe Lines which they deſcribe in entering, if produced back, would croſs each other, tho' it muſt not be aſſerted that this is the preciſe Diſtance; becauſe the Brightneſs, Diſtinctneſs, and apparent Magnitude of the Object, on which its apparent Diſtance in ſome Meaſure depends, will alſo ſuffer an Alteration by the Refraction of the Rays in paſſing through that *Medium*.

3. That we eſtimate the *Magnitude* of an Object by that of the *optic Angle*.

4. That Viſion is the *brighter*, the greater the Number of Rays is which enter the Pupil. And,

5. That, in ſome Caſes, the *apparent Brightneſs*, *Diſtinctneſs*, and *Magnitude* of an Object are the only Means whereby our Judgment is determined in eſtimating the Diſtance of it.

Prop. I. An Object placed within a *Medium* terminated by a plain Surface on that Side which is next the Eye, if the *Medium* be denſer than that in which the Eye is (as we ſhall always ſuppoſe it to be, unleſs where the contrary is expreſſed) appears *nearer* to the Surface of the *Medium* than it is.

Thus, if A be a Point of an Object placed within the *Medium* BCDE (*Fig.* 36.) and A*b*, A*c* be two Rays proceeding from thence, theſe Rays paſſing out of a denſer into a rarer *Medium*, will be refracted from their reſpective Per-

Perpendiculars *bd*, *ce*, and will enter the Eye at H, ſuppoſe in the Directions *bf*, *cg*, let then theſe Lines be produced back till they meet in F; this will be the apparent Place of the Point A; and becauſe the refracted Rays *bf*, *cg*, will diverge more than the incident ones A*b*, A*c* (Chap. III. Prop. 3.) it will be nearer to the Points *b* and *c*, than the Point A; and as the ſame is true of each Point in the Object, the Whole will appear to an Eye at H, *nearer* to the Surface BC than it is *.

Prop. II.

* From hence it is, that when one End of a ſtrait Stick is put under Water, and the Stick is held in an oblique Poſition, it appears bent at the Surface of the Water; *viz.* becauſe each Point that is under Water appears nearer the Surface, and conſequently higher than it is.

From hence likewiſe it is, that an Object at the Bottom of a Veſſel may be ſeen when the Veſſel is filled with Water, though it be ſo placed with Reſpect to the Eye, that it cannot be ſeen when the Veſſel is empty. To explain this; let ABCD (*Fig.* 37.) repreſent a Veſſel, and let E be an Object lying at the Bottom of it. This Object, when the Veſſel is empty, will not be ſeen by an Eye at F, becauſe HB, the upper Part of the Veſſel, will obſtruct the Ray EH; but when it is filled with Water to the Height GH, the Ray EK being refracted at the Surface of the Water into the Line KF, the Eye at F ſhall ſee the Object by Means of that.

In like Manner, an Object ſituated in the Horizon appears above its true Place, upon Account of the Refraction of the Rays which proceed from it in their Paſſage through the *Atmoſphere* of the Earth. For firſt, if the Object be ſituated beyond the Limits of the *Atmoſphere*, its Rays in entering it will be refracted towards the Perpendicular, that is, towards a Line drawn from the Point where they enter, to the Center of the Earth which is the Center of the *Atmoſphere*, and as they paſs on they will be continually refracted the ſame Way, becauſe they are all along entering a denſer Part, the Center of whoſe Convexity is ſtill the ſame Point; upon which Account the Line they deſcribe will be a Curve bending downwards; and therefore

Prop. II. An Object ſeen through a *Medium* terminated by plain and parallel Surfaces, appears *nearer*, *brighter*, and *larger*, than with the naked Eye.

For Inſtance, let AB (*Fig.* 38.) be the Object, CDEF the *Medium*, and GH the Pupil of an Eye, which is here drawn large to prevent Confuſion in the Figure. And 1ſt let RK, RL be two Rays proceeding from the Point R, and entering the denſer *Medium* at K and L; theſe Rays will here by Refraction be made to diverge leſs (Chap. III. Prop. 2.) and to pro-

fore none of the Rays that come from that Object can enter an Eye upon the Surface of the Earth, except what enter the *Atmoſphere* higher than they need to do, if they could come in a right Line from the Object; conſequently the Object muſt appear above its proper Place. Secondly, If the Object be placed within the *Atmoſphere*, the Caſe is ſtill the ſame; for the Rays which flow from it muſt continually enter a denſer *Medium* whoſe Center is below the Eye, and therefore being refracted towards the Center, that is, downwards as before, thoſe which enter the Eye muſt neceſſarily proceed as from ſome Point above the Object, wherefore the Object will appear above its proper Place.

From hence it is, that the Sun, Moon, and Stars appear above the *Horizon*, when they are juſt below it, and higher than they ought to do, when they are above it: Likewiſe diſtant Hills, Trees, *&c.* ſeem to be higher than they are.

Further, the lower theſe Objects are in the *Horizon*, the greater is the Obliquity with which the Rays which flow from them enter the *Atmoſphere*, or paſs from the rarer into the denſer Parts of it, and therefore they appear to be the more elevated by Refraction; upon which Account the lower Parts of them are *apparently* more elevated than the other. This makes their upper and under Parts ſeem nearer together than they ought to do, as is evident in the Sun and Moon, which appear of an oval Form when they are in the *Horizon*, their *horizontal* Diameters appearing of the ſame Length they would do if the Rays ſuffered no Refraction, while their *vertical* ones are ſhortened thereby.

ceed

ceed afterwards, ſuppoſe in the Lines K*a*, L*b*; at *a* and *b*, where they paſs out of the denſer *Medium*, they will be as much refracted the contrary Way, proceeding in the Lines *ac*, *bd*, parallel to their firſt Directions (ſee Chap. IV.) produce theſe Lines back till they meet in *c*, this will be the apparent Place of the Point R, and it is evident from the Figure that it muſt be nearer the Eye than that Point; and becauſe the ſame is true of all other Pencils flowing from the Object AB, the Whole will be ſeen in the Situation *f g*, *nearer* to the Eye than the Line AB. 2d, As the Rays RK, RL would not have entered the Eye, but have paſſed by it in the Directions K*r*, L*t*, had they not been refracted in paſſing through the *Medium*, the Object appears *brighter*. 3d, The Rays A*b*, B*i*, will be refracted at *b* and *i* into the leſs converging Lines *bk*, *il*, and at the other Surface into *k*M, *l*M, parallel to A*b* and B *i*, produced (ſee Chap. 4.) ſo that the Extremities of the Object will appear in the Lines M*k*, M*l* produced, *viz.* in *f* and *g*, and under as large an Angle *f* M *g*, as the Angle A *q*B under which an Eye at *q* would have ſeen it, had there been no *Medium* interpoſed to refract the Rays; and therefore it appears *larger* to the Eye at GH, being ſeen through the interpoſed *Medium*, than otherwiſe it would have done. But it is here to be obſerved, that the nearer the Point *e* appears to the Eye on

Ac-

Account of the Refraction of the Rays RK, RL, the ſhorter is the Image *f g*, becauſe it is terminated by the Lines M*f* and M*g*, upon which Account the Object is made to appear leſs; and therefore the apparent Magnitude of an Object is not much augmented by being ſeen through a *Medium* of this Form.

Farther, it is apparent from the Figure, that the Effect of a *Medium* of this Form depends wholly upon its Thickneſs; for the Diſtance between the Lines R*r* and *ec*, and conſequently the Diſtance between the Points *e* and R, depends upon the Length of the Line K*a*: Again, the Diſtance between the Lines AM and *f*M, depends on the Length of the Line *hk*; but both K*a* and *hk* depend on the Diſtance between the Surfaces CE and DF, and therefore the Effect of this *Medium* depends upon its Thickneſs.

Prop. III. An Object ſeen through a convex *Lens*, appears *larger*, *brighter*, and *more diſtant*, than with the naked Eye.

To illuſtrate this, let AB (*Fig.* 39.) be the Object, CD the *Lens*, and EF the Eye. 1. From A and B, the Extremities of the Object, draw the right Lines AY*r*, BX*r*, croſſing each other in the Pupil of the Eye; the Angle A*r*B comprehended between theſe Lines, is the Angle under which the Object would be ſeen with the naked Eye. But by the Interpoſition of a *Lens* of this Form, whoſe Property it is to render converging Rays more ſo (ſee Chap. IV.) the Rays AY and BX will be made

to

to croſs each other before they reach the Pupil. Therefore the Eye at E, will not perceive the Extremities of the Object by Means of theſe Rays, (for they will paſs it without entering) but by ſome others which muſt fall without the Points Y and X, or between them; but if they fall between them, they will be made to concur ſooner than they themſelves would have done, and therefore if the Extremities of the Object could not be ſeen by them, it will much leſs be ſeen by theſe. It remains therefore, that the Rays which will enter the Eye from the Points A and B after Refraction, muſt fall upon the *Lens* without the Points Y and X; let then the Rays AO and BP be ſuch. Theſe, after Refraction, entering the Eye at *r*, the Extremities of the Object will be ſeen in the Lines *r*Q, *r*T produced, and under the *optic Angle* Q*r*T, which is larger than A*r*B, and therefore the *apparent Magnitude of the Object will be increaſed.* 2. Let GHI be a Pencil of Rays flowing from the Point G; as it is the Property of this *Lens* to render diverging Rays leſs diverging, parallel or converging, (ſee Chap. IV.) it is evident, that ſome of thoſe Rays, which would proceed on to M and N, and miſs the Eye, were they to ſuffer no Refraction in paſſing through the *Lens*, will now enter it; by which Means the Object will appear *brighter.* 3. As to the apparent Diſtance of the Object, that will vary according to the Situation

of

of it with reſpect to the *Focus* of parallel Rays of the *Lens*. 1. Then let us ſuppoſe the Object placed ſo much nearer the *Lens* than its *Focus* of parallel Rays, that the refracted Rays KE and LF, though rendered leſs diverging by paſſing through it, may yet have a conſiderable Degree of Divergency, ſo that we may be able to form a Judgment of the Diſtance of the Object thereby. In this Caſe, the Object ought to appear where EK, FL produced back concur, which, becauſe they diverge leſs than the Rays GH, GI, will be beyond G, that is, at a greater Diſtance from the *Lens* than the Object is. But becauſe both the Brightneſs and Magnitude of the Object will at the ſame Time be augmented, Prejudice will not permit us to judge it quite ſo far off as the Point where thoſe Lines meet, but ſomewhere between that Point and its proper Place. 2. Let the Object be placed in the *Focus* of parallel Rays, then will the Rays KE and LF become parallel (ſee Chap. IV.) and though in this Caſe the Object would appear at an immenſe Diſtance, if that Diſtance were to be judged of by the Direction of the Rays KE and LF, yet upon Account of the Brightneſs and Magnitude of it, we ſhall not think it much *farther* from us, than if it were ſeen by the naked Eye. 3. If the Object be ſituated beyond the *Focus* of parallel Rays, as in AB (*Fig.* 40.) the Rays flowing from thence, and falling upon the *Lens* CD,

CD, will be collected into their reſpective *Foci* at *a* and *b*, and the intermediate Points *m*, *n*, &c. and will there form an Image of the Object AB; and after croſſing each other in the ſeveral Points of it, as expreſſed in the Figure, will paſs on diverging as from a real Object. Now if an Eye be ſituated at *c*, where *Ac*, *Bc*, Rays proceeding from the extreme Points of the Object, make not a much larger Angle *Ac*B, than they would do if there were no *Lens* interpoſed; and the Rays belonging to the ſame Pencil do not converge ſo much as thoſe the Eye would receive, if it were placed nearer to *a* or *b*, the Object upon theſe Accounts appearing very little larger or brighter than with the naked Eye, is ſeen nearly in its proper Place; but if the Eye recedes a little Way towards *ab*, the Object then appearing both brighter and larger, ſeems to approach the *Lens* †: which is an evident Proof of what has

† That the Object ſhould ſeem to approach the *Lens* in this Caſe, was a Difficulty that exceedingly puzzled the learned *Barrow*, and which he pronounces inſuperable, and not to be accounted for by any Theory we have of Viſion. *Molineux* alſo leaves it to the Solution of others, as that which will be inexplicable, till a more intimate Knowledge of the viſive Faculty, as he expreſſes it, be obtained by Mortals.

They imagined, that ſeeing an Object appears *farther off*, the *leſs* the Rays *diverge* which fall upon the Eye; if they ſhould proceed *parallel* to each other, it ought to appear *exceedingly remote*, and if they ſhould *converge*, it ſhould then appear *more diſtant ſtill*: The Reaſon of this was, becauſe they looked upon the apparent Place of an Object, as owing only to the Direction of the Rays whatever it was, and not at all to its apparent Magnitude or Splendor.

been

been ſo often aſſerted, *viz.* that we judge of the Diſtance of an Object in ſome Meaſure by its Brightneſs and Magnitude *; for the Rays converge the more the farther the Eye recedes from the *Lens*; and therefore if we judged of the Diſtance of the Object by the Direction of the Rays which flow from it, we ought in this Caſe to conceive it at a greater Diſtance, than when the Rays were parallel, or diverged at their Entrance into the Eye.

Prop. IV. If an Object be placed farther from a convex *Lens*, than its *Focus* of parallel Rays, and the Eye be ſituated farther from it on the other Side, than the Place where the Rays of the ſeveral Pencils are collected into their reſpective *Foci*, the Object appears *inverted*, and *pendulous in the Air*, between the Eye and the *Lens*.

To explain this, let AB (*Fig.* 40.) repreſent the Object, CD the *Lens*, and let the Rays of the Pencil ACD be collected in *a*, and thoſe of BCD in *b*, forming there an inverted Image of the Object AB, and let the Eye be placed in F: It is apparent from the Figure, that ſome of the refracted Rays which paſs through each Point of the Image, will enter

* Perhaps it may proceed from our judging of the Diſtance of an Object in ſome Meaſure by its Magnitude, that that Deception of Sight commonly obſerved by Travellers may ariſe; *viz.* that upon the firſt appearing of a Building larger than uſual, as a *Cathedral* Church, or the like, it generally ſeems nearer to them, than they afterwards find it to be.

the

the Eye as from a real Object in that Place, and therefore the Object AB will appear there, as the Propoſition aſſerts. But we are ſo little accuſtomed to ſee Objects in this Manner, that it is very difficult to perceive the Image with one Eye; but if both Eyes are ſituated in ſuch a Manner, that Rays flowing from each Point of the Image may enter both, as at G and H, and we direct our optic Axes to the Image, 'tis eaſy to be perceived.

If the Eye be ſituated in *a* or *b*, or very near them on either Side, the Object appears exceedingly confuſed, *viz.* if at *d*, the Rays which proceed from the ſame Point of the Object converge ſo very much, and if at *e*, they diverge ſo much, that they cannot be collected together upon the *Retina*, but fall upon it as if they were the Axes of ſo many diſtinct Pencils coming through every Point of the *Lens*; wherefore little more than one ſingle Point of the Object is ſeen at a Time, and that appears all over the *Lens*; from whence nothing but Confuſion ariſes.

If the *Lens* be ſo large that both Eyes may be applied to it, as in *h* and *k*, the Object will appear double; for 'tis evident from the *Figure*, that the Rays which enter the Eye at *h* from either Extremity of the Object A or B, do not proceed as from the ſame Point with that from whence thoſe which enter the other at *k* ſeem to flow; the Mind therefore is here deceived,

and looks upon the Object as ſituated in two different Places, and therefore judges it to be double.

Prop. V. An Object ſeen through a concave *Lens* appears *nearer*, *ſmaller*, and leſs *bright*, than with the naked Eye.

Thus, let AB (*Fig.* 41.) be the Object, CD the Pupil of an Eye, and EF the *Lens*. Now, as it is the Property of a *Lens* of this Form, to render diverging Rays more ſo, and converging ones leſs ſo, the diverging Rays GH, GI, proceeding from the Point G, will be made to diverge more, and ſo to enter the Eye as from ſome nearer Point *g*; and the Rays AH, BI, which converge, will be made to converge leſs, and to enter the Eye as from the Points *a* and *b*; wherefore the Object will appear in the Situation *agb*, *leſs* and *nearer* than without the *Lens*. Farther, as the Rays which proceed from G, are rendered more diverging, ſome of them will be made to paſs by the Pupil of the Eye, which otherwiſe would have entered it, and therefore each Point of the Object will appear *leſs bright* *.

Prop. VI.

* From what has been obſerved about the Properties of convex and concave *Lenſes*, we may ſee the Reaſon why the former Sort are made Uſe of by *old People* to help their Sight; and the latter by thoſe who are *purblind*. *Old People*, as was obſerved before, having the *Tunica Cornea* of their Eyes too flat, require that the Object be placed at a greater Diſtance from them, than other People whoſe Eyes are of a juſt Form, that the Rays which enter the Pupils of their Eyes from the ſame

Prop. VI. An Object ſeen through a *poligonous* Glaſs, that is, ſuch as is terminated by ſeveral plain Surfaces, is multiplied thereby.

For Inſtance, let A (*Fig.* 42.) be an Object, and BC a poligonous Glaſs terminated by the plain Surfaces BD, DE, &c. and let the Situation of the Eye F be ſuch, that the Rays AB being refracted in paſſing thro' the Glaſs, may enter it in the Direction BF, and the Rays AC in the Direction CF. Then will the Eye, by Means of the former, ſee the Object in G, and by the latter in H; and by Means of the Rays AI, the Object will appear alſo in its proper Situation A.

Thus much for the Principles of *Dioptrics*, and the Solution of ſome obvious *Phænomena* which tend to confirm the ſame: Thoſe which yet remain to be accounted for, ſhall, according to the Method we have hitherto obſerved, be treated of in the Diſſertation of this Part.

ſame Point of the Object, may not diverge too much. Now a convex *Lens* makes thoſe Rays diverge leſs, as they would naturally do if the Object was placed farther off. Again, thoſe who are *purblind*, having the *Tunica Cornea* too protuberant, require ſuch a *Lens* as may render thoſe Rays more diverging, leſt they ſhould be collected into their reſpective *Foci* before they fall upon the *Retina*; and therefore *Lenſes* of the concave Sort are of Uſe to them.

DISSERTATION I.

Of the Horizontal Moon.

THE *Phænomenon* of the *Horizontal Moon* is this: When the Moon is juſt above the Surface of the Earth, either immediately after ſhe is riſen, or juſt before ſhe ſets, ſhe appears four or five Times greater in Diameter, than when ſhe is in her Meridian Altitude: And yet her apparent Diameter, if taken by an Inſtrument, is found to ſubtend the ſame Angle in either Situation *.

The Moon's apparent Diameter being found to ſubtend the ſame Angle, whether ſhe be in the Horizon or Meridian, it is evident the Image of her, projected upon the *Retina* of an Eye, is of the ſame Dimenſions in either Caſe; and therefore that ſhe ſhould appear of a different Magnitude in one Situation from what

* What is ſaid here of the Moon's Diameter, as taken by an Inſtrument, muſt be underſtood of her *horizontal* Diameter, and not of her *vertical* one, for the Length of this is diminiſhed by Refraction (as explained Chap. VII. Note the firſt) and, therefore, if it be taken by an Inſtrument, it will not be found to ſubtend the ſame Angle in the Horizon as in the Meridian: But notwithſtanding this, it appears longer to the naked Eye when in the former, than in the latter Situation, as well as the horizontal Diameter.

ſhe does in the other, has always been Matter of great Speculation among the Learned both in *Optics* and *Aſtronomy*. *Des Cartes* was of Opinion, that we think the Moon larger when ſhe is in the Horizon, than when ſhe is in the Meridian, becauſe in the former Caſe, by comparing her Diſtance with that of interpoſed Objects, we imagine it greater than when ſhe is elevated: And that as we judge her Diſtance greater in that Situation, we of Courſe think her Diameter longer, becauſe it ſubtends the ſame Angle in either Caſe. But more of this by and by, when we come to the Explication Dr. *Wallis* has given of this Matter.

Gaſſendus was of Opinion, that becauſe the Moon appears leſs bright when in the Horizon than in the Meridian, we view her in the former Situation with a larger Pupil, than we do in the latter; and from thence he concludes, that the Image of her upon the *Retina* muſt be larger. But this is contrary to the Laws of *Optics*; for if the refractive Power of the Humours of the Eye collects the Rays of the ſeveral Pencils into their reſpective *Foci* upon the *Retina* (and there is no Reaſon to ſuppoſe the contrary in this Caſe) the Breadth of the Pupil makes no Alteration in the Magnitude of the Image; becauſe the Situation of thoſe *Foci* is determined by the *Axes* of the ſeveral Pencils, which, croſſing each other in the Cen-

ter of the Pupil, (as was ſhewn Chap. VI. pag. 331. in the Note) paſs on to the ſame Points of the *Retina*, whether the Pupil be broad or narrow.

Molineux, in the Philoſophical Tranſactions, N° 187. tells us of a certain *French Abbé*, that revived the forementioned Suppoſition of *Gaſſendus*, and adding two others of his own, endeavoured to account for this *Phænomenon*. His Suppoſitions were theſe, *viz.* "That this "contracting and enlarging the Pupil (ſup-"poſed by *Gaſſendus*) cauſeth a different "Shape in the Eye; an open Pupil making "the Chryſtalline flatter, and the Eye longer, "and the narrower Pupil ſhortening the Eye, "and making the Chryſtalline Humour more "convex. The firſt attends our looking at "Objects that are remote, or which we think "ſo; the latter accompanies the viewing Ob-"jects nigh at Hand. Likewiſe an open Pu-"pil and flat Chryſtalline attends Objects of "a more ſedate Light, whilſt Objects of more "forcible Rays require a greater Convexity, "and narrower Pupil. From theſe Poſitions, "continues *Molineux*, the *Abbé* endeavoured "to give an Account of our *Phænomenon*, as "follows; When the Moon is nigh the Ho-"rizon, by Compariſon with interpoſed Ob-"jects, we are apt to imagine her much far-"ther from us than when more elevated, and "therefore we order our Eyes as for viewing

"an

"an Object farther from us; that is, we ſome-
"thing enlarge the Pupil, and thereby make
"the Chryſtalline flatter; moreover the Duſk-
"iſhneſs of the Moon in that Poſture does
"not ſo much ſtrain the Sight; and conſe-
"quently the Pupil will be more large, and
"the Chryſtalline more flat; hence a larger
"Image ſhall be projected on the Fund of the
"Eye, and, therefore, the Moon ſhall appear
"larger. Theſe two fore-mentioned Accidents,
"*viz.* the Moon's imaginary Diſtance and
"Duſkiſhneſs, gradually vaniſhing as ſhe riſes,
"a different *Species* is hereby introduced in
"the Eye, and conſequently ſhe ſeems gradu-
"ally leſs and leſs, till again ſhe approaches
"nigh the Horizon."

As to what is taken for granted in this Solution concerning a Change in the Chryſtalline Humour and Form of the Eye, upon viewing an Object in a *duſky* or faint Light, that ſeems to be very ill grounded. We know of no ſuch Connection between the Muſcles of the *Iris* and thoſe of the *Ligamenta Ciliaria*, as is neceſſary to produce this Effect. And the Coats of the Eye are not ſo pliable, as eaſily to admit of an Alteration in their Form *. Could the Author have made good his other Suppoſition, *viz.* *That by Compariſon with*

* See what has been obſerved concerning the Power we have of making an Alteration in the Eye, in order to ſee *diſtinctly*, (Chap. VI. pag. 332.)

interpoſed Objects, we are apt to imagine her much farther from us, than when more elevated, he need not have had Recourſe to any other; this alone would have been ſufficient; but *hic Labor eſt.* This alone, I ſay, would have been ſufficient; for if by comparing her Diſtance with that of interpoſed Objects, we imagine it greater when ſhe is in the Horizon, than when ſhe is in the Meridian; as ſhe ſubtends an equal Angle in both Caſes, we muſt, in Conſequence thereof, (agreeably to *Des Cartes*'s Notion above-mentioned) imagine her to be bigger in the former Situation than in the latter; becauſe a diſtant Object cannot ſubtend the ſame Angle at the Eye that one which is nearer does, unleſs it be proportionally larger *.

The famous *Hobbes* endeavoured at a Solution of this *Phænomenon*, but it is hardly worth mentioning; The Figure he has drawn to explain his Solution by, ſeems to have been the Occaſion of his Error. He draws a Circle to repreſent that blue Surface commonly called the Sky, in which the heavenly Bodies ſeem to be fixed, and concentric to this, a leſſer, to repreſent the Surface of the Earth, but vaſtly too big in Proportion; ſo that a Spectator upon the Surface of this Earth, is conſiderably nearer to the upper Part of the

* See Chap. VI. pag. 341.

other

other Circle than to the Sides of it: Wherefore an Object that ſubtends the ſame Angle at different Heights muſt neceſſarily hide a greater Portion of that Ark when it is in the Horizon, than when it is in the Meridian; becauſe that Ark is farther behind the Object in the former than in the latter Situation; from whence he concludes, that the Moon muſt appear bigger in that Situation than in the Meridian. Had he drawn his Circles in any tolerable Proportion to that which he deſigned them to repreſent, he would eaſily have ſeen his Miſtake.

A few Years ago Mr. *De Veil* publiſhed a Treatiſe upon the Subject of the *Horizontal Moon*, which he dedicates to the Ladies of *Northampton*. If I remember right, his Solution of it was in the following Manner: 1. When an Object is placed beyond the *Focus* of parallel Rays of a convex *Lens*, the farther the Eye (ſituated on the other Side of the *Lens*) recedes from it towards the *Focus* of the Rays which flow from that Object, the larger that Object appears. 2. Rays of Light flowing from the Moon, and paſſing thro' the *Atmoſphere* of the Earth, are collected into a *Focus* on the other Side of it. 3. When the Moon is in the Horizon, we are nearer to this *Focus* by almoſt a *Semidiameter* of the Earth, than when ſhe is in the Meridian: And therefore, the Moon ought to appear larger when in the former than in the latter Situation.

The

The Propoſitions in this Solution are all true, but the ſecond is not applicable in the preſent Caſe; for unleſs we conſider the Refraction that Rays of Light which flow from the Moon, and paſs through the Atmoſphere of the Earth, ſuffer in their Emerſion, that is, while they paſs through the latter Half of it, as well as that which they ſuffer in their Immerſion, or while they paſs through the former Half, we ſhall find that they will not be collected into their reſpective *Foci* on the other Side the Earth, as this Gentleman imagines: Which, if it can be ſhewn, his Solution falls to the Ground of Courſe; for the Refraction which the Rays ſuffer in their Emerſion is not to be taken into Conſideration, becauſe they reach the Eye of a Spectator upon the Earth as ſoon as they have paſſed through the firſt Half of the Atmoſphere, when the Moon is in his Horizon; and before they have paſſed through that Half, when ſhe is in his Meridian.

Let us then imagine two Rays flowing from one and the ſame Point of the Surface of the Moon, it being neceſſary, in order to conſtitute a *Focus*, that ſuch Rays ſhould, after Refraction, meet in a Point; the meeting of ſuch as flow from different Points in the ſame Surface is not ſufficient; if it were, we might then have *Foci* where we pleaſed, and that as well without refracting or reflecting Surfaces as with them. And let the firſt of thoſe two Rays fall perpendicularly upon the Atmoſphere of the Earth, and

be

be ſuppoſed to paſs through the Center of it; and let the other, after Refraction, paſs by the Surface of the Earth. Now the Moon's *Parallax*, that is, the Angle under which the Semidiameter of the Earth is ſeen from the Moon, being about one Degree, it is evident that theſe Rays muſt, before their Incidence upon the Earth's Atmoſphere, diverge the one from the other by ſuch an Angle. But it appears from Sir *Iſaac Newton*'s Table of Refractions, publiſhed by Dr. *Halley*, in the Philoſophical Tranſactions, N° 368. that, when any of the heavenly Bodies appears in the Horizon, the Rays by which it is ſeen are refracted but by an Angle of thirty-three Minutes and forty-five Seconds; and therefore the Ray which we have ſuppoſed after Refraction to paſs by the Surface of the Earth, will be refracted only by ſuch an Angle; which falling conſiderably ſhort of one Degree, the Angle by which it diverged from the perpendicular one before Refraction, it will be ſo far from being made to converge towards it thereby, that it will ſtill be in a State of Divergency from it. And therefore Rays flowing from the Moon, and refracted only in their Immerſion into the Atmoſphere of the Earth, will not be collected into their reſpective *Foci* on the other Side: Which was to be ſhewn.

Dr. *Wallis*, in the Philoſophical Tranſactions, N° 187. gives us a Solution of the *Horizontal Moon* (or rather an Explication of what *Des Cartes* had given before) which is as follows:

He

He aſcribes this *Phænomenon* to the Deception of the Imagination, and accounts for that Deception in the following Manner: He obſerves, firſt, That the Imagination doth not eſtimate the Greatneſs of an Object ſeen, by the *optic Angle* only, but by this compared with the ſuppoſed Diſtance. So that if two Things are ſeen under the ſame or equal Angles, and if, upon any Account whatever, we apprehend one of theſe to be farther from us than the other, that which we apprehend to be farther from us, will to the Imagination appear greater. Secondly, That one great Advantage for eſtimating the apparent Diſtance of any Thing, is from the Variety of intermediate Objects between the Eye and the Thing ſeen; for then the Imagination muſt allow Room for all thoſe Things.

" Now, ſays he, when the Sun * or Moon " is near the Horizon, the Proſpect we have of " Hills and Valleys, Plains and Woods, &c. re- " preſent to our Imagination a great Diſtance, " capable of receiving all theſe: Or, if it hap- " pens that theſe interpoſed Objects are not " actually ſeen, yet, having been accuſtomed " to ſee them, the Memory ſuggeſts to us a " View as large as is the viſible Horizon.

" But when the Sun or Moon is in an high- " er Poſition, we ſee nothing between us and " them (unleſs, perhaps, ſome Clouds) and

* For the Sun appears larger in the *Horizon*, as well as the Moon.

" there-

" therefore nothing that can preſent to our " Imagination ſo great a Diſtance as the other " is. And therefore, though both be ſeen un- " der the ſame Angle, they do not appear (to " the Imagination) of the ſame Bigneſs, be- " cauſe not fancied at the ſame Diſtance: But " that near the Horizon is judged bigger (be- " cauſe ſuppoſed farther off) than the ſame, " when at a greater Altitude."

If I might be allowed to mention any Thing of my own, after theſe ingenious Gentlemen have given their Opinions upon this Matter, it ſhould be this, *viz.* That I have often thought, that he who would give a rational Account, why the Sun or Moon appears farther from us in the Horizon than in the Meridian (for that is all that is requiſite towards a Solution of the *Horizontal Moon*, as has been already obſerved) ſhould firſt ſhew why that apparent azure Surface we call the Sky, does not ſeem to be an entire concave *Hemiſphere*, but only a Portion of ſuch a one: For our judging the Heavens to be no more than ſuch a Portion, is undoubtedly the Cauſe why we judge both the Sun, Moon, and Stars to be farther from us when in the Horizon than in the Meridian; becauſe we have nothing elſe we can refer their Places to, but that.

Now, poſſibly, the Cauſe why we think the Heavens of that Form, may, after all, be only this, *viz.* That, as the Rays which come from the upper Parts of that imaginary Surface, the Sky,

Sky, paſs through a leſs Portion of the *Atmoſphere* than ſuch as come from the horizontal Parts of it, the Sky appears to us more *diſtinctly*, and generally more *bright* in thoſe Parts than in the latter; and therefore, ſince we daily obſerve that thoſe Objects which appear moſt *diſtinct* are generally ſuch as are *neareſt* to us, and alſo as *bright* Objects, when we have nothing but bare Imagination to determine us in eſtimating the Diſtance of them, appear *nearer* to us than the ſame Objects when leſs ſo *, we think the upper Parts of the Sky *nearer* us than the lower. Wherefore, ſince we refer all the heavenly Bodies to this Surface, we neceſſarily imagine them farther from us, and conſequently larger, and alſo more diſtant from each other †, when near the Horizon, than when they are arrived at their meridian Altitude.

* See what has been ſaid concerning the Brightneſs of an Object being a Means whereby it appears nearer us, under Prop. 3. of the 7th Chapter.

† The apparent horizontal Diſtance of two Stars from one another is obſerved to be greater when they are in the Horizon, than in the Meridian.

The END of VOL. I.